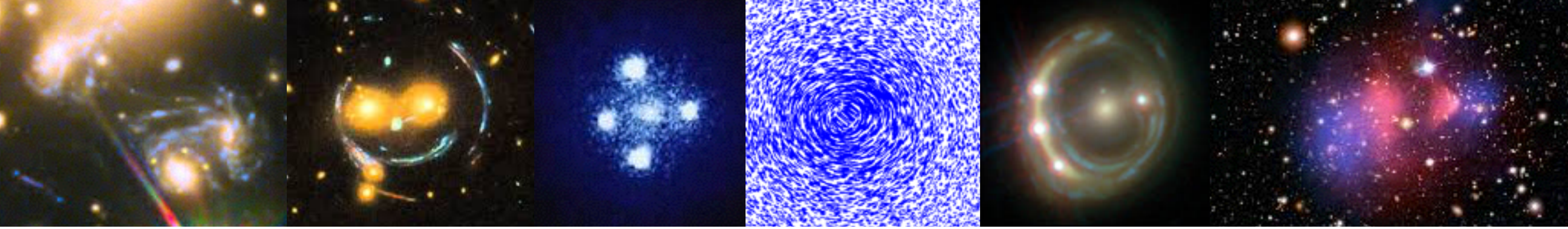

PHY-765 SS19 Gravitational Lensing Week 7

Magnifying Sources

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Last week - what did we learn?

- Expressed the time delay between multiple images of a lens:

$$\Delta t = \Delta t_{\mathrm{Geometry}} + \Delta t_{\mathrm{Shapiro}}$$

$$\Delta t = \frac{D_{\mathrm{L}} D_{\mathrm{S}}}{c D_{\mathrm{LS}}} \left[\frac{(\boldsymbol{\theta} - \boldsymbol{\beta})^2}{2} - \frac{\Phi(\boldsymbol{\theta})}{c^2} \right]$$

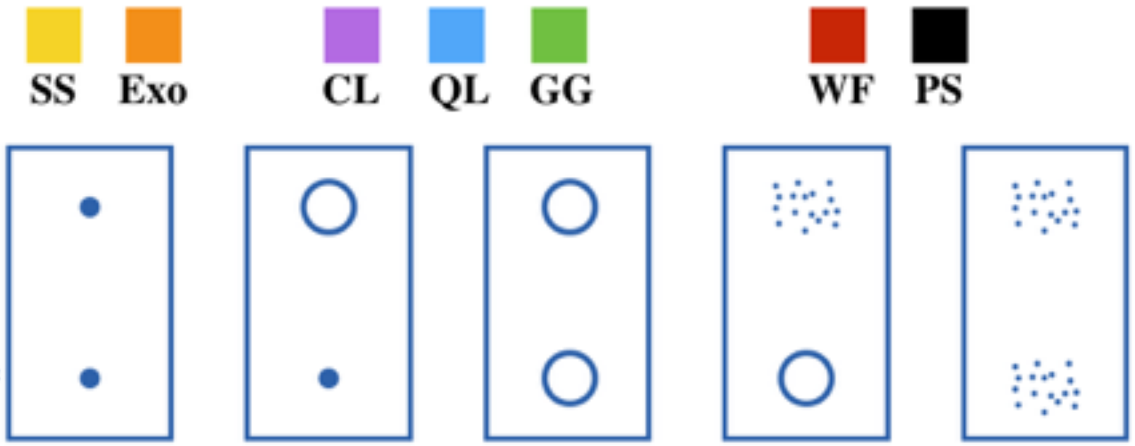
- For the point mass lens we saw that the difference between two images is

$$t_+ - t_- \simeq -(1 + z_{\mathrm{L}}) \frac{D_{\mathrm{L}} D_{\mathrm{S}}}{c D_{\mathrm{LS}}} 2c^2 \theta_E \beta$$

- This enabled us to predict the right order of appearance of SN Refsdal
  - Looked at the full lens model precisions and actual re-appearance
- Described how time delays are also useful for
  - Lens model improvements
  - Determining cosmological parameters (H_0)

The aim of today

- Explore the third consequence of the lens equation: **magnification**
- Describe magnification in terms of the Jacobian
- Define magnification, shear, convergence (again), and parity of images
- Consider how images are magnified in the point mass lens and SIS/CIS
- Applications of magnification
  - Mapping the mass distribution in lenses
  - Finding high-redshift galaxies



Surface Brightness Conservation

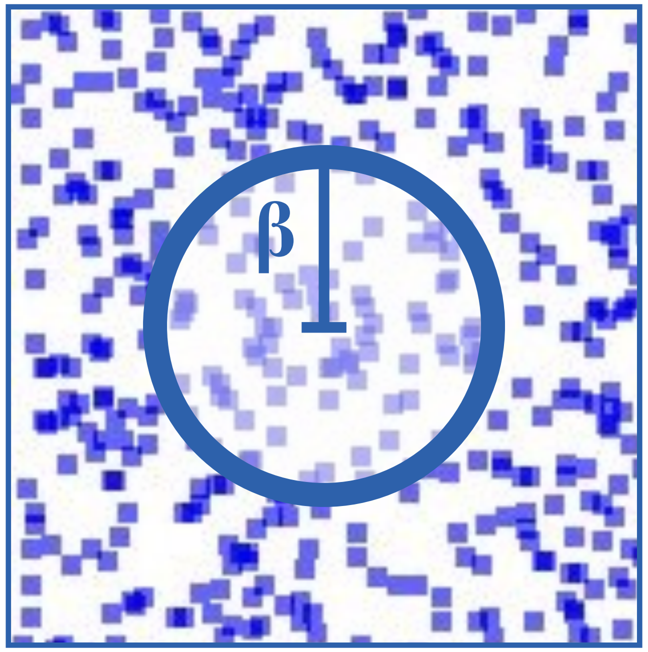
- Surface brightness: the flux density per solid angle

$$\mathcal{S} = \frac{F[\mathrm{Jy}]}{d\Omega[\mathrm{deg}^2]}$$

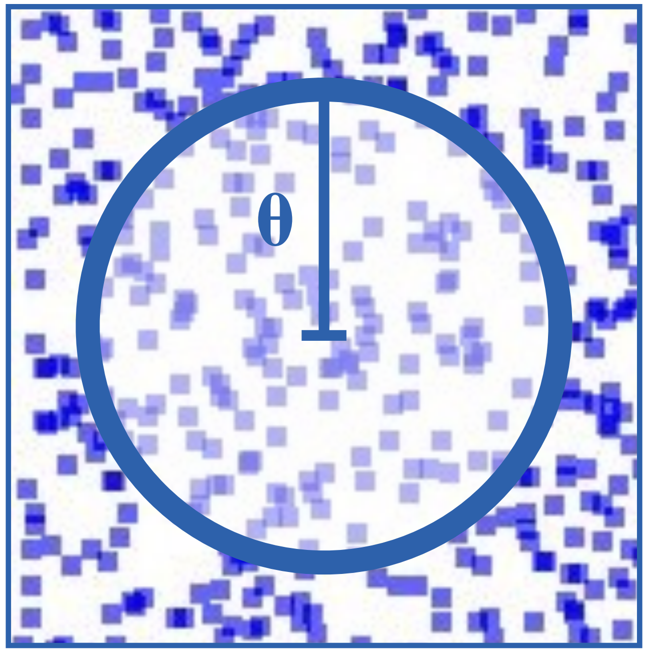
- Received flux goes down for distant sources with r^2
- Physical area observed in solid angle increases with r^2
- So surface brightness of any source is independent of r
- Gravitational lensing does not change surface brightness
  - but can change the *effective* area observed of the source
- Lenses focus light from the background source
- Hence, $\mathcal{S}(\boldsymbol{\beta}) = \mathcal{S}(\boldsymbol{\theta})$
- And the increase in observed brightness is given by

$$\frac{\mathcal{S}(\boldsymbol{\theta})d\Omega_{\text{lens plane}}}{\mathcal{S}(\boldsymbol{\beta})d\Omega_{\text{source plane}}} = \frac{\mathcal{S}(\boldsymbol{\theta})d\theta^2}{\mathcal{S}(\boldsymbol{\beta})d\beta^2} = \frac{F(\boldsymbol{\theta})}{F(\boldsymbol{\beta})} = \frac{d\theta^2}{d\beta^2} = \mu$$

Source Plane



Lens (Image) Plane



The Jacobian Matrix and Magnification

- The Jacobian Matrix is defined with indices $\boldsymbol{J}_{ij} \equiv \partial f_i / \partial x_j$
- It describes the mapping between coordinate systems via its determinant

$$|\det \boldsymbol{J}|\, du\, dv = dx\, dy$$

- $\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta})$ describes the mapping between source and lens plane
- So the Jacobian matrix for gravitational lensing (linearized locally) is

$$\mathcal{A}(\boldsymbol{\theta}) = \frac{\partial \boldsymbol{\beta}}{\partial \boldsymbol{\theta}} = \begin{pmatrix} \frac{\partial \beta_i}{\partial \theta_i} & \frac{\partial \beta_i}{\partial \theta_j} \\ \frac{\partial \beta_j}{\partial \theta_i} & \frac{\partial \beta_j}{\partial \theta_j} \end{pmatrix}$$

- Hence, the magnification μ (the ratio of the solid angles dθ & dβ - or fluxes) can be seen as a coordinate transformation, and expressed in terms of the Jacobian matrix:

$$\mu \equiv \det M(\boldsymbol{\theta}) = \frac{1}{\det \mathcal{A}(\boldsymbol{\theta})}$$

  - where the *magnification tensor is* defined as $M(\boldsymbol{\theta}) = \frac{1}{\mathcal{A}(\boldsymbol{\theta})}$

The Jacobian Matrix and Magnification

- In week 3 we saw that $\boldsymbol{\alpha} = \nabla\psi$
- Inserting this and the lens equation into the Jacobian matrix we get

$$\mathcal{A}(\boldsymbol{\theta}) = \begin{pmatrix} 1 - \frac{\partial \alpha_i}{\partial \theta_i} & -\frac{\partial \alpha_i}{\partial \theta_j} \\ -\frac{\partial \alpha_j}{\partial \theta_i} & 1 - \frac{\partial \alpha_j}{\partial \theta_j} \end{pmatrix} = \begin{pmatrix} 1 - \frac{\partial^2 \psi}{\partial \theta_i^2} & -\frac{\partial^2 \psi}{\partial \theta_i \partial \theta_j} \\ -\frac{\partial^2 \psi}{\partial \theta_j \partial \theta_i} & 1 - \frac{\partial^2 \psi}{\partial \theta_j^2} \end{pmatrix} \equiv (\delta_{ij} - \boldsymbol{\Psi}_{ij})$$

