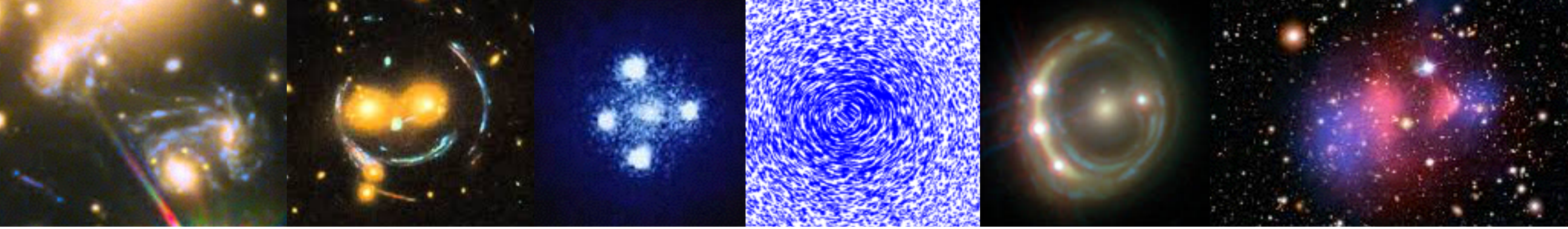

PHY-765 SS19 Gravitational Lensing Week 6

Time Delays

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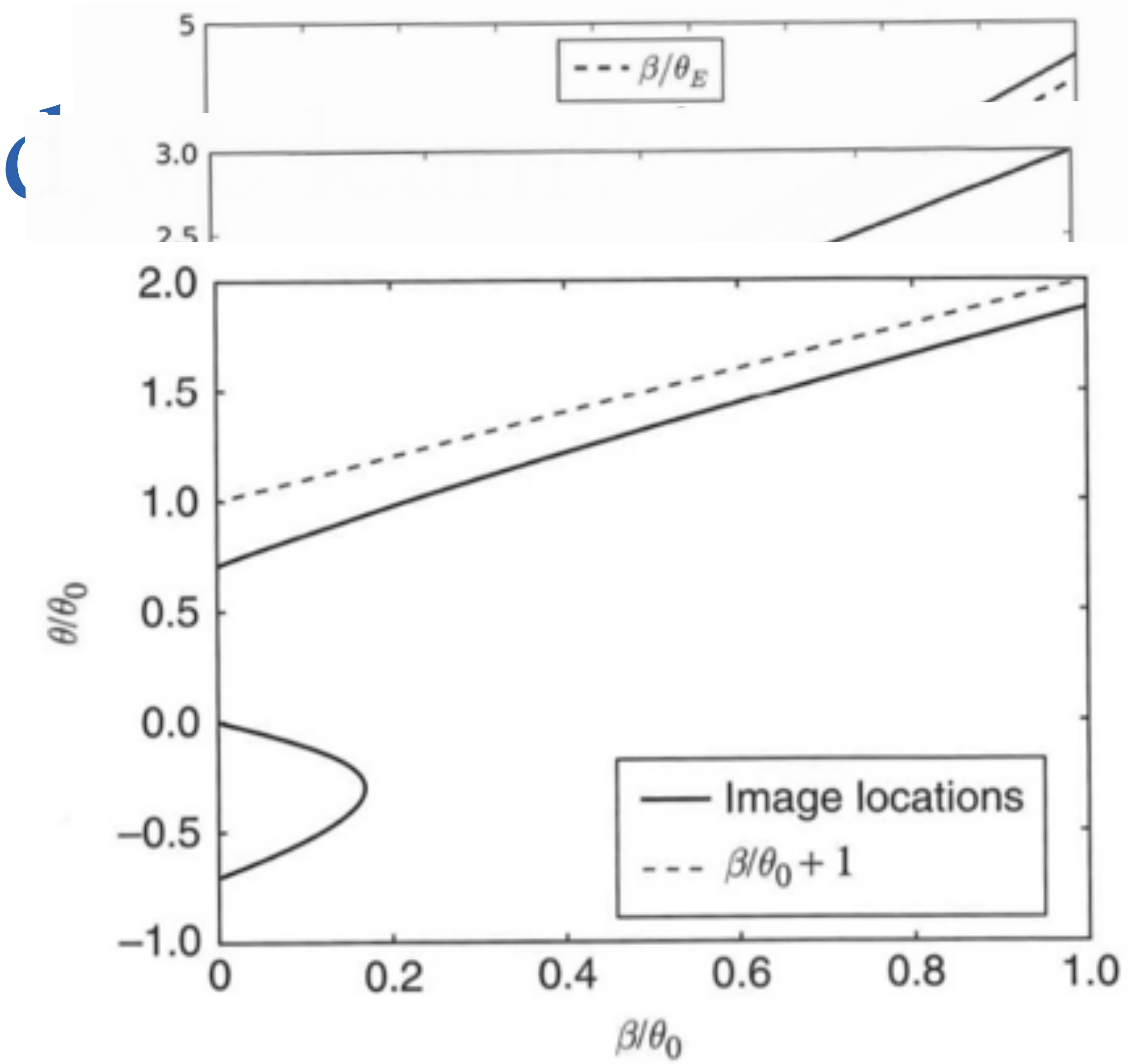
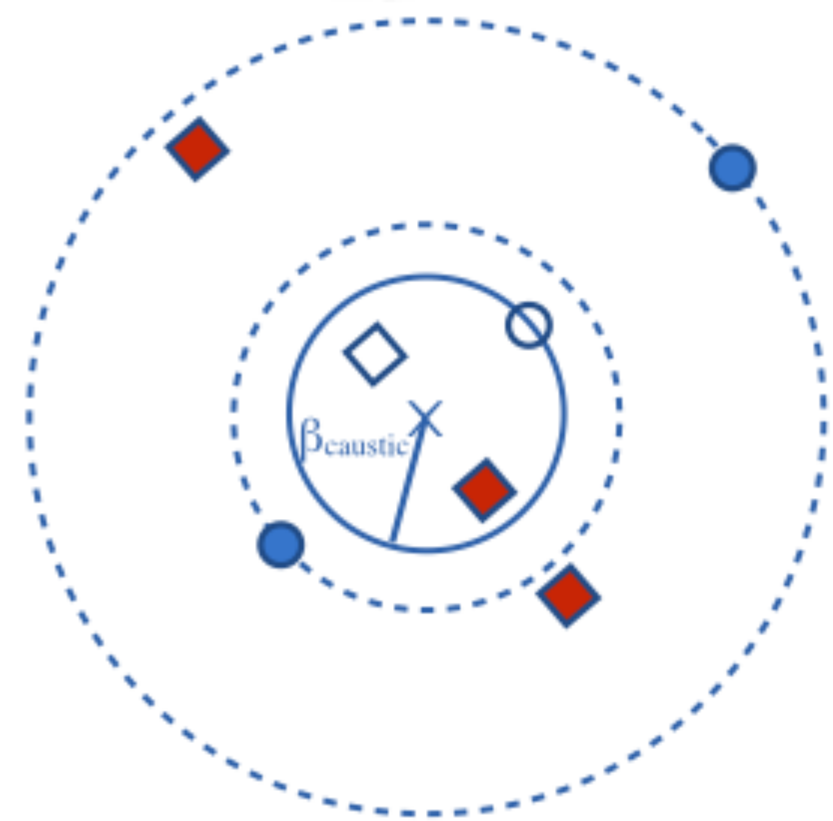
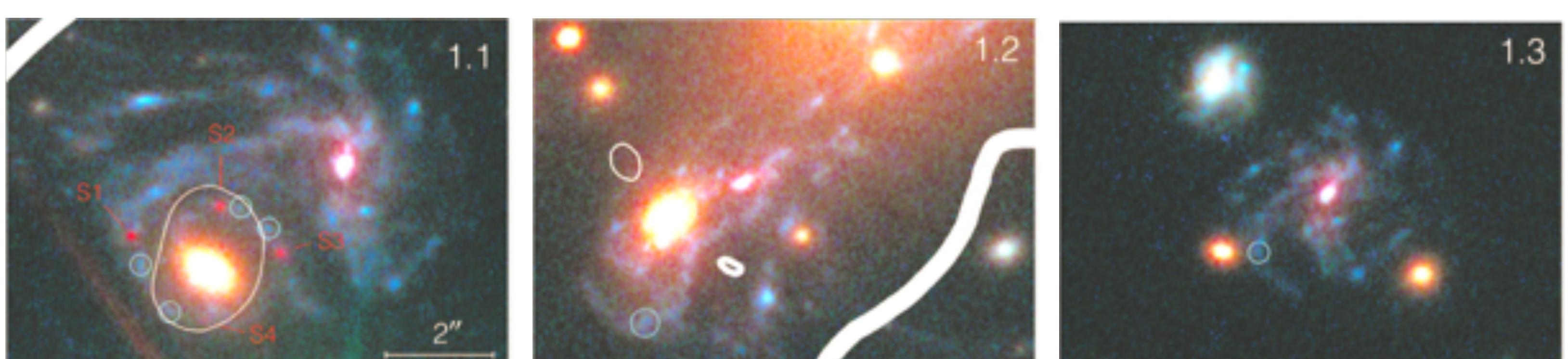
Last week - what di



