PHY-765 SS19: Gravitational Lensing. Worksheet Week 3

1 Finish the Scientific Poster for Presentation May 8th

Finish your poster presentation for week 5. If you didn't already send the URL of the ADS page of the paper you plan to prepare a poster on to kbschmidt@aip.de, please do so.

2 Determine Characteristic Sizes of the Schwarzschild Lens

In the following you will estimate the characteristic sizes for a point source lens, giving you an idea of the quantities involved.

2.1

Using the geometric relation between ξ and θ, express the lens equation in terms of β, θ, R_{S} and length scales for the point source deflector (considering α and not $\hat{\alpha}$).

2.2

Show that the characteristic angle α_0 (when $\beta = 0$) in the lens plane is given by the Einstein radius expressed in terms of R_{S} and the involved distances.

2.3

In a similar way, determine the characteristic length scale in the lens plane, ξ_0.

2.4

In terms of R_{S} and the involved distances, determine the characteristic length scale in the source plane, η_0.

2.5

Assuming that you have a lens of $M = 5 \times 10^{11} M_\odot$ at a redshift $z_{\mathrm{L}} = 0.54$ and a source at $z_{\mathrm{S}} = 1.49$, what is the characteristic deflection angle α_0. What if the mass of the lens is instead $M = 10^{14} M_\odot$. (Hint: use Ned Wright's calculator to get the distances involved).

2.6

For a lens much closer to the observer than the source, i.e., $D_{\mathrm{L}} \ll D_{\mathrm{LS}} \sim D_{\mathrm{S}}$, determine the constants K_α and K_ξ in the expressions

$$\alpha_0 = K_\alpha \left(\frac{D_{\mathrm{L}}}{1\mathrm{kpc}}\right)^{-1/2} \left(\frac{M}{M_\odot}\right)^{1/2} \mathrm{arcsec}$$

and

$$\xi_0 = K_\xi \left(\frac{D_{\mathrm{L}}}{1\mathrm{kpc}}\right)^{1/2} \left(\frac{M}{M_\odot}\right)^{1/2} \mathrm{cm}$$

2.7

Is the approximation $D_{\mathrm{L}} \ll D_{\mathrm{LS}} \sim D_{\mathrm{S}}$ used in 2.6 a fair description for the setup described in 2.5?

3 Some algebra

The following let you derive some of the relations quoted in this week's slides.

3.1

From the geometry of a gravitational lens, show that

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \frac{D_{\mathrm{LS}}}{D_{\mathrm{S}}} \hat{\boldsymbol{\alpha}}(D_{\mathrm{L}}\boldsymbol{\theta})$$

3.2

Using the realtions provided in this week's slides, express the scaled deflection angle in terms of $\boldsymbol{\theta}$:

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) = \frac{1}{\pi} \int d^2\theta' \kappa(\boldsymbol{\theta}') \frac{\boldsymbol{\theta} - \boldsymbol{\theta}'}{|\boldsymbol{\theta} - \boldsymbol{\theta}'|^2}$$

3.3

For any vector, show that

$$\nabla \ln |\boldsymbol{r}| = \boldsymbol{r}/|\boldsymbol{r}|^2$$