- Where $\boldsymbol{\Psi}_{ij}$ is the *distortion tensor* defined as

$$\boldsymbol{\Psi}_{ij} \equiv \begin{pmatrix} \kappa + \gamma_1 & \gamma_2 \\ \gamma_2 & \kappa - \gamma_1 \end{pmatrix}$$

  - using the *convergence* (κ) and *shear* (γ)
  - and that $\boldsymbol{\psi}$ is symmetric, i.e., $M_{ij} = M_{ji}$

Convergence & Shear

- In week 3:
  - we defined the convergence as $\kappa(\boldsymbol{\theta}) \equiv \frac{\Sigma(D_\mathrm{L}\boldsymbol{\theta})}{\Sigma_\mathrm{cr}}$
  - and noted that the convergence satisfies the Poisson equation $\nabla^2\psi = 2\kappa$
- So from the definition of $\boldsymbol{\Psi}_{ij}$ we can define:

$$\kappa \equiv \frac{1}{2}\left(\frac{\partial^2\psi}{\partial\theta_i^2} + \frac{\partial^2\psi}{\partial\theta_j^2}\right) \qquad \gamma_1 \equiv \frac{1}{2}\left(\frac{\partial^2\psi}{\partial\theta_i^2} - \frac{\partial^2\psi}{\partial\theta_j^2}\right) \qquad \gamma_2 \equiv \frac{\partial^2\psi}{\partial\theta_i\partial\theta_j}$$

- Giving that the magnification of a lensed source is given by

$$\mu = \frac{1}{(1-\kappa)^2 - \gamma^2} \quad ; \quad \gamma^2 \equiv \gamma_1^2 + \gamma_2^2$$ (Exercise 2)

- From this the *tangential* and *radial* critical curves are defined

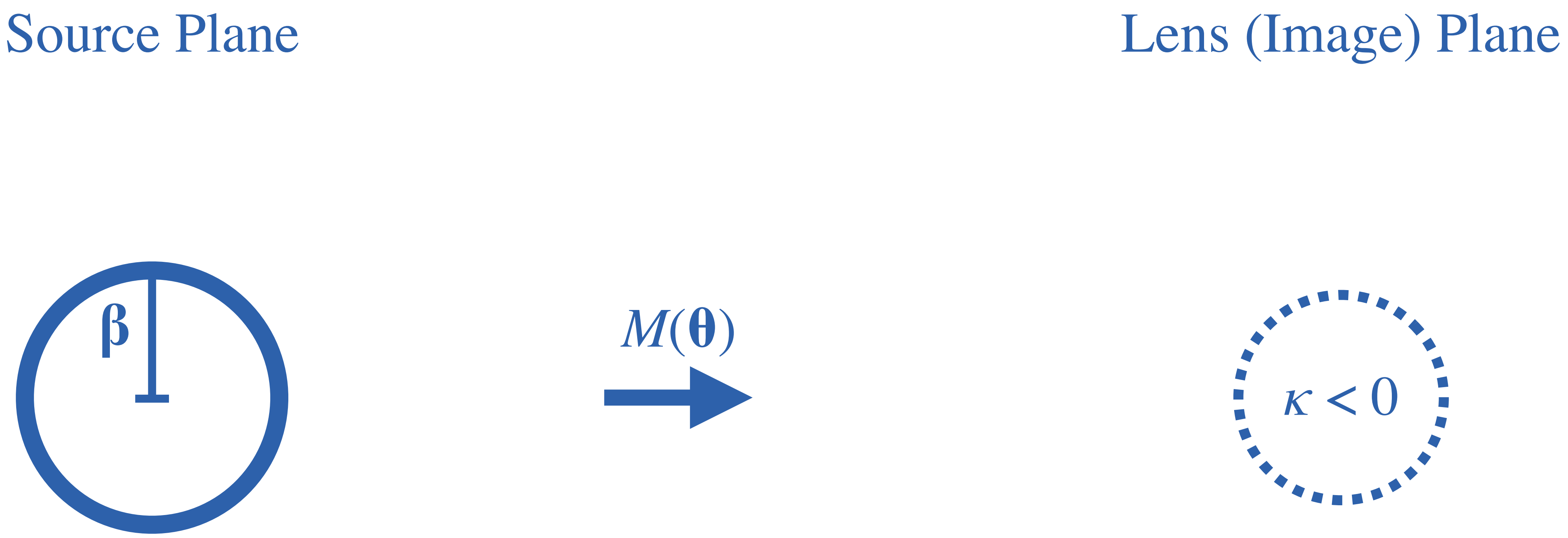
$$\mu_\mathrm{t} = \frac{1}{\lambda_\mathrm{t}} = \frac{1}{1-\kappa-\gamma} \qquad \mu_\mathrm{r} = \frac{1}{\lambda_\mathrm{r}} = \frac{1}{1-\kappa+\gamma}$$

Convergence & Shear

- Consider a line of sight along which $\gamma_1 = \gamma_2 = 0$ and $\kappa \neq 0$ and small, then

$$\mathcal{A}(\boldsymbol{\theta}) = \delta_{ij}(1-\kappa) \quad \Rightarrow \quad \boldsymbol{\beta} = (1-\kappa)\boldsymbol{\theta}$$

- Dividing by (1-κ) and Taylor expanding (κ is small) give $\boldsymbol{\theta} = (1+\kappa)\boldsymbol{\beta}$



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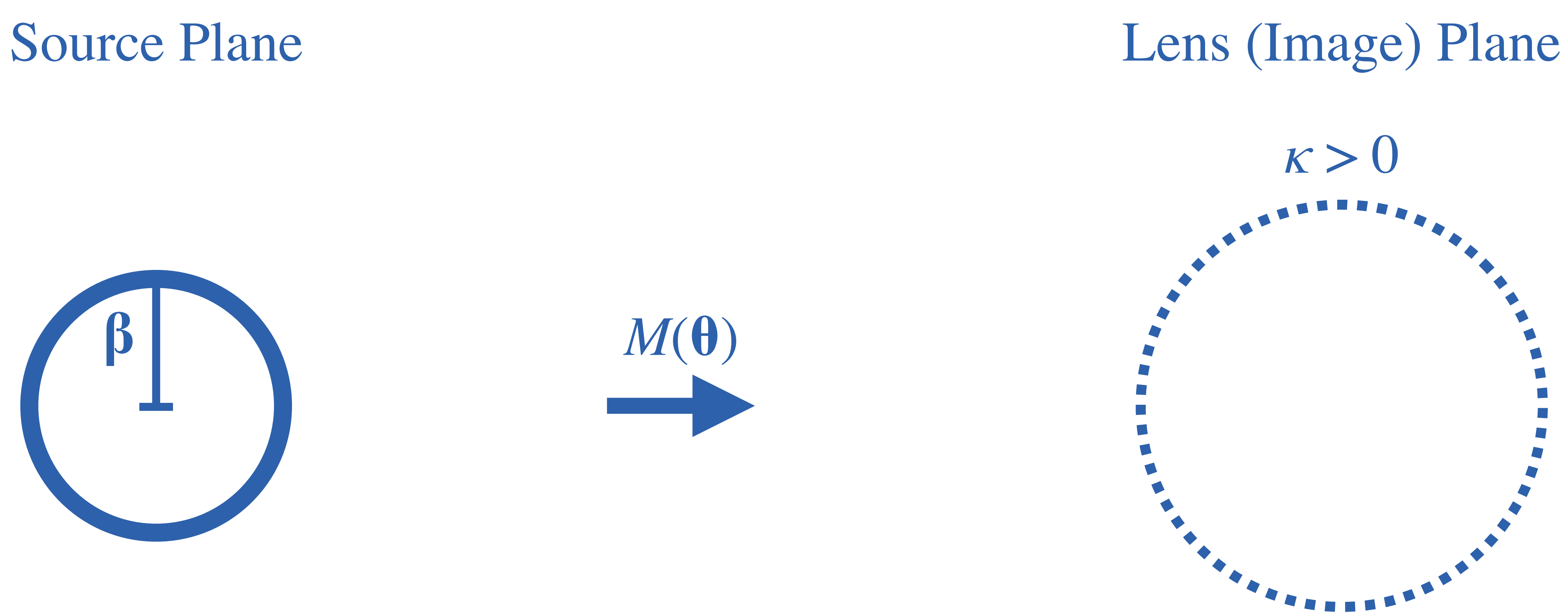

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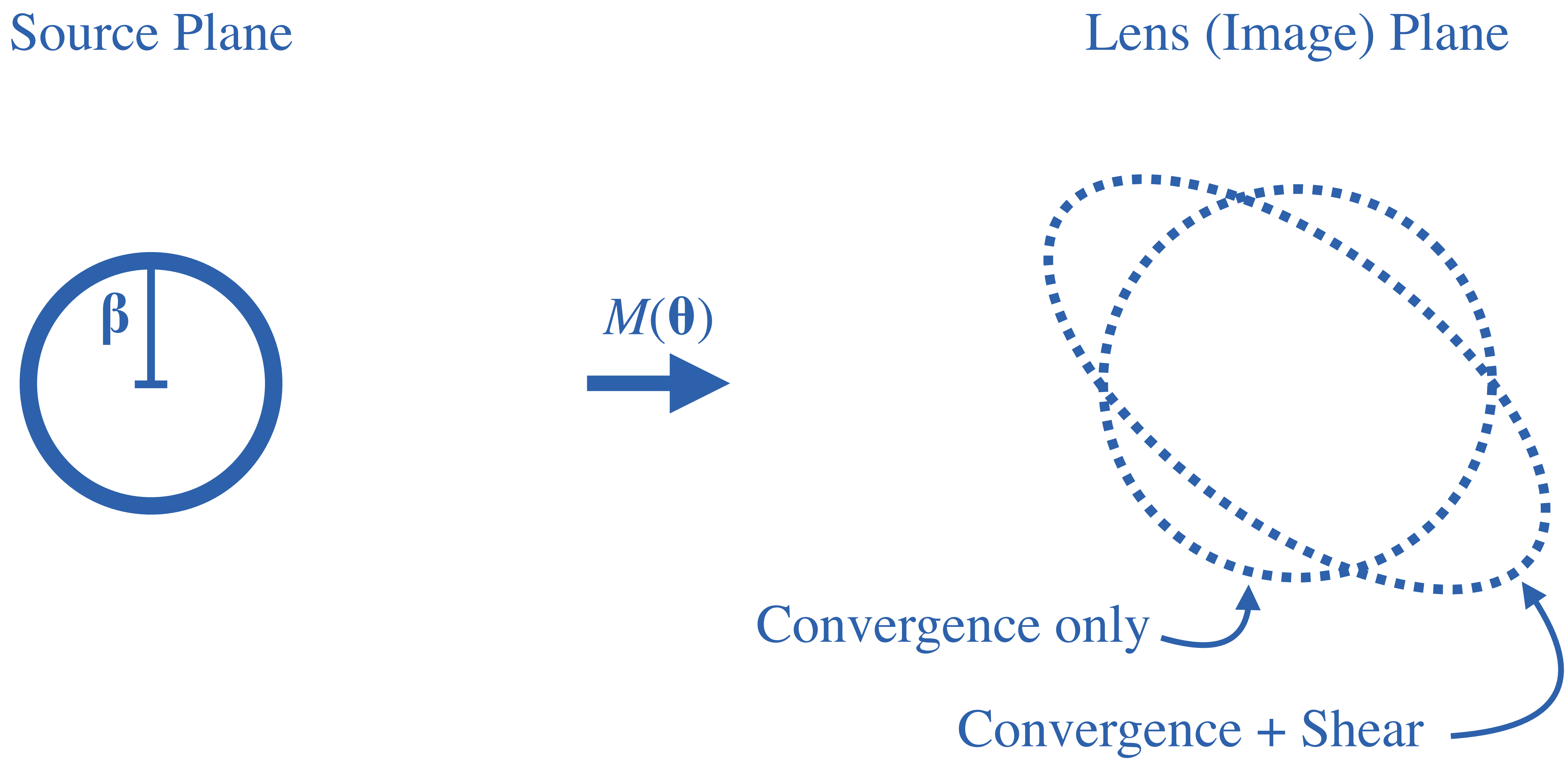