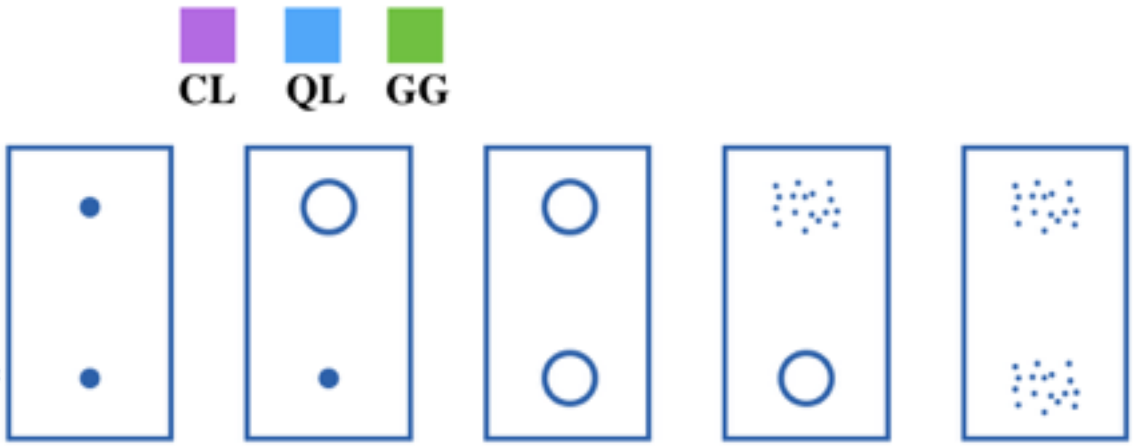
- Looked at multiple images for:
  - Point mass lens
  - Isothermal Sphere (IS)
  - Singular Isothermal Sphere (SIS)
  - Cored Isothermal Sphere (CIS)
- Introduced caustics (source plane) and critical curves (lens plane)
- Multiple images of SN refsdal and its host





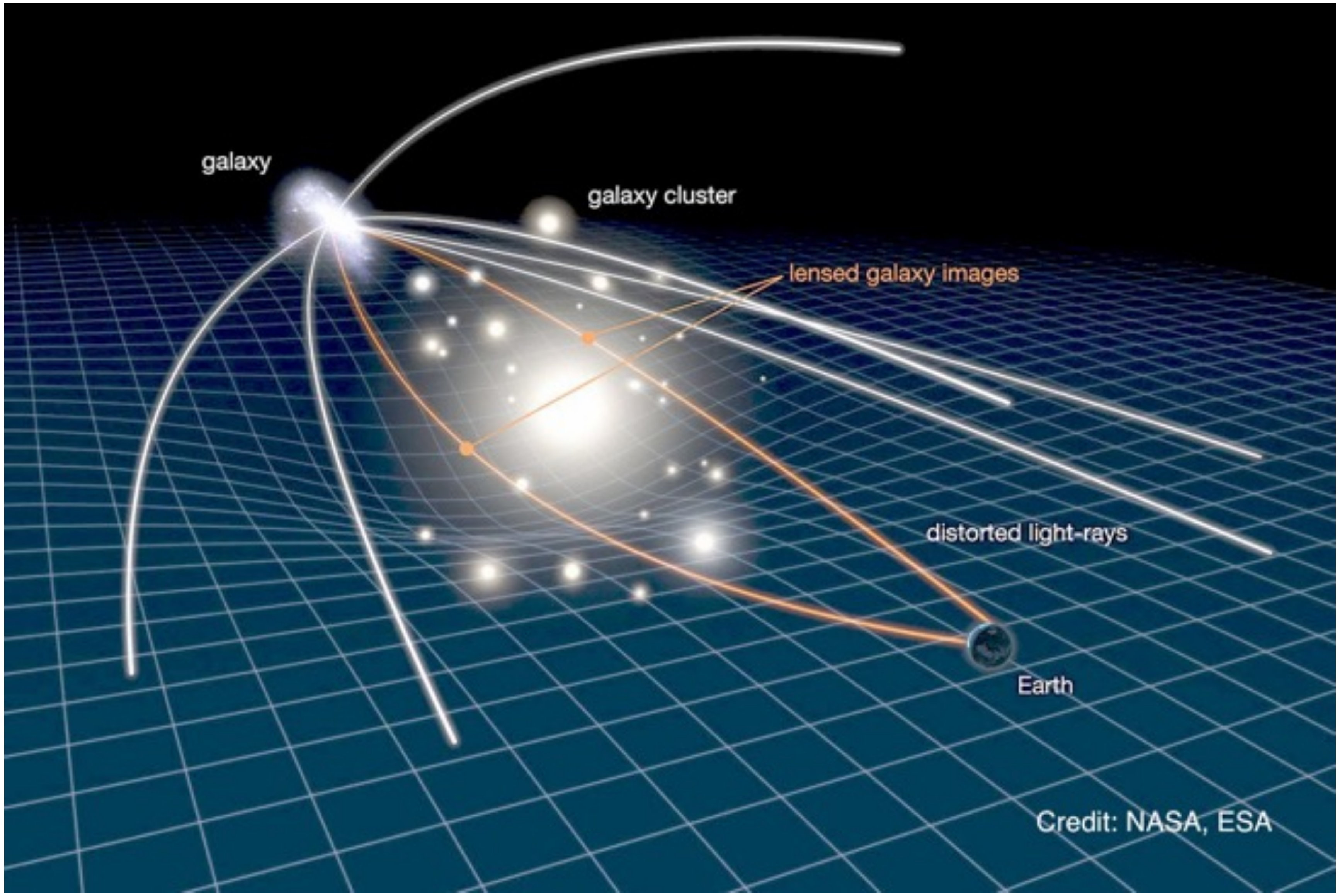
The aim of today

- Explore the second consequence of the lens equation: **time delays**
- Express time delay between the two images of the point mass lens
- Present examples of the usefulness of time delays
  - Testing GR predictions
  - The SN Refsdal re-appearance
  - Determining H_0 (COSMOGRAIL & H0LiCOW)