Magnification for the point mass lens

- From the point mass lens we have that (week 3): $\boldsymbol{\psi}(\boldsymbol{\theta}) = \frac{\theta_{\rm E}^2}{c^2} \ln \theta$
- Away from the origin of the point mass κ disappears so μ only depends on

$$\begin{aligned} \gamma_1 &= -\frac{\theta_{\rm E}^2}{\theta^4}\left(\theta_x^2 - \theta_y^2\right) \\ \gamma_2 &= -\frac{2\theta_{\rm E}^2 \theta_x \theta_y}{\theta^4} \end{aligned}$$

(Exercise 3.1)

- Resulting in

$$\mu = \frac{1}{1 - \frac{\theta_{\rm E}^4}{\theta^4}}$$

(Exercise 3.2)

- For perfect alignment $\beta=0$ the Einstein ring formally has $\mu \rightarrow \infty$
  - Practically neither source nor lens is ever a point mass
  - Total magnification is not infinite but large

Magnification for the Isothermal Sphere

- For the IS with a core we have that (week 4):

$$\boldsymbol{\alpha} = \frac{\theta_0}{\theta^2}\left[\sqrt{\theta^2+\theta_{\rm core}^2}-\theta_{\rm core}\right]\boldsymbol{\theta}$$

- But M($\boldsymbol{\theta}$), i.e. κ, γ_1 and γ_2 are just (linear comb.) of first derivatives of $\boldsymbol{\alpha}$ so we can derive the magnification calculating these

$$\mu = \left[\left(1-\frac{\theta_0}{2\sqrt{\theta^2+\theta_{\rm core}^2}}\right)^2 - \frac{\theta_0^2\left(2\theta_{\rm core}\sqrt{\theta^2+\theta_{\rm core}^2}-2\theta_{\rm core}^2-\theta^2\right)^2}{4\theta^4\left(\theta^2+\theta_{\rm core}^2\right)}\right]^{-1}$$

- Using that

$$\kappa = \frac{\theta_0}{2\sqrt{\theta^2+\theta_{\rm core}^2}}$$

$$\gamma^2 = \frac{\theta_0^2\left(2\theta_{\rm core}\sqrt{\theta^2+\theta_{\rm core}^2}-2\theta_{\rm core}^2-\theta^2\right)^2}{4\theta^4\left(\theta^2+\theta_{\rm core}^2\right)}$$

Magnification for the SIS

- If $\theta_{core} = 0$ in the expression of μ for the (C)IS we get that

$$\kappa \rightarrow \frac{\theta_0}{2|\theta|}$$

$$\gamma^2 \rightarrow \frac{\theta_0^2}{4\theta^2}$$

- Resulting in

$$\mu = \frac{1}{1 - \frac{\theta_0}{|\theta|}}$$

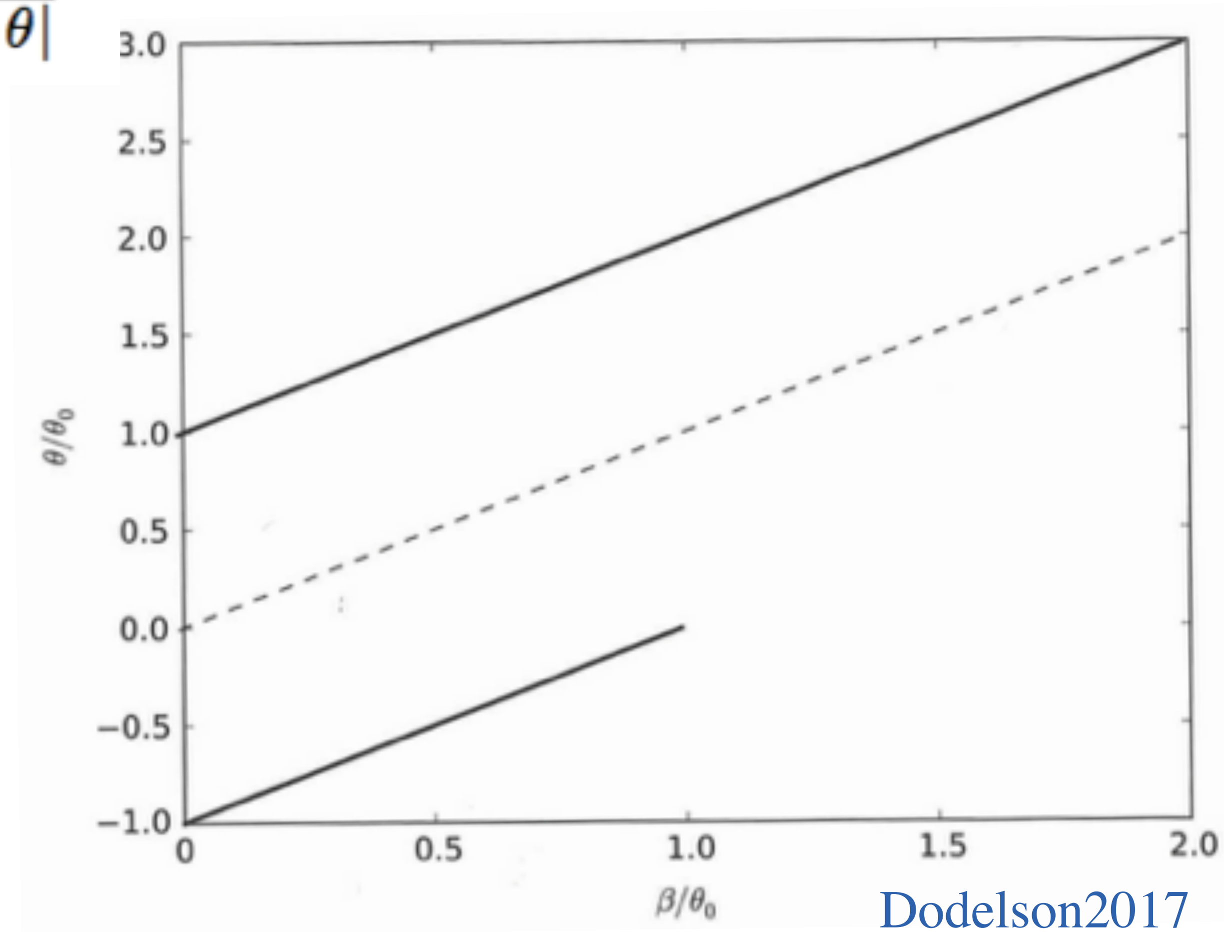
- So if $|\theta| < \theta_0$ then $\mu < 0$
- For $\beta < \theta_0$ two images appear (week 4)

$$\theta_+ = \beta + \theta_0 \qquad \theta_- = \beta - \theta_0$$

- With

$$\mu(\theta_+) = +\frac{\theta_0 + \beta}{\beta}$$

$$\mu(\theta_-) = -\frac{\theta_0 - \beta}{\beta}$$

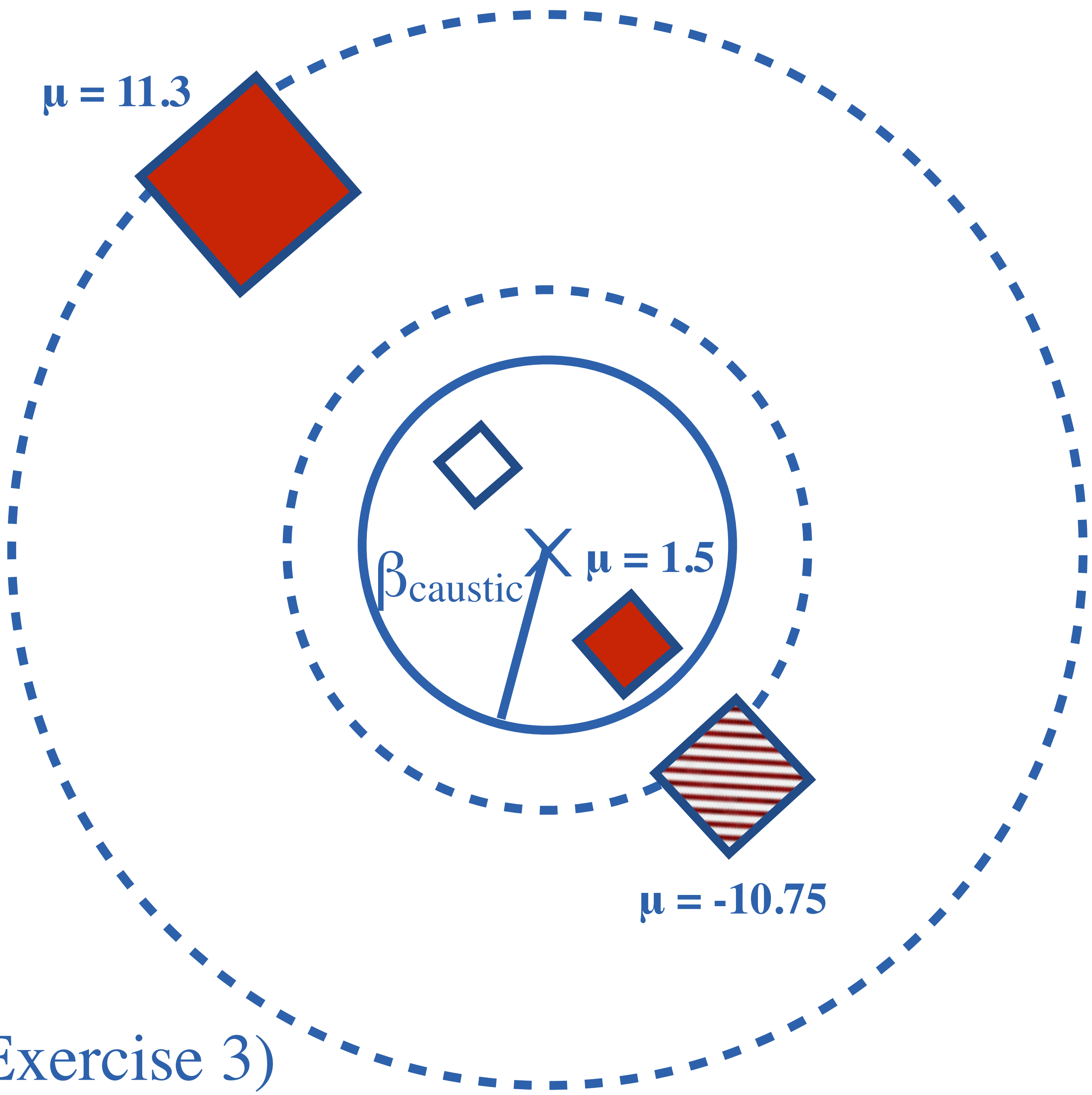


Dodelson2017

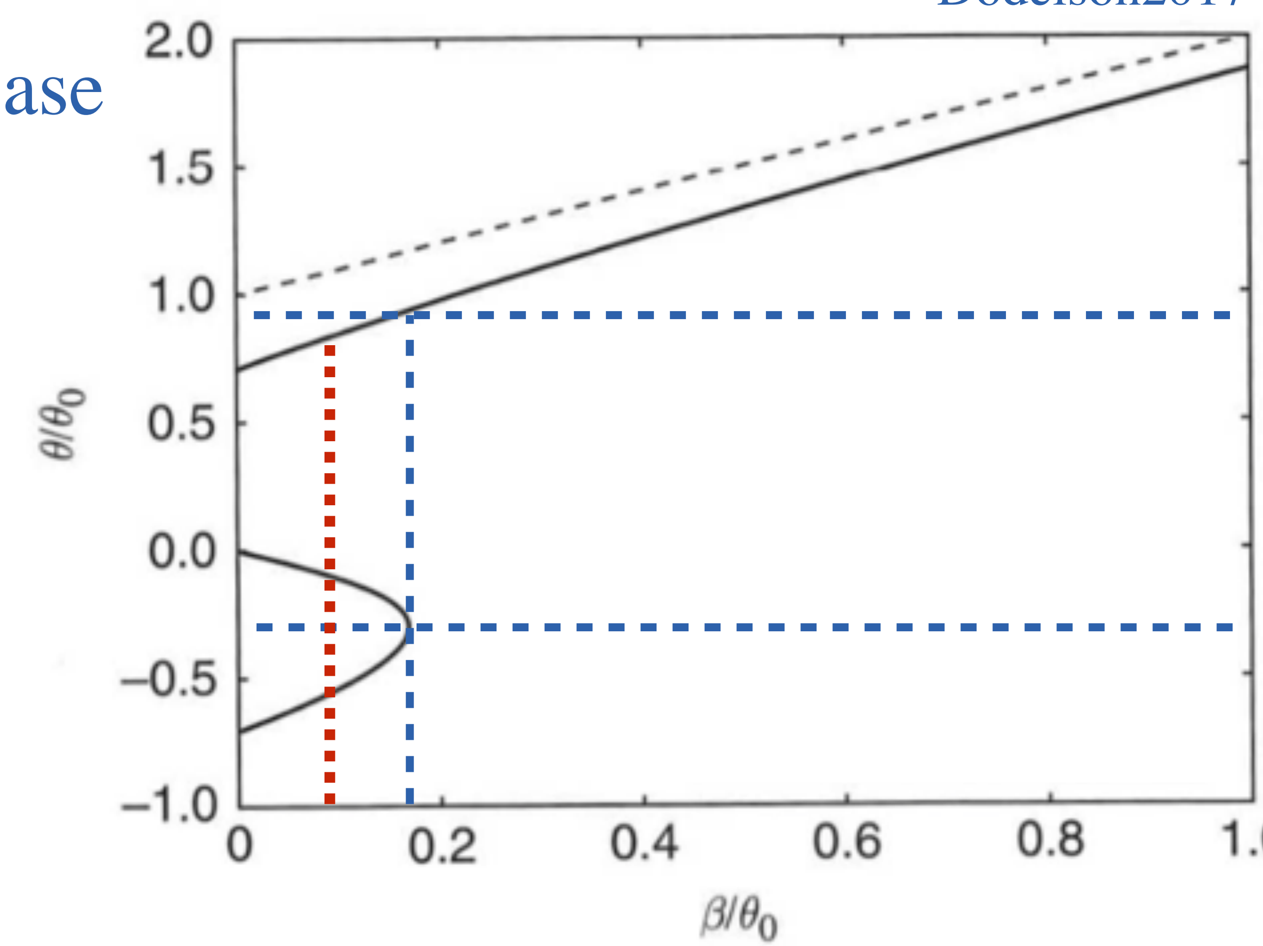
Caustics and Critical Curves Pt. 2

Dodelson2017

- For the CIS we were considering the case
- The number of images changes by 2 when crossing the caustic



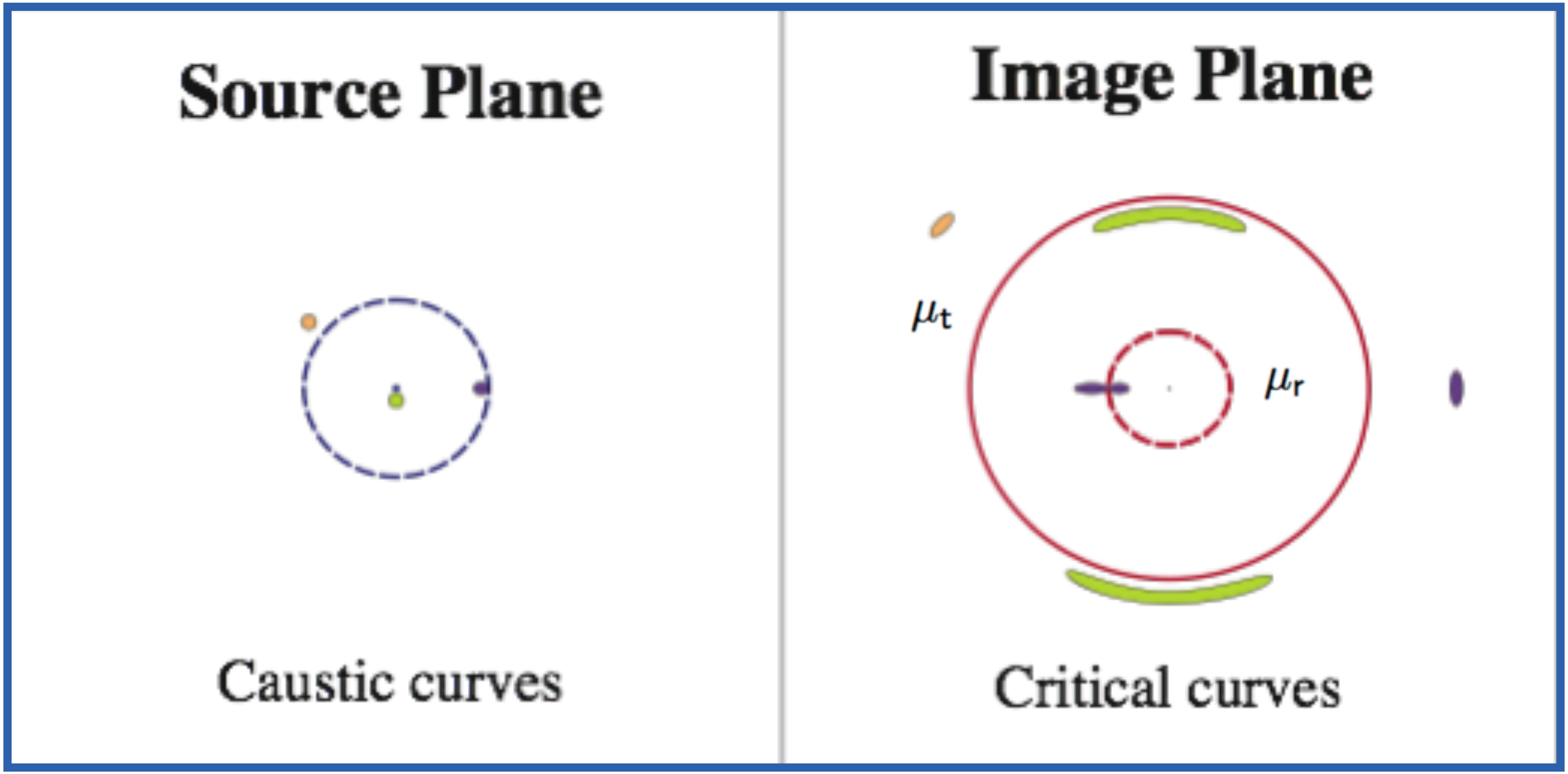
(Exercise 3)



- Calculating magnifications of images and scaling accordingly
- Absolute magnitude not known
- So the *observables* are image positions and flux ratios for multiple images - not μ