Time Delays

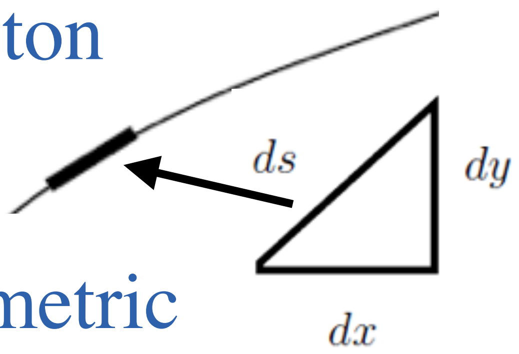
- Time delay is a natural consequence of the appearance of multiple images



$$\Delta t = \Delta t_{\mathrm{Geometry}} + \Delta t_{\mathrm{Shapiro}}$$

Shapiro time delay

- "The delay of light as it passes through a gravitational potential well"
- In week 1 we were considering the GR line element for a photon



$$ds^2 = g_{00}\,dt^2 + g_{ij}\,dx^i\,dx^j = 0$$

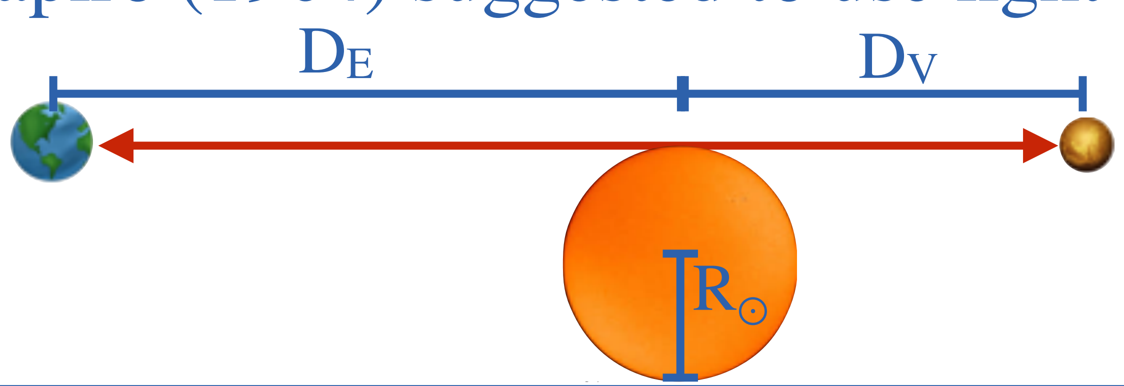
- Aligning light rays along the *z*-direction and again using the metric

$$g_{00} = c^2\left(1 - \frac{2GM}{rc^2}\right) \qquad g_{ij} = -\delta_{ij}\left(1 + \frac{2GM}{rc^2}\right)$$

- By Taylor expansion we find that

$$dz = c\,dt\left[\frac{1 - 2GM_\odot/rc^2}{1 + 2GM_\odot/rc^2}\right]^{1/2} \simeq c\,dt\left[1 - \frac{2GM_\odot}{rc^2}\right]$$

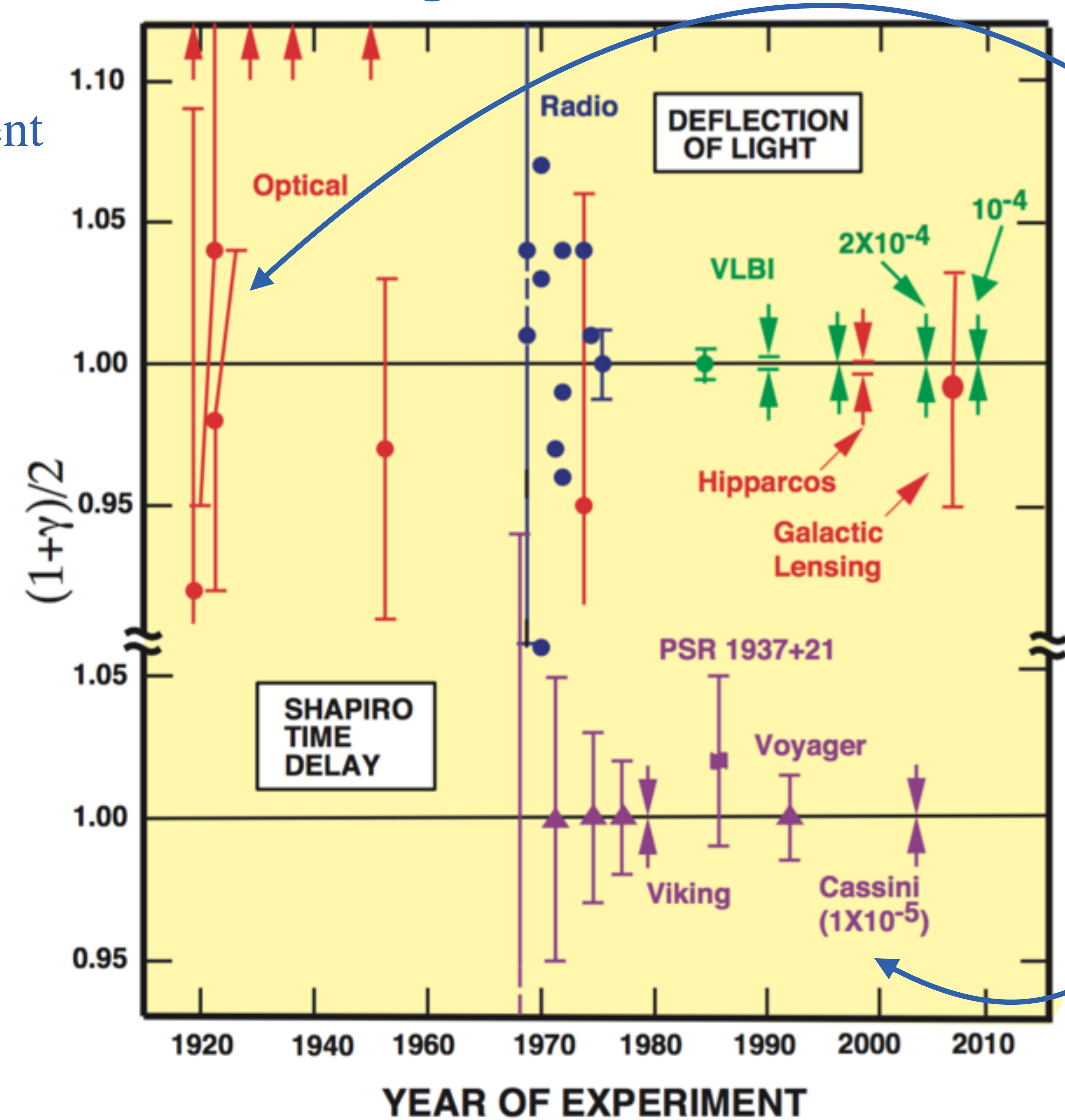
- So in the absence of gravity $dz/dt = c$, but with gravity $dz/dt < c$
- Shapiro (1964) suggested to use light deflection off Venus to measure this



$$\Delta t_{\rm Shapiro} = 200\mu s \qquad \text{(Exercise 2)}$$

Confirming General Relativity

Quantity describing (dis)agreement with GR.
For GR: $(1+\gamma)/2 = 1$



Solar eclipse measurements (week 1)

Shapiro time delay measurements using reflection off Venus, Mercury, satellites etc.

Will+2014

Shapiro time delay

- Considering the 3D gravitational potential $\phi(z) = -MG/r$ we have

$$\Delta t_{\rm Shapiro} = \frac{-2}{c^3}\int dz\,\phi(z) = -\frac{\Phi(\boldsymbol{\theta})}{c^2} \times \frac{D_{\rm S}D_{\rm L}}{D_{\rm LS}\,c}$$

- Where the expression $dz \simeq c\,dt\left[1 - \frac{2GM_\odot}{rc^2}\right]$ was divided by $1\text{–}2MG/rc^2$, Taylor expanded & integrated (see Exercise 2)