Caustics and Critical Curves Pt. 2

- The CIS with shear and convergence



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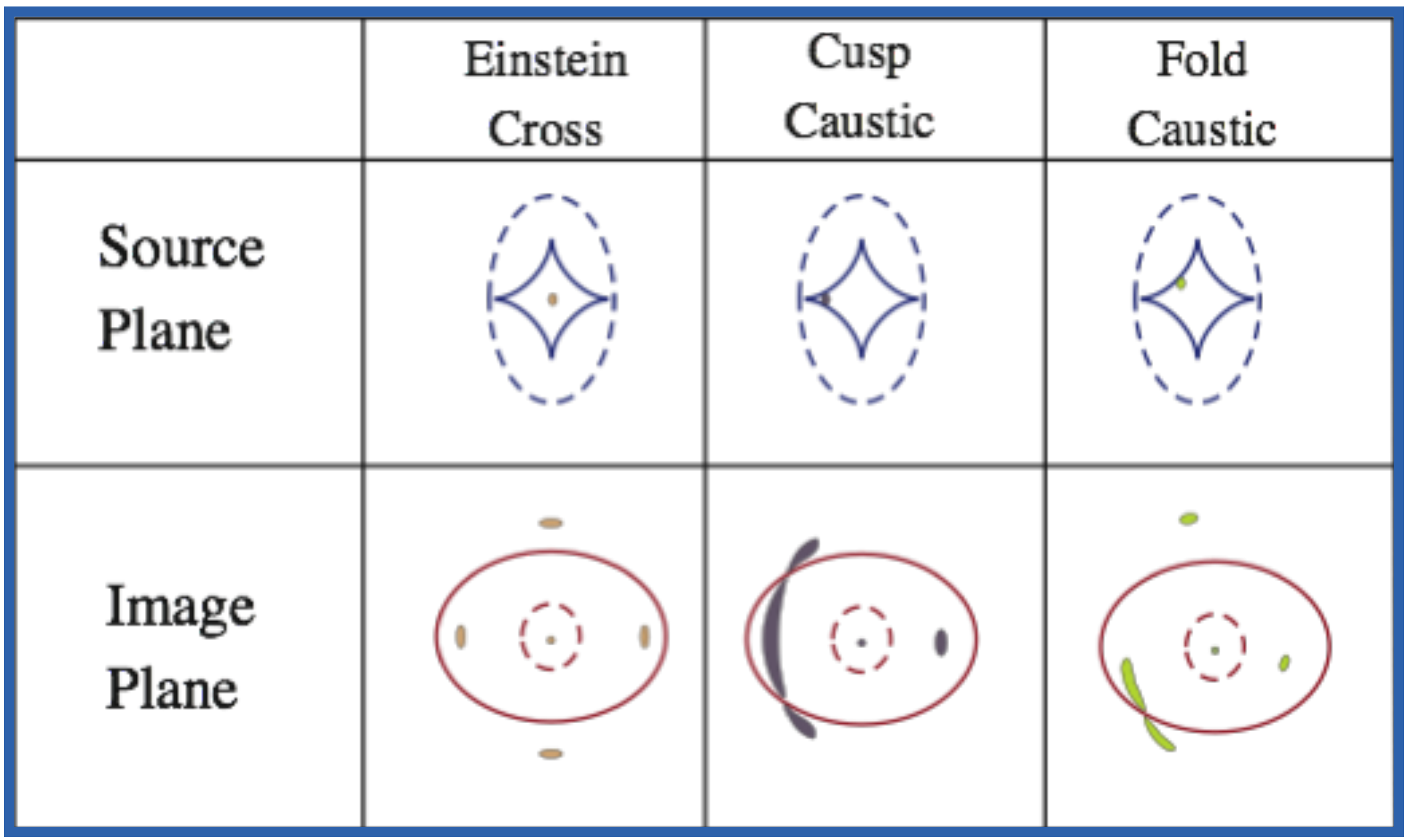
Caustics and Critical Curves Pt. 2

- So what about all the quads (and doubles) we are seeing?
- Consequence of asymmetric mass distribution
  - The Singular Isothermal Ellipsoid (SIE) attempts to account for this
  - Roughly the SIS added an asymmetric potential (or external shear)



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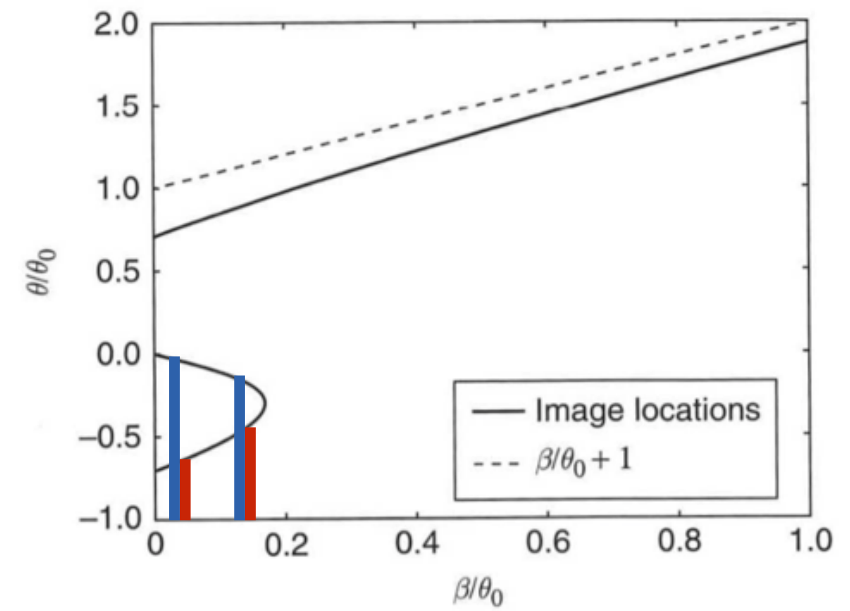
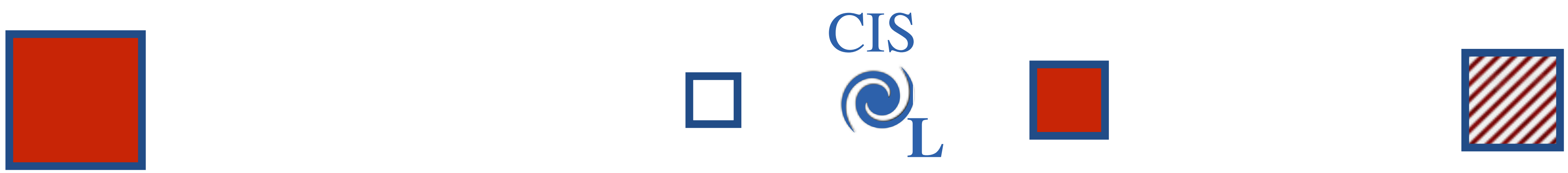
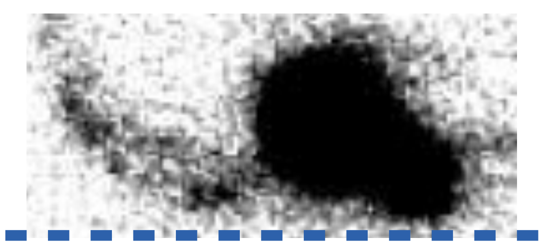
Parity

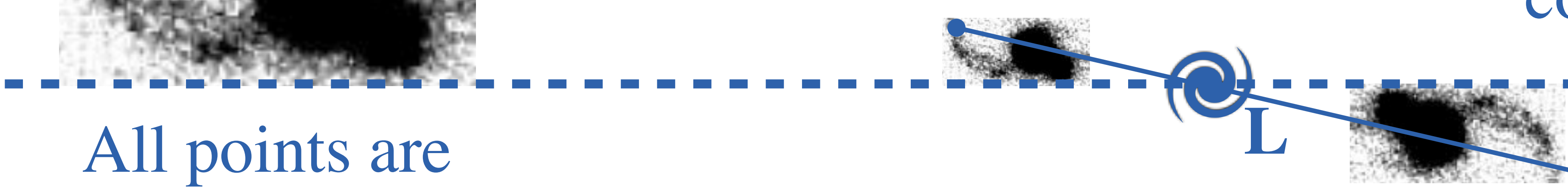


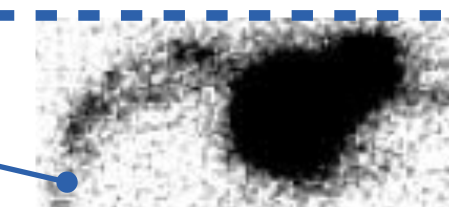
- Critical curves are the dividing lines between
  - Images with μ = detM < 0 have *negative parity*
  - Images with μ = detM > 0 have *positive parity*



Symmetrical lens so bottom of tail touches connecting (dashed) line







All points are deflected with α > 0

Symmetrical lens so also tail on connecting (solid) line
→ image **horizontally flipped** (around dashed axis)

The **vertical axis flip**:
An image closer to the lens (in the source plane) appear farther away in the lens plane
This flip is captured by μ<0

Direct Determination of Magnification

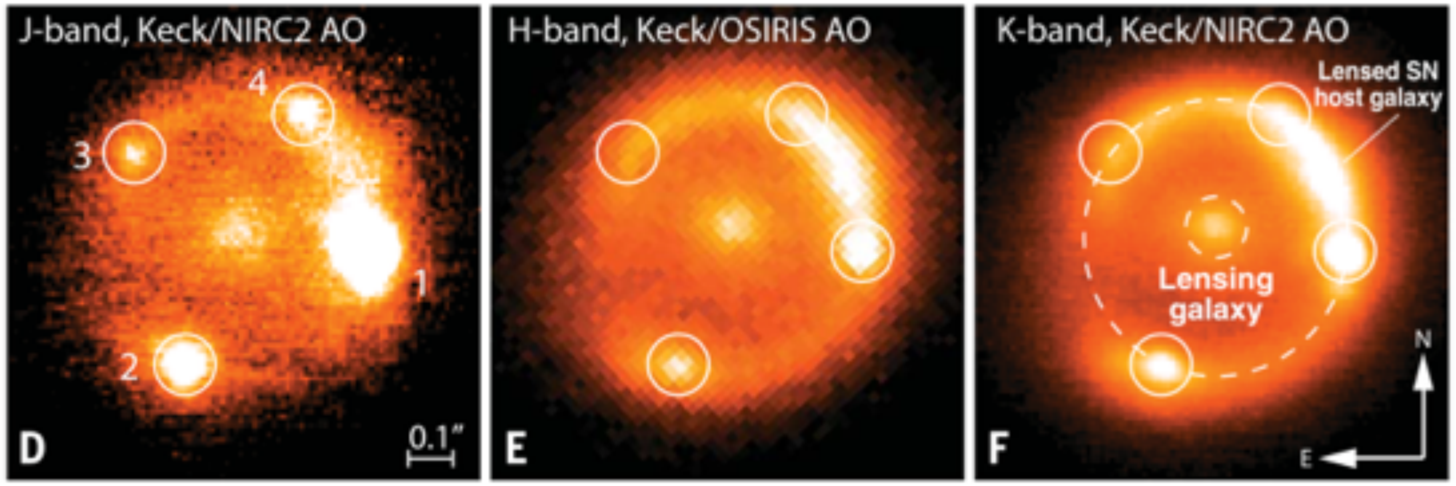
- The observable for magnification is the flux ratio
- This translates into a difference in magnitudes

$$m_1 - m_2 = -2.5 \log_{10}\left(\frac{F_1}{F_2}\right) = -2.5 \log_{10}(\mu)$$

- To directly detect magnification you want to compare $m_{intrinsic}$ & $m_{observed}$

- Goobar et al. (2017) presented a lensed SNIa (as mentioned in Week 5)
- Using that SNIa is standardizable they get $m_{intrinsic,SNIa}$
  - Leading to $\mu \sim 52$ ($\Delta m = -4.3 \pm 0.2$ mag)
  - Independent of lens model and cosmology ($m_{intrinsic,SNIa}$ at $z_{SNIa} = 0.409$)

Goobar et al. (2017)
$z_{SNIa} = 0.409$
$z_{lens} = 0.216$



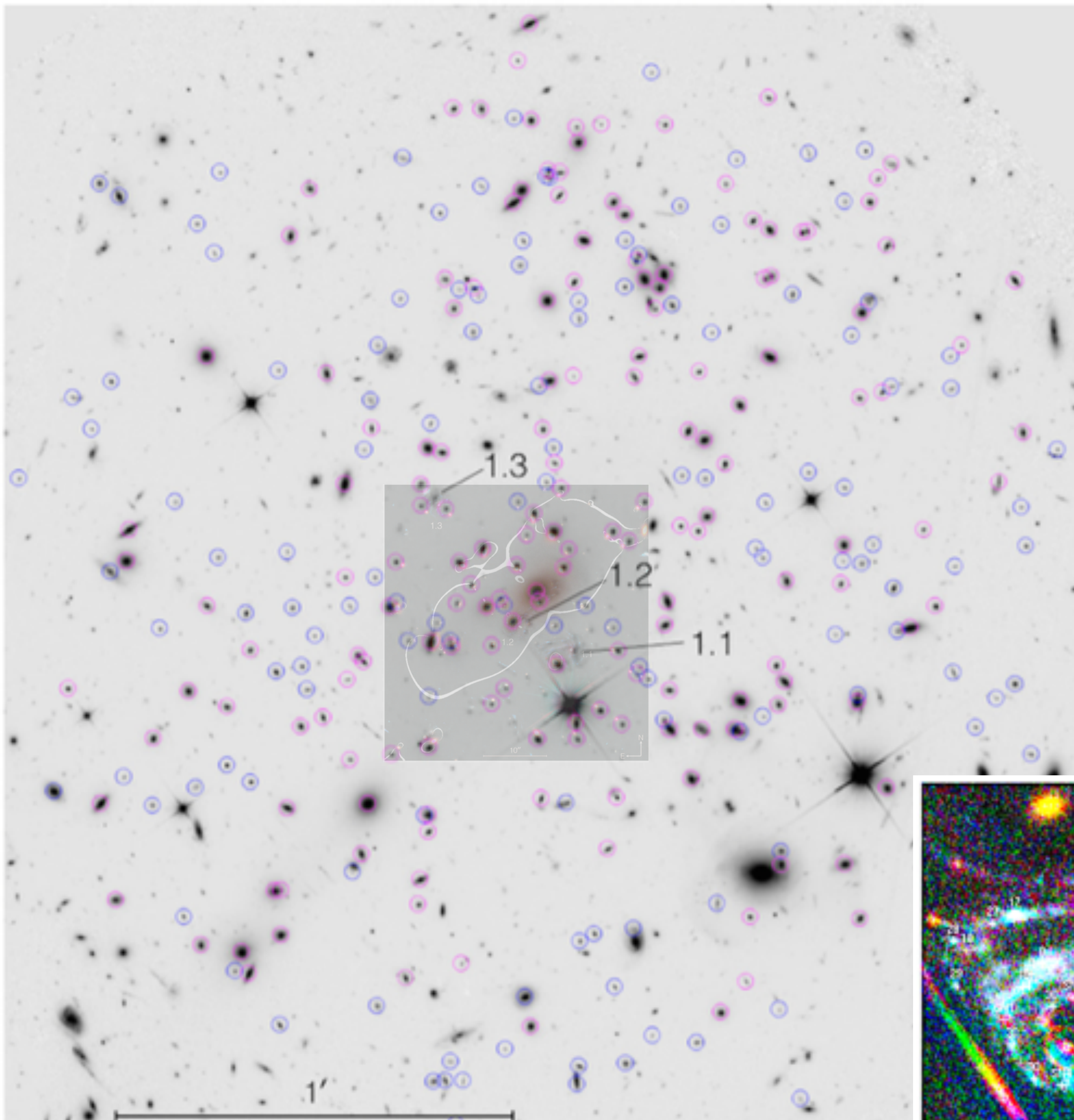
Determining the Mass of Lenses

- Determining the mass of a lens: use μ as predictor of the lens potentail
  1. Lens standard candle → Not many lensed standard candles
  2. Remove the lens → Lenses stationary (except for sun & microlenses)
  3. Compare samples of galaxies
  4. Use sets of multiple images in large (massive) systems

- To use sample statistics (3) you need
  - Magnitudes of galaxies being lensed
  - Magnitude of a similar sample of galaxies not being lensed
- This provides Δm between the samples estimating $\sim\mu$ and hence M_{lens}
- However, numbers of strongly lensed ($\theta \lesssim \theta_E$) galaxies per lens is small
- Hence, you have to observe galaxies outside θ_E, i.e., weak lensing regime
- Increasing area to O(arcmin^2) pushes S/N above a few
  - Annuli give estimates of surface density $\Sigma_{lens} = \Sigma_{cr}\,\boldsymbol{\kappa}$ as a function of r

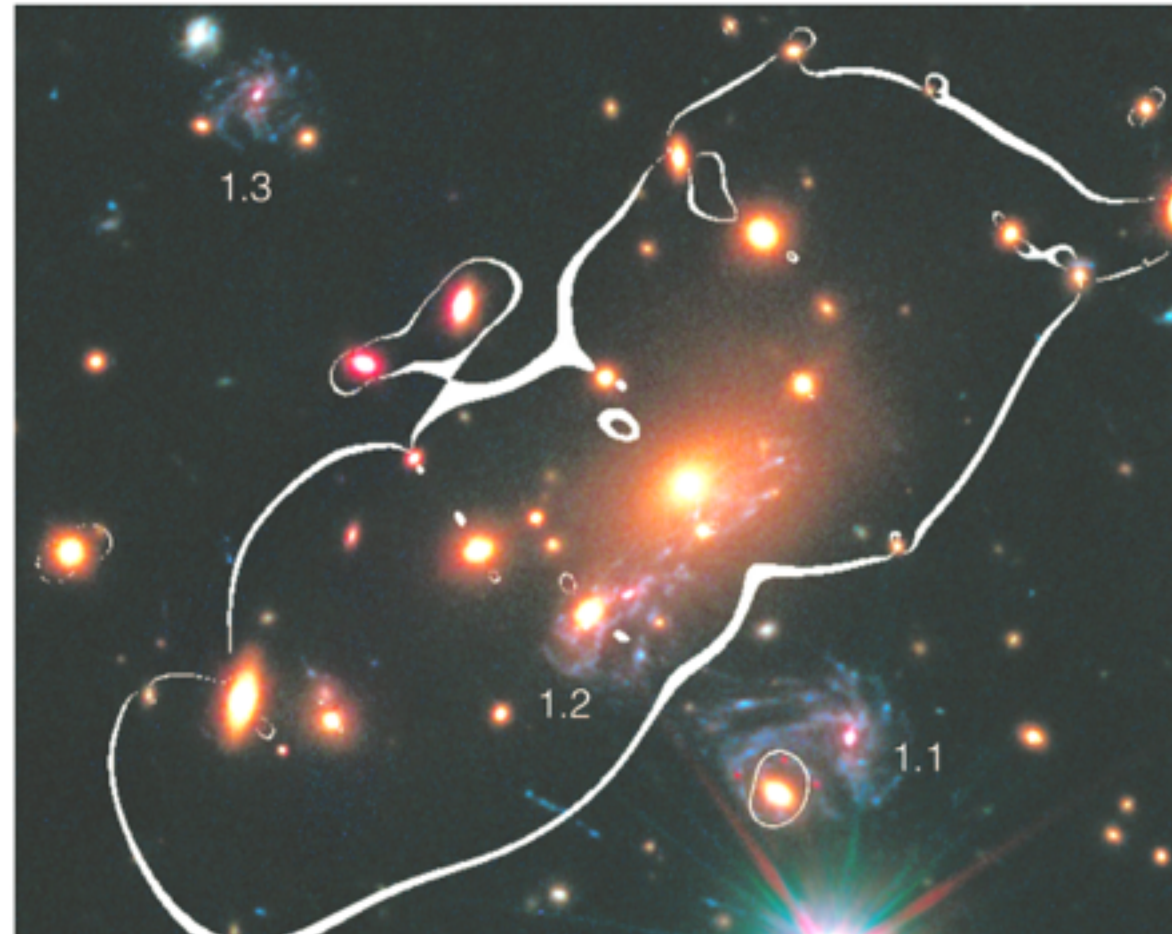
Determining the Mass of Lenses

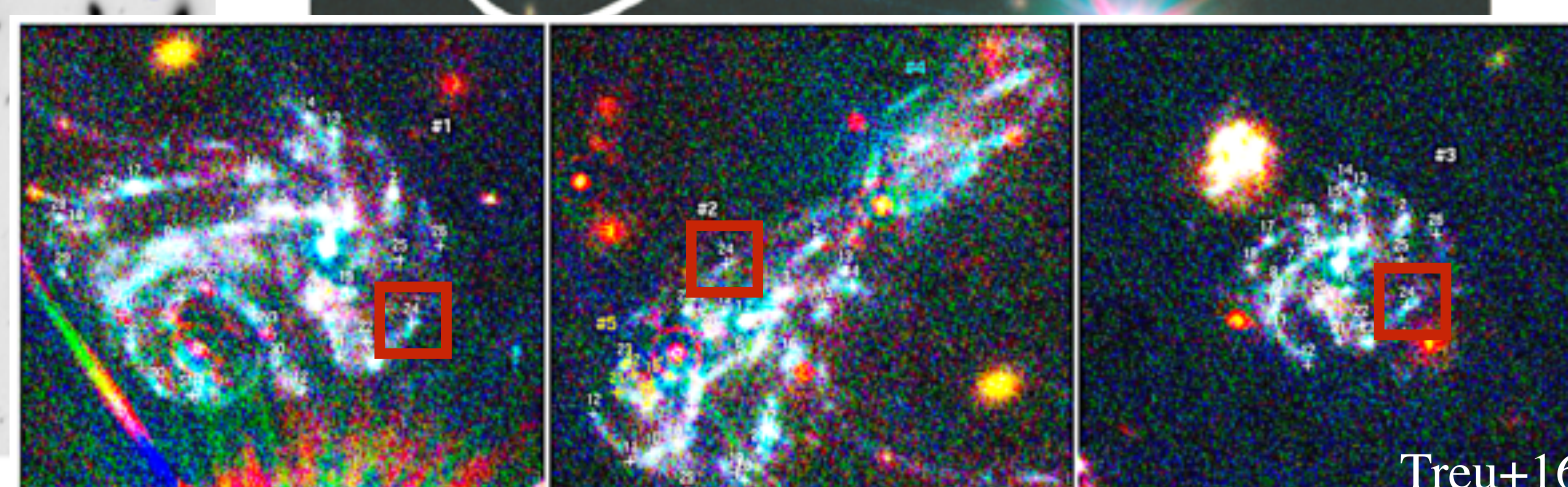
- The position of multiple images (4), their magnification and morphology, can be used to constrain mass maps of lenses (no reference sample needed).