- And we introduced the *projected gravitational potential*

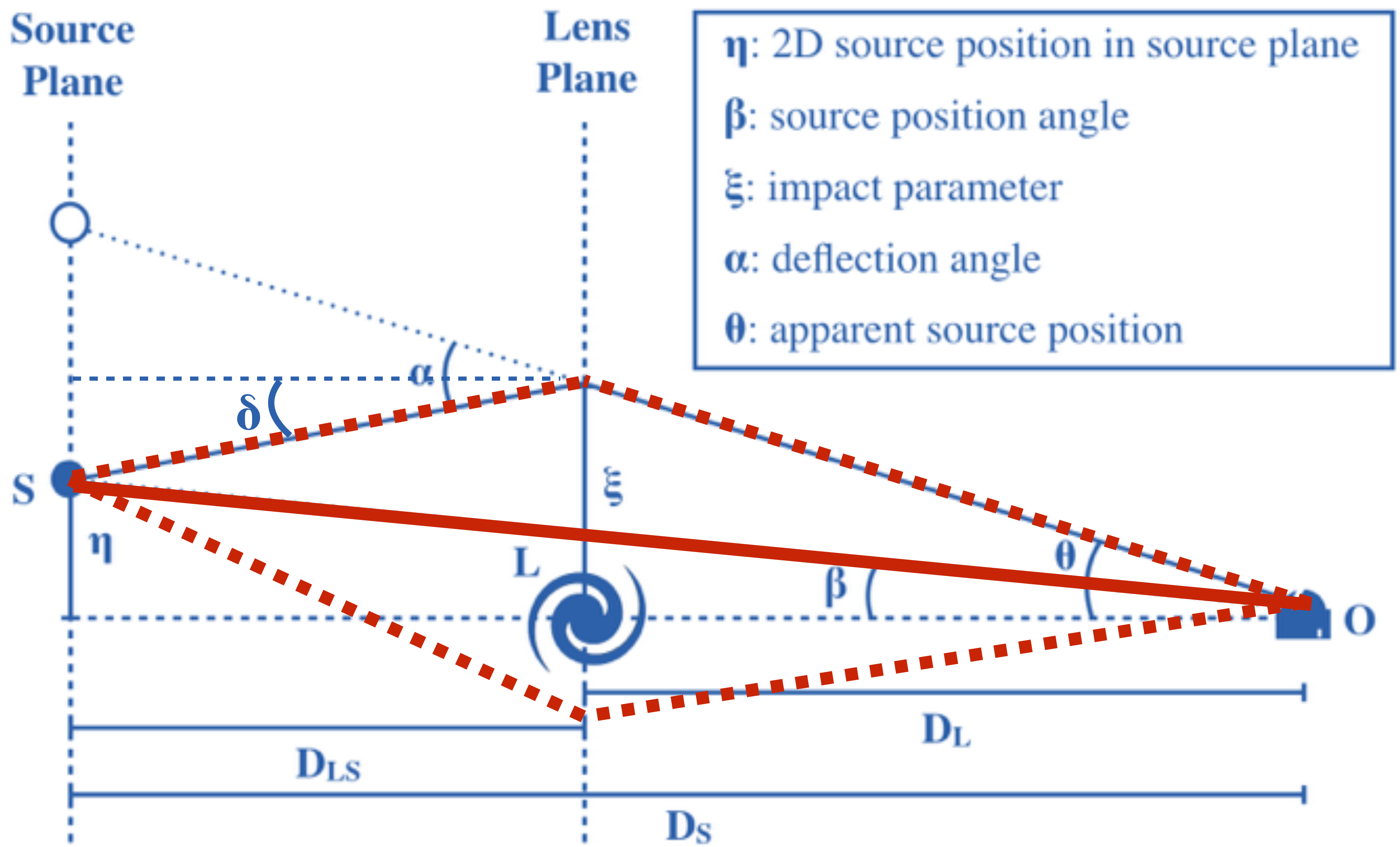
$$\Phi(\boldsymbol{\theta}) = \boxed{\boldsymbol{\psi}(\boldsymbol{\theta})c^2 = \frac{4MGD_{\rm LS}}{D_{\rm S}D_{\rm L}}\ln|\boldsymbol{\theta}|}$$

Depends on image position (light path)

Defined for the point mass in week 3

Geometric time delay

- The Geometric time delay caused by different path lengths for images



- By geometry the undeflected light path is just $D_u = D_S / \cos(\beta)$
- and the deflected light path is $D_d = D_L / \cos(\theta) + D_{LS} / \cos(\delta)$

Geometric time delay

- Combining these two expressions (and Taylor expanding cosines) we get

$$D_{\rm d} - D_{\rm u} = \frac{D_{\rm L} D_{\rm S}}{D_{\rm LS}} \frac{(\boldsymbol{\theta} - \boldsymbol{\beta})^2}{2}$$

- Dividing by c turns this light path difference into a time difference

$$\Delta t_{\rm Geometry} = \frac{D_{\rm L} D_{\rm S}}{c D_{\rm LS}} \frac{(\boldsymbol{\theta} - \boldsymbol{\beta})^2}{2}$$

Time delay

$$\Delta t = \Delta t_{\text{Geometry}} + \Delta t_{\text{Shapiro}}$$

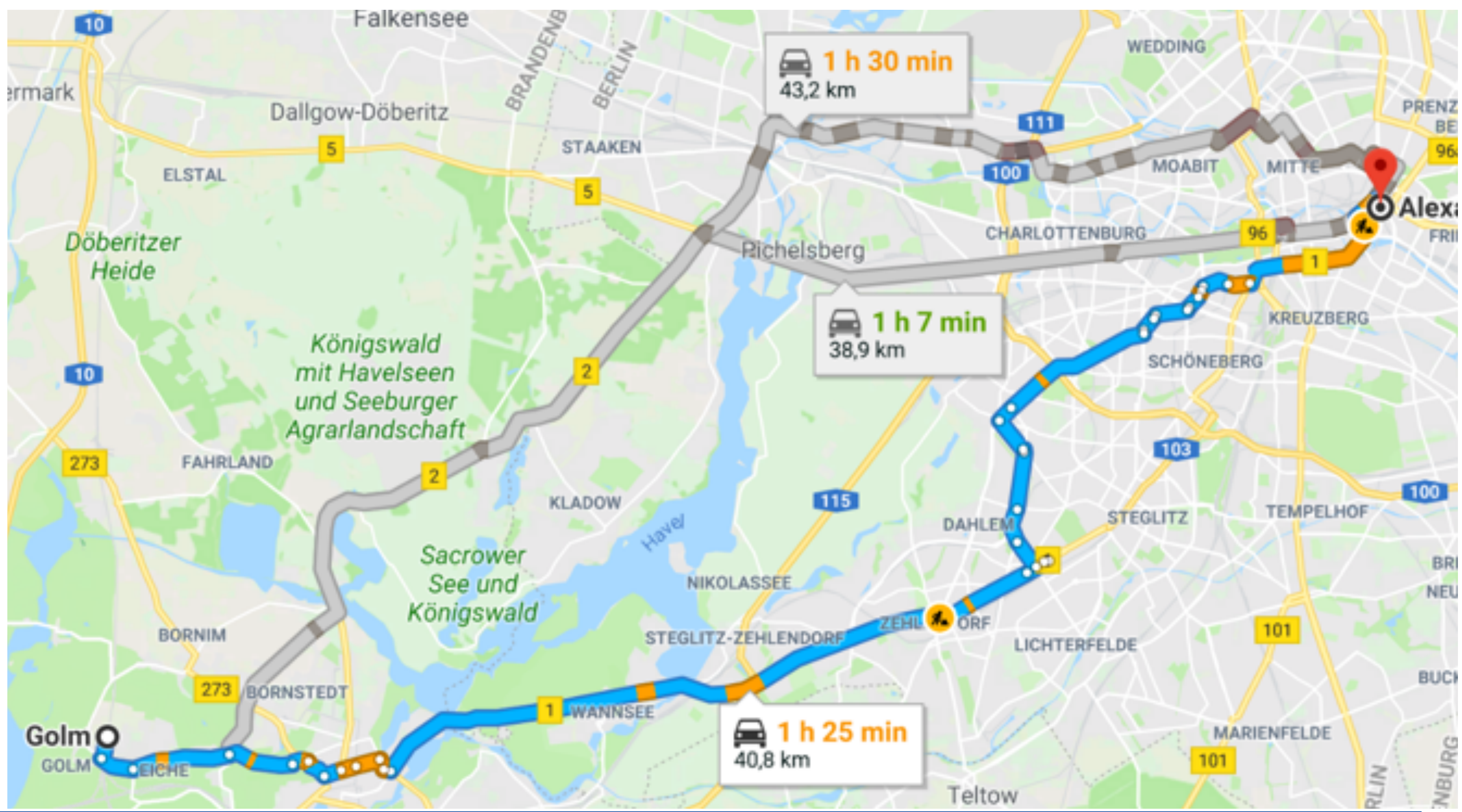
- Inserting the expressions we have the combined time delay

Only depends on distances; no lens details

$$\Delta t = \frac{D_{\rm L} D_{\rm S}}{c D_{\rm LS}} \left[\frac{(\boldsymbol{\theta} - \boldsymbol{\beta})^2}{2} - \frac{\Phi(\boldsymbol{\theta})}{c^2} \right]$$

Only depends on lens mass distribution

- Where the distances are given in co-moving distances
  - Add $(1+z_{\rm L})$ factor in front to have angular diameter distances



- GL time delay analogy:
  - **θ-β** is "route"
  - Φ(**θ**) is "traffic"

side note

Time delay reveals Lens Equation

- If we insist on minimizing the time traveled by light rays
  - Light travels on null-geodesics
- Or in other words we invoke *Fermat's Principle*:

  If S is the source and O the observer in a space time defined by a metric $g_{\mu\nu}$ and mass M, then a smooth null curve γ from S to O is a light ray (null geodesic) if, and only if, its arrival time $\boldsymbol{\tau}$ *at O is stationary under first-order variations of γ within the set of null curves from S to O, i.e.,* $\delta\boldsymbol{\tau} = 0$

- So differentiating Δt with respect to the angle we have:

$$\frac{d}{d\theta^i}\left[\frac{(\boldsymbol{\theta}-\boldsymbol{\beta})^2}{2} - \frac{\Phi(\boldsymbol{\theta})}{c^2}\right] = 0$$

- And since $\boldsymbol{\alpha} = \nabla\psi$ we have the lens equation $0 = \boldsymbol{\theta} - \boldsymbol{\beta} - \boldsymbol{\alpha}(\boldsymbol{\theta})$
- Hence, the lens equation is a 'manifestation' of Fermat's principle.

Time delay for the point mass lens

- For images 1 and 2 of a background source being lensed we have

$$t_1 - t_2 = (1+z_\mathrm{L})\frac{D_\mathrm{L}D_\mathrm{S}}{cD_\mathrm{LS}}\left(\left[\frac{(\boldsymbol{\theta}_1-\boldsymbol{\beta})^2}{2}-\frac{\Phi(\boldsymbol{\theta}_1)}{c^2}\right]-\left[\frac{(\boldsymbol{\theta}_2-\boldsymbol{\beta})^2}{2}-\frac{\Phi(\boldsymbol{\theta}_2)}{c^2}\right]\right)$$

- If the lens is a point mass and β much smaller than the Einstein radius

$$\theta_\pm \simeq \pm\theta_\mathrm{E} + \frac{\beta}{2} \qquad (\beta \ll \theta_\mathrm{E})$$

(cf. last week)

- And the geometrical time delay is insignificant (deflections ~identical)
  - I.e., $(\theta_+ - \beta)^2 \sim (\theta_- - \beta)^2$
- The main contribution to the time delay comes from the potential $\Phi(\boldsymbol{\theta})$
- Using the point mass lens expression for the gravitational potential

$$\Phi(\boldsymbol{\theta}) = \boldsymbol{\psi}(\boldsymbol{\theta})c^2 = \frac{4MGD_\mathrm{LS}}{D_\mathrm{S}D_\mathrm{L}}\ln|\boldsymbol{\theta}| = c^2\theta_E^2\ln|\boldsymbol{\theta}|$$

Time delay for the point mass lens

- We find that in the limit of small β (Taylor expanding ln)

$$\Phi(\boldsymbol{\theta}_+) - \Phi(\boldsymbol{\theta}_-) = c^2\theta_E^2 \ln\left|\frac{\theta_+}{\theta_-}\right| \simeq 2c^2\theta_E\beta$$

- Therefore, for a point mass lens the time difference between two images (when the geometric time delay is insignificant) is

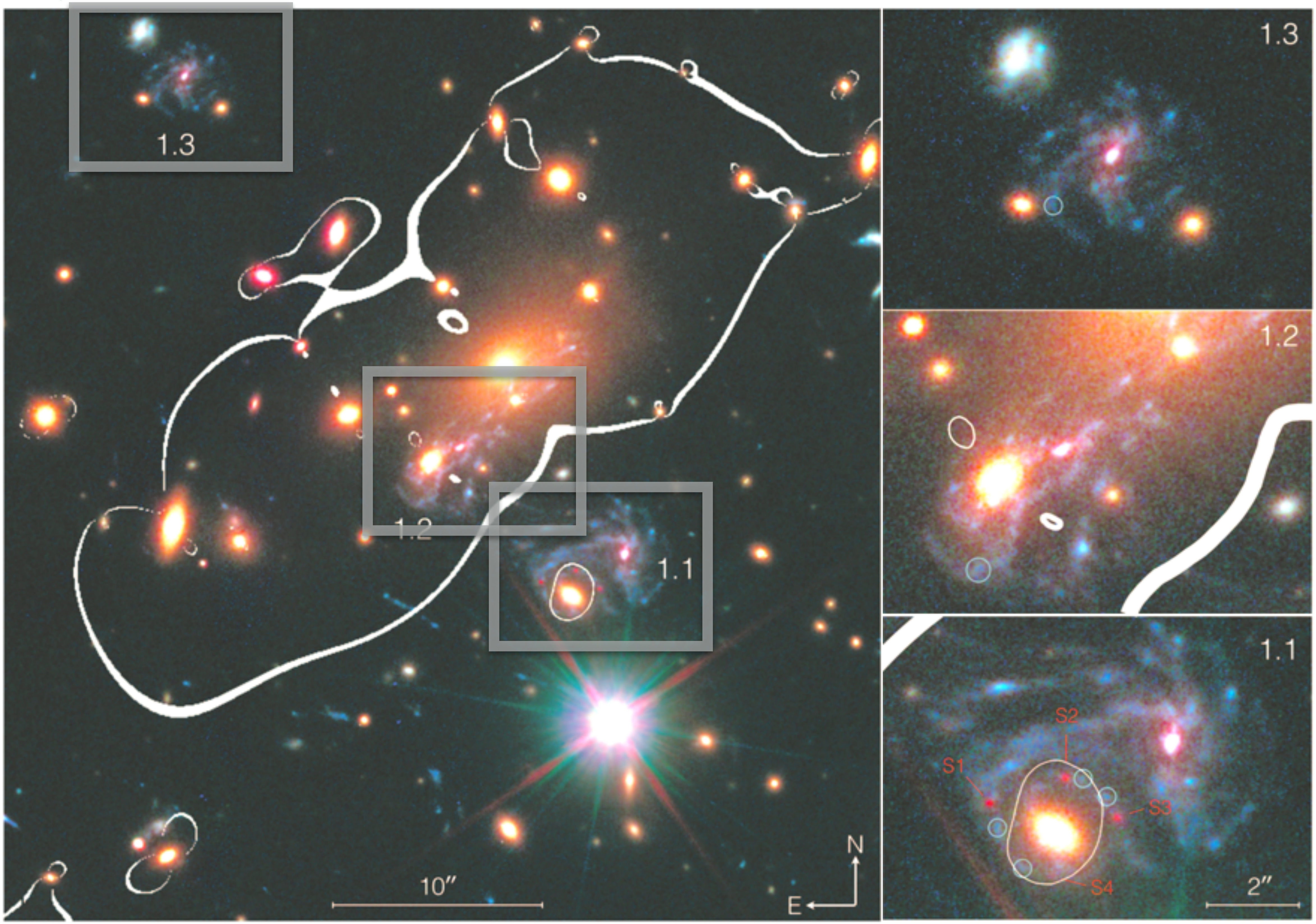
$$t_+ - t_- \simeq -(1+z_\mathrm{L})\frac{D_\mathrm{L}D_\mathrm{S}}{cD_\mathrm{LS}}2\theta_E\beta$$

- Light passing closest to the lens (t_-) is delayed the most
- Thus, light from image θ_+ arrive first
- Characteristic light delays between the two images are of the order months to years for cosmological lens geometries

(Exercise 3)