Treu+16

Kelly+15

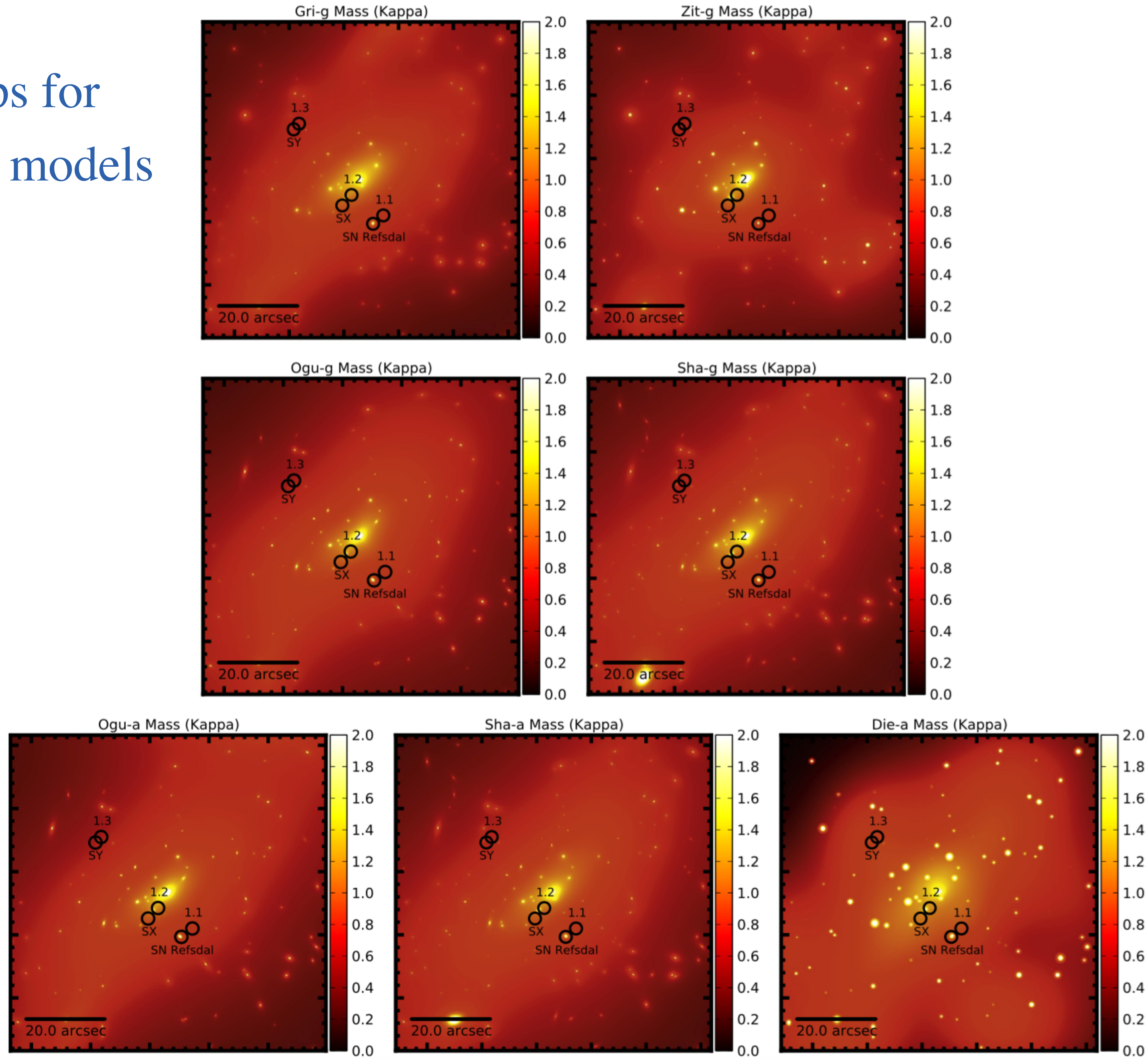




Treu+16

Determining the Mass of Lenses

Mass (κ) maps for different lens models

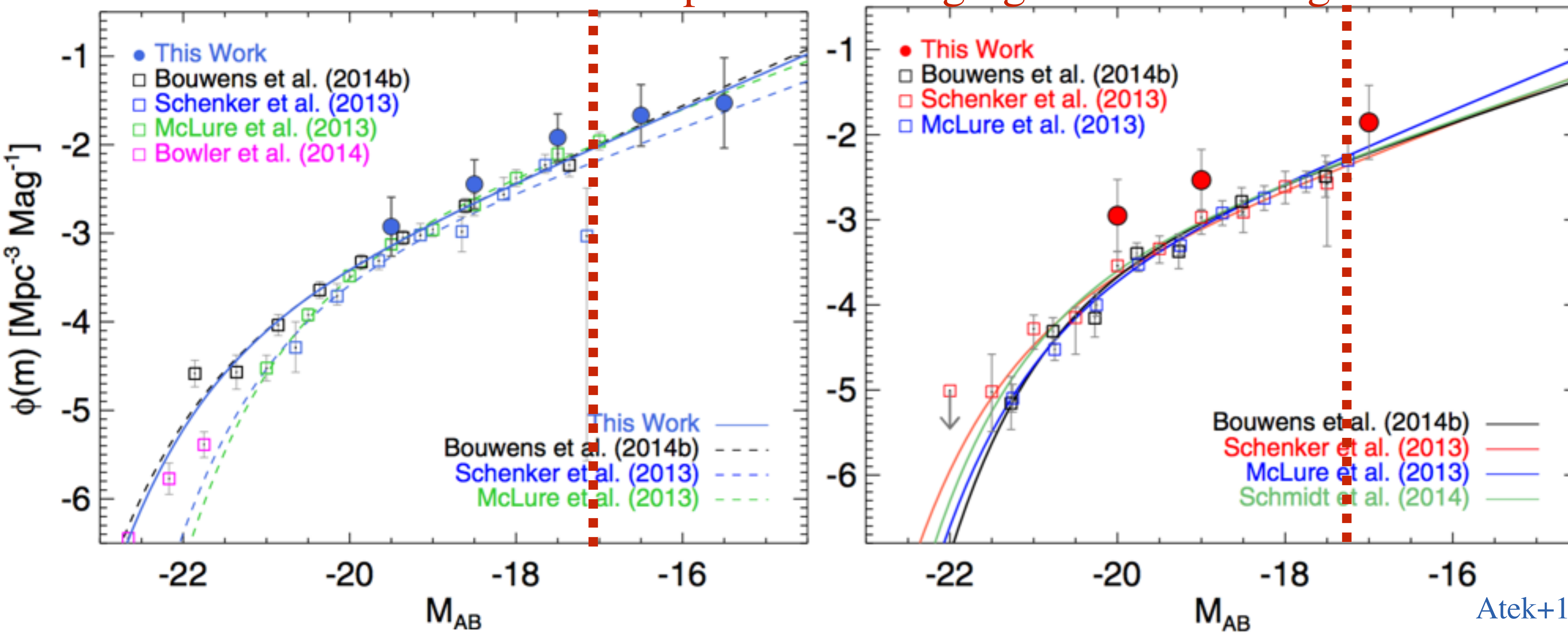


Treu+16

Finding Lensed High-z Galaxies

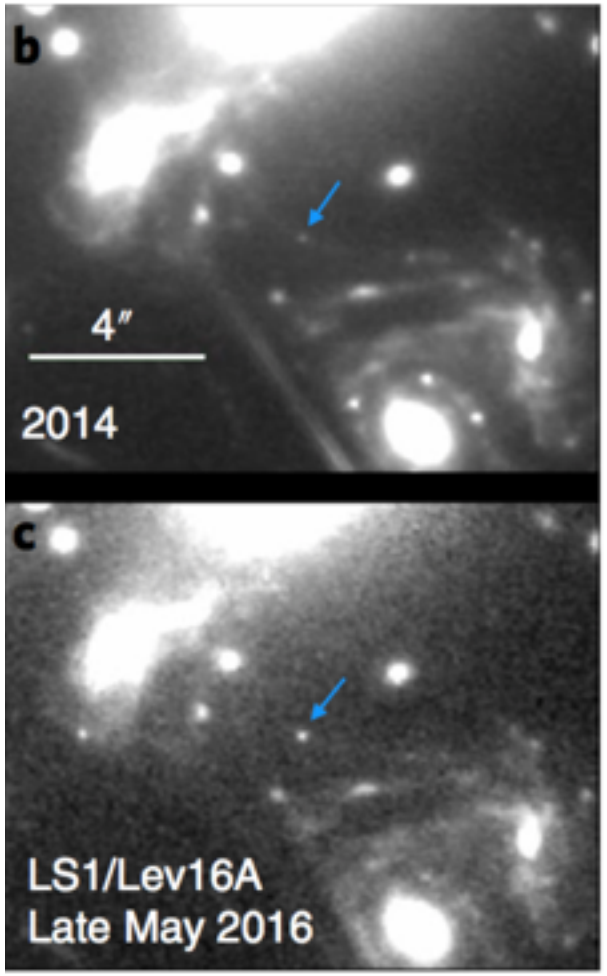
- The magnification of sources enable detection of faint & distant sources
- High-z galaxies are faint both because of intrinsic brightness and distances
- Even the deepest Hubble images (m~29) do not reach these depths
- GL to the rescue: $\mu \sim 10$ implies $\Delta m = 2.5$ mag

z~7 Deepest HST imaging without lensing z~8



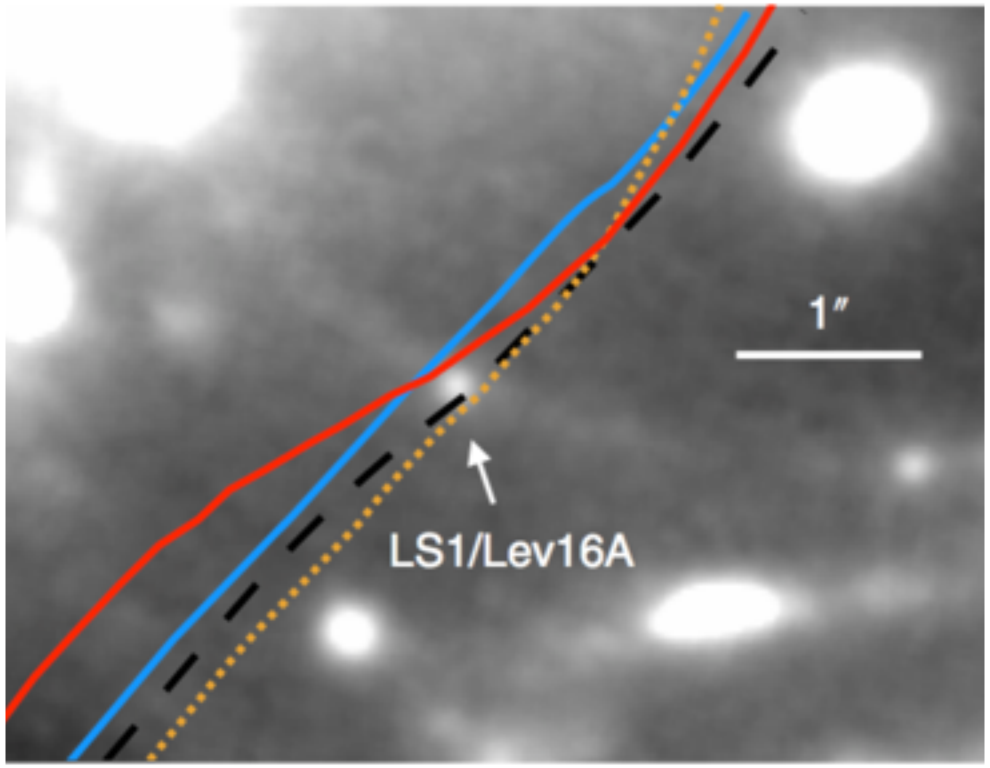
Atek+15

Lensed Star 1



- Dense monitoring of MACS1149 due to SN Refsdal
- New transient appeared in the host of SN Refsdal in 2016
  - Not associated with SN Refsdal
  - Light curve different from any known (super)nova
- True critical curve is indicated by $\mu \rightarrow \infty$
  - hence extreme magnification can occur
- Leading theory:
  LS1 is a blue super giant crossing the caustic at z=1.49 behind MACS1149 being magnified by more than a factor 2000.

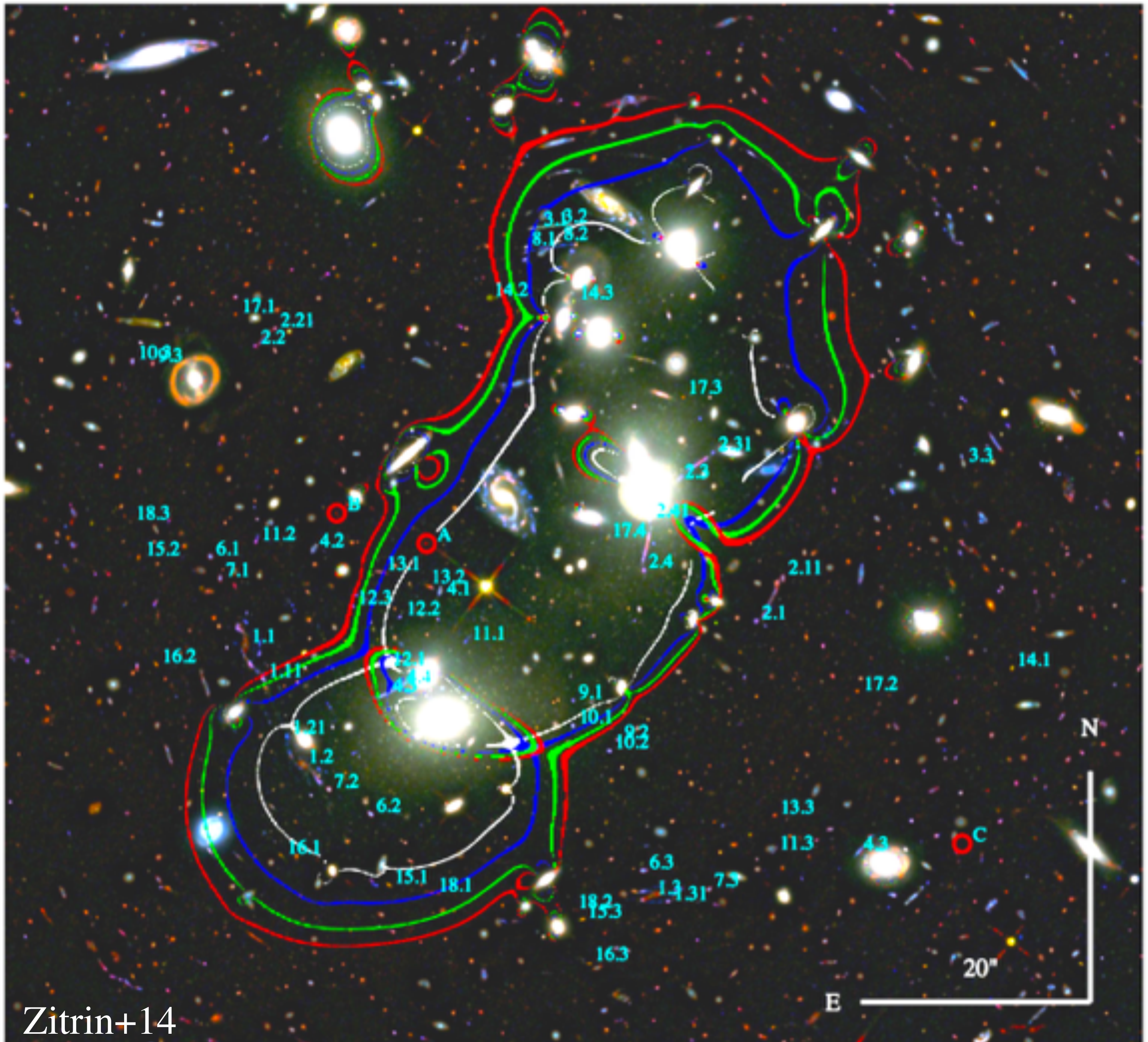
Lens model critical curves



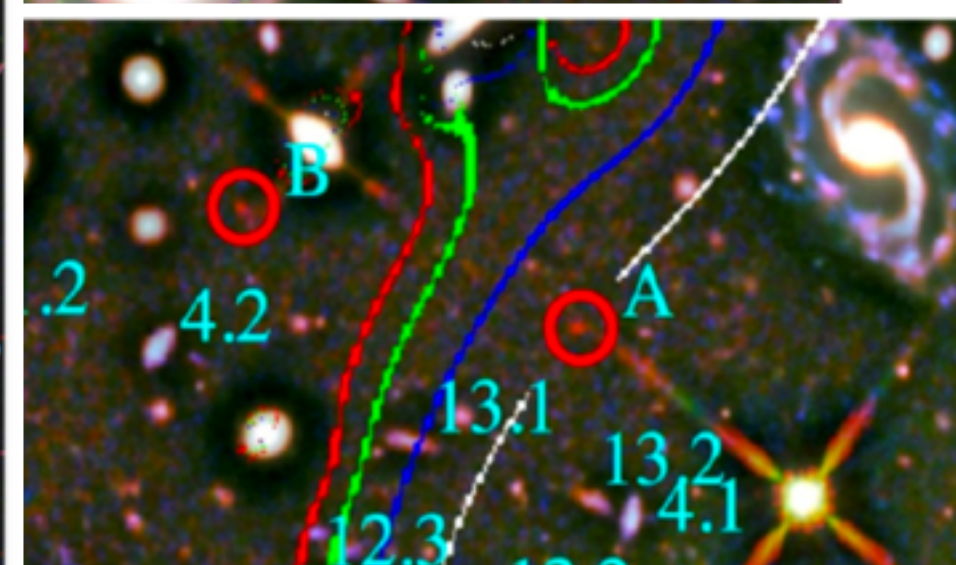
Kelly+18

Confirming High-z Galaxies

- The standard way to confirm high-z galaxies is Lyα in spectrum
- But the lensing geometry can also provide confirmation



Critical curves:
z = 1.3 (white)
z = 2.0
z = 3.6
z = 10.0



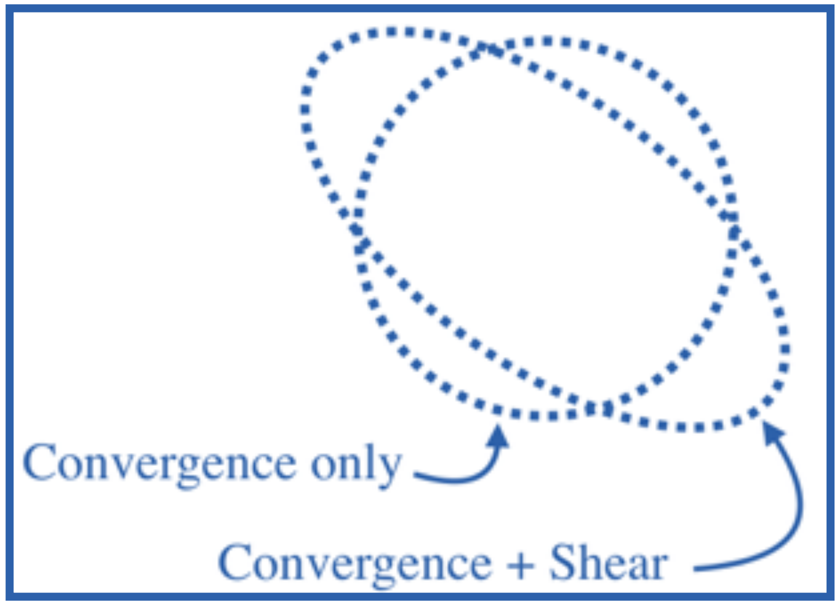
So in summary…

- Lensed sources are magnified, i.e. apparent fluxes increased (light focused)
- The magnification is as the inverse determinant of the Jacobian matrix

$$\frac{F(\boldsymbol{\theta})}{F(\boldsymbol{\beta})} = \mu \equiv \det M(\boldsymbol{\theta}) = \frac{1}{\det \mathcal{A}(\boldsymbol{\theta})}$$

- It can be expressed in terms of the convergence, **κ**, and the shear, γ

$$\mu = \frac{1}{(1-\kappa)^2 - \gamma^2} \quad ; \quad \gamma^2 \equiv \gamma_1^2 + \gamma_2^2$$



- The magnification for standard lens models are:
  - Point mass lens: $$\mu = \frac{1}{1 - \frac{\theta_E^4}{\theta^4}}$$
  - SIS: $$\mu = \frac{1}{1 - \frac{\theta_0}{|\theta|}}$$
  - CIS: $$\mu = \left[\left(1 - \frac{\theta_0}{2\sqrt{\theta^2 + \theta_{\rm core}^2}} \right)^2 - \frac{\theta_0^2 \left(2\theta_{\rm core}\sqrt{\theta^2 + \theta_{\rm core}^2} - 2\theta_{\rm core}^2 - \theta^2 \right)^2}{4\theta^4 \left(\theta^2 + \theta_{\rm core}^2 \right)} \right]^{-1}$$
- Magnification and geometry useful for lens mass measurements and modeling, and for faint object (high-*z*) searches.