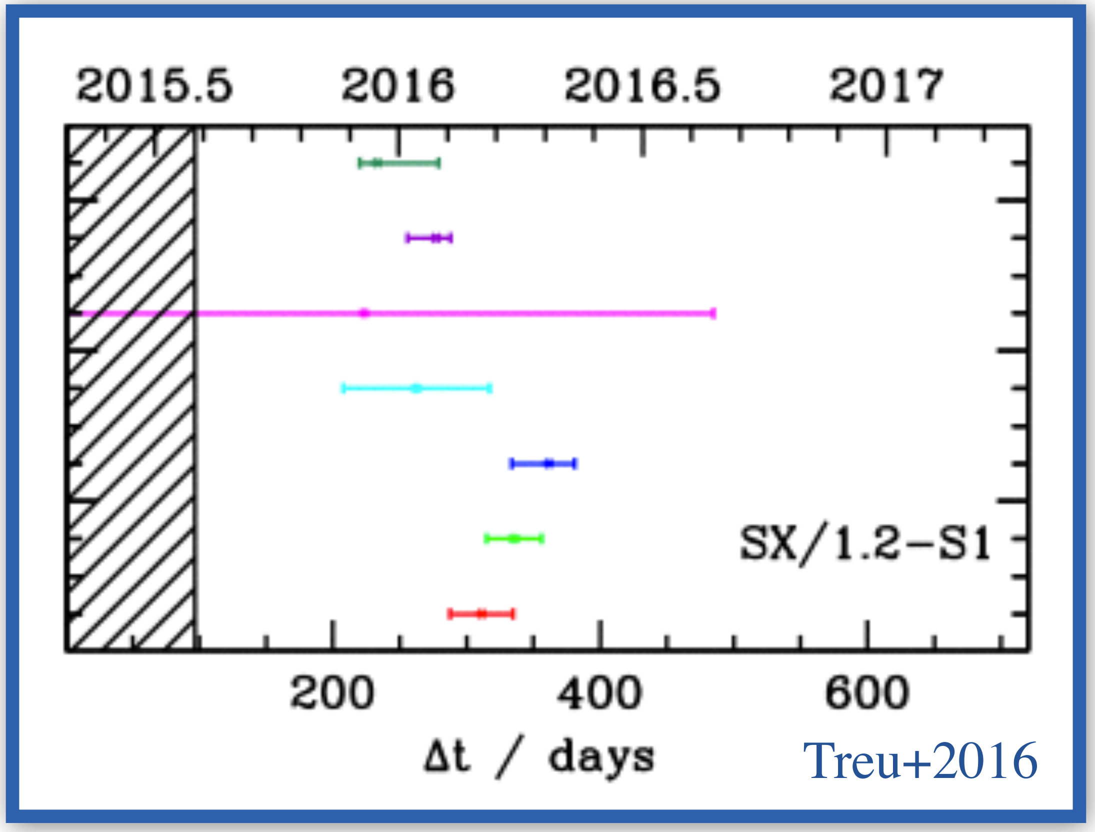
SN Refsdal - Time delay

Kelly+2015

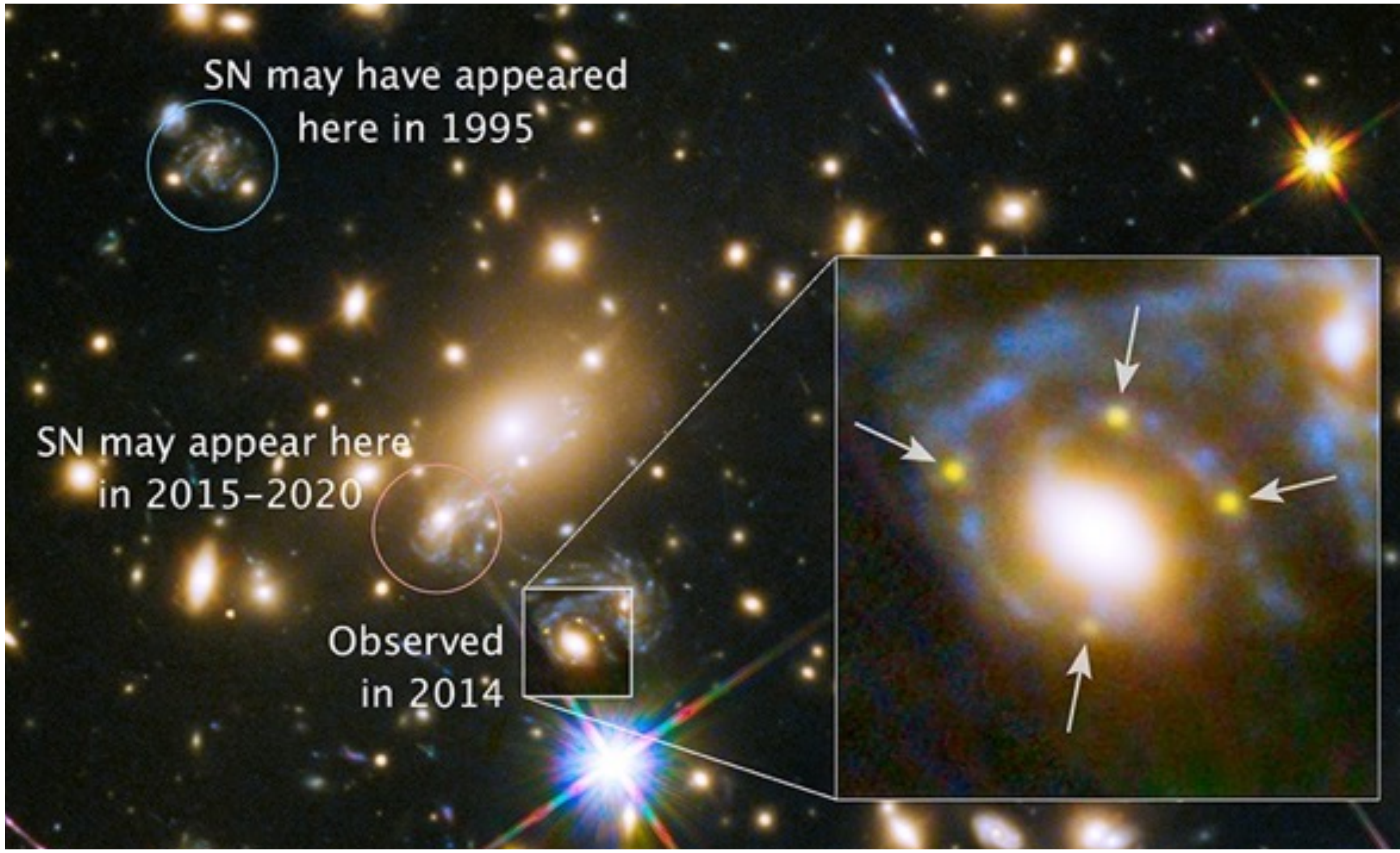


SN Refsdal - Time delay

- The SN Refsdal data can be used to strengthen/improve cluster lens models
- Treu+2016 coordinated a "blind" modeling of the MACS1149 cluster
  - Models from Zitrin+, Diego+, Oguri+, Sharon+, Grillo+
- Prediction of re-appearance of SN Refsdal and time-delay estimates

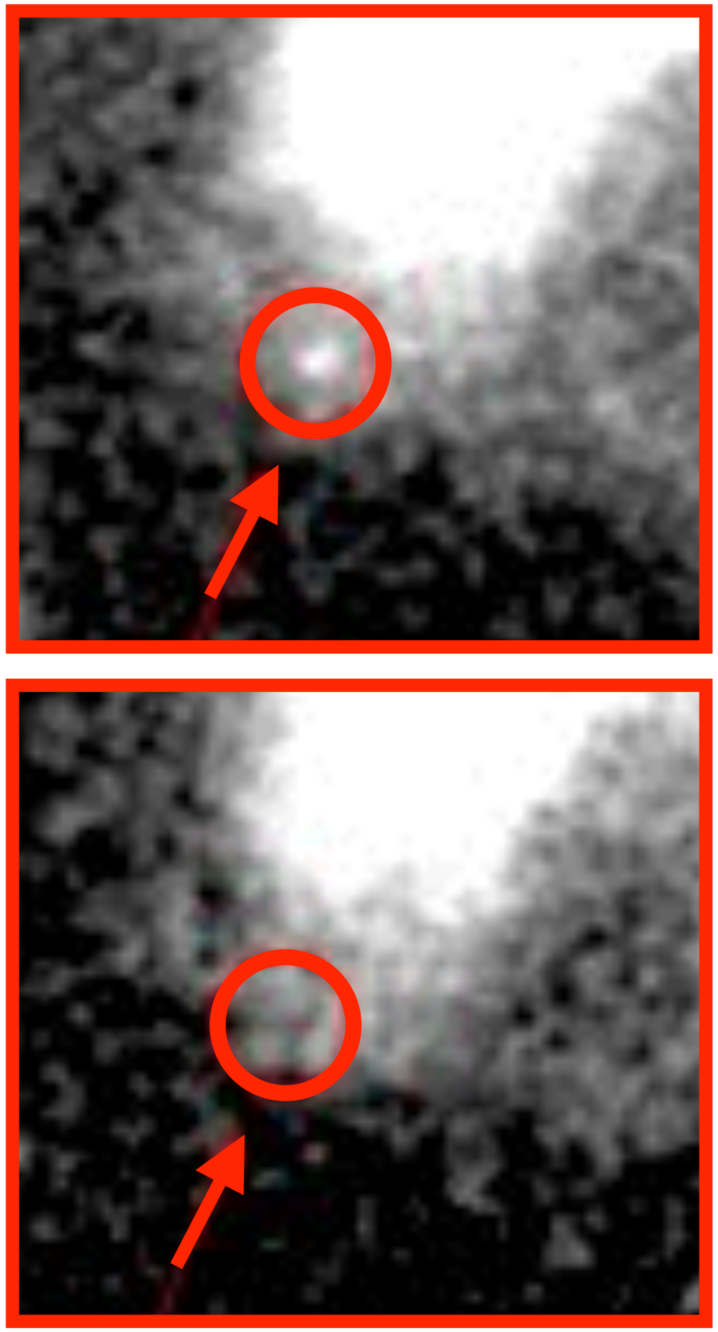


Treu+2016

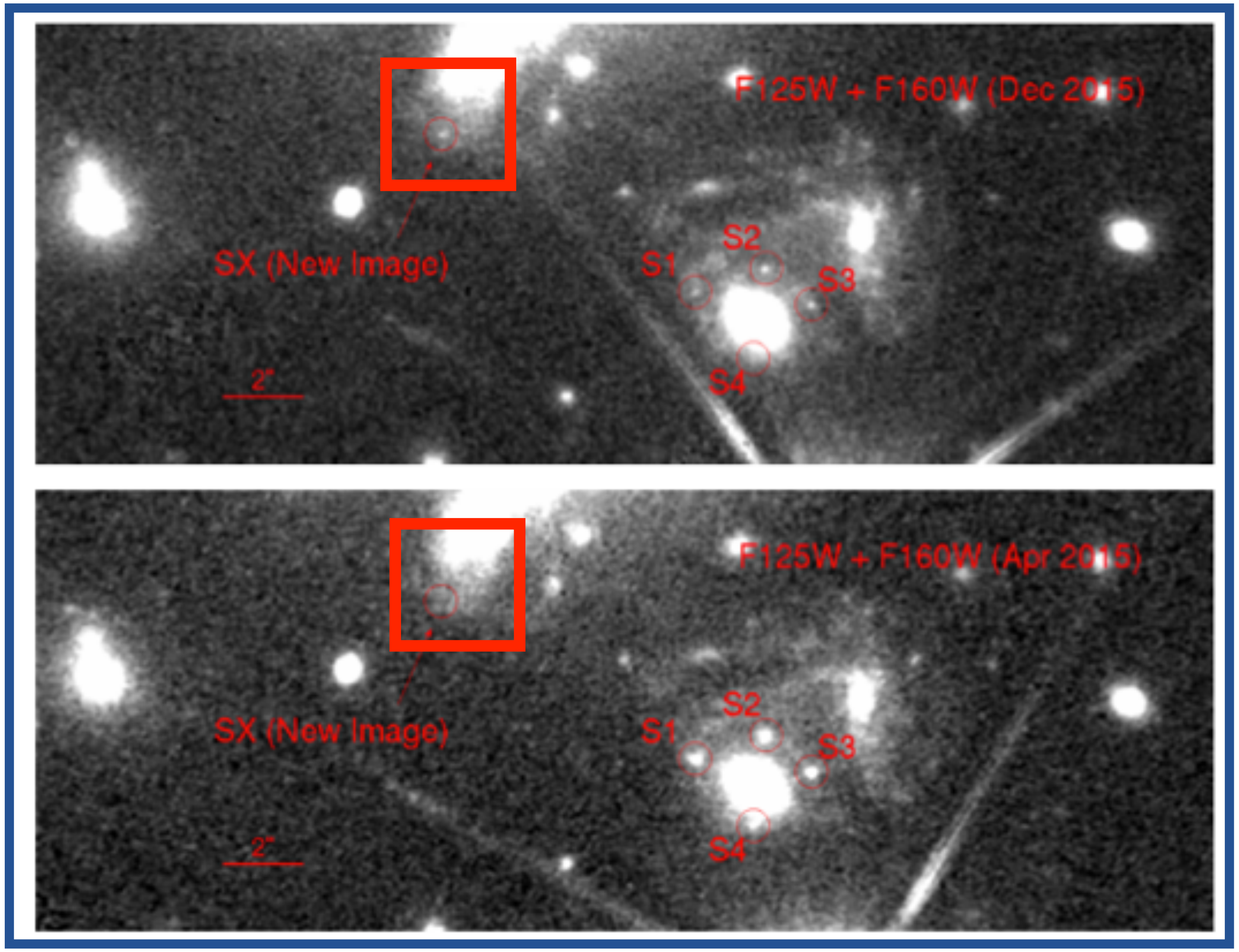


SN Refsdal - Time delay

- When MACS1149 Became re-observable ~Oct. 30 2015 (after Treu+2016 came out!) the hunt for the predicted re-appearance began.



On December 11 SN Refsdal re-appeared



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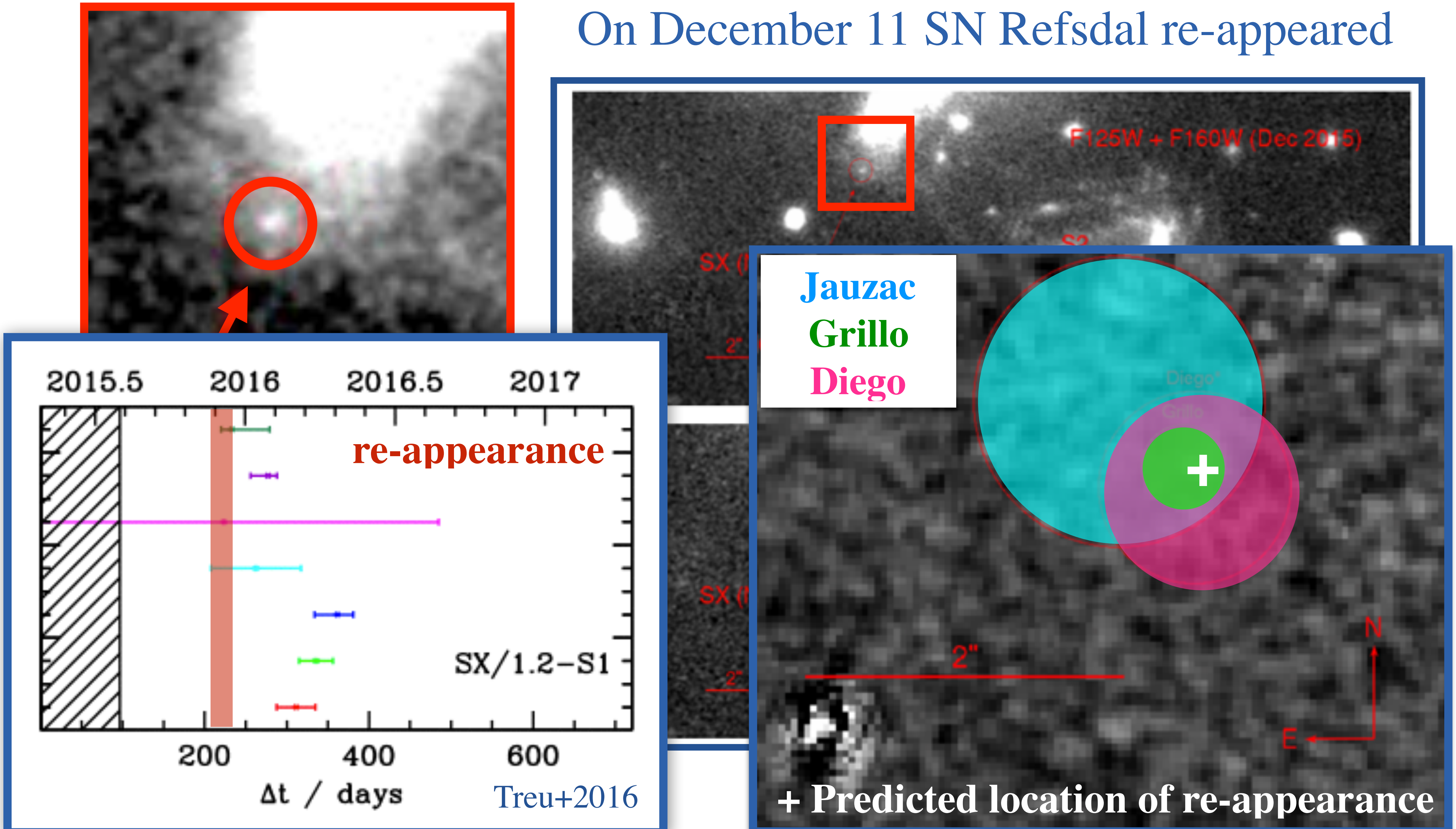
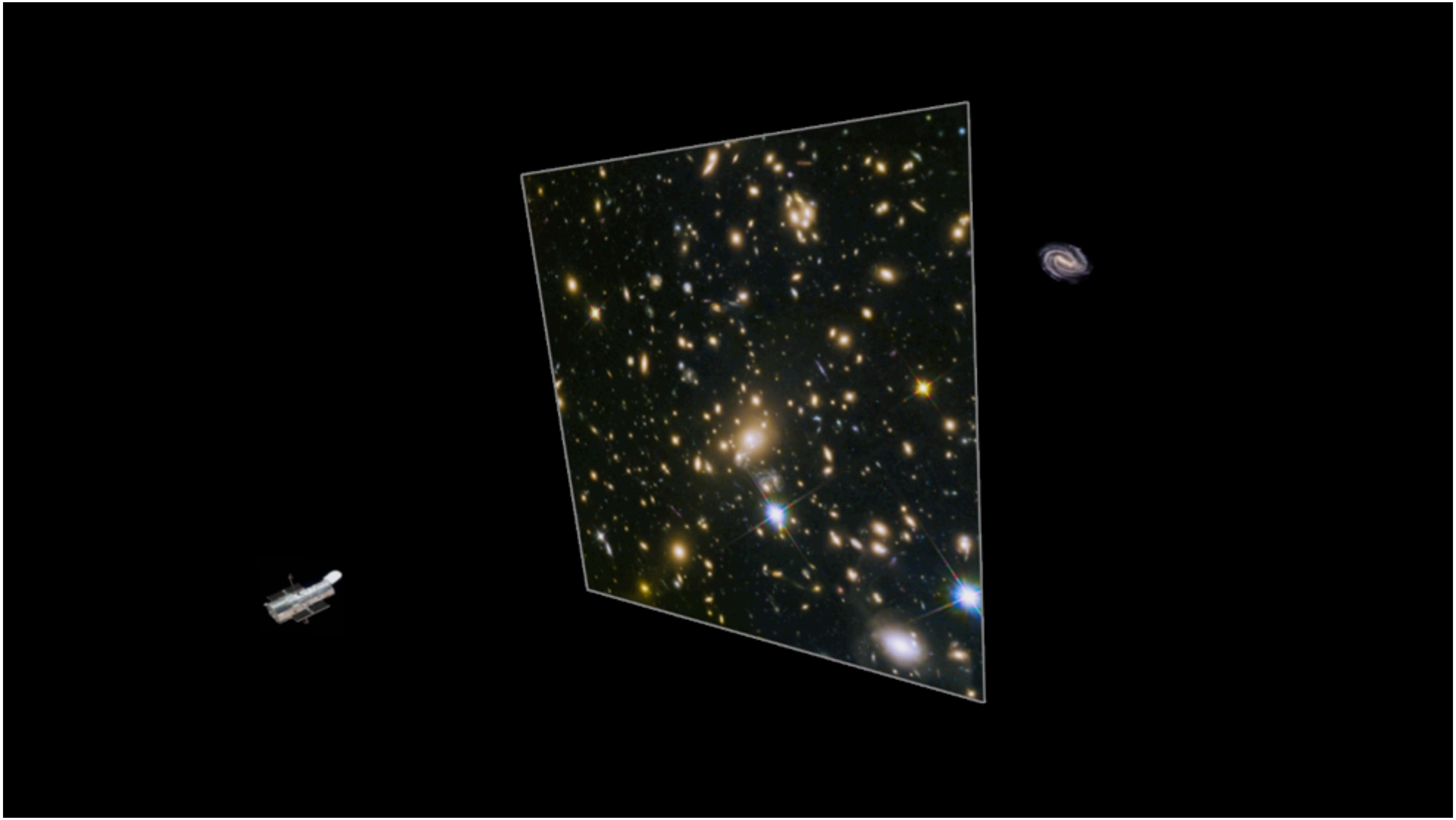


SN Refsdal - Time delays



http://hubblesite.org/newscenter/archive/releases/2015/08/video/

Estimating H_0 from time delays

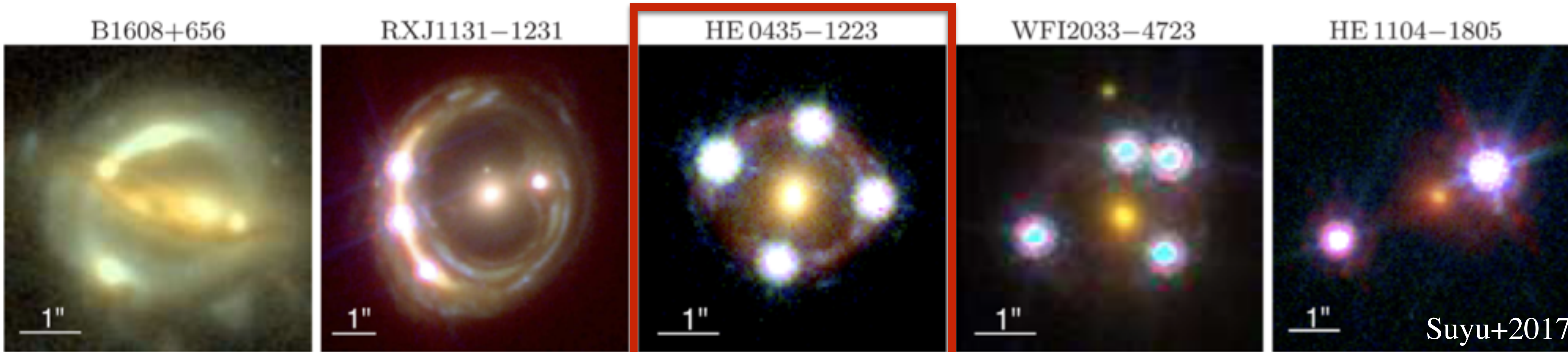
- Time delay lenses can also be used for estimating H_0
- The $\Delta t_{Geometry}$ is proportional to path lengths, i.e., scales with $1/H_0$
- The $\Delta t_{Shapiro}$ is also proportional to the path lengths, i.e., scales with $1/H_0$
- Hence, for any gravitational lens $H_0(t_1 - t_2)$ depends only on geometry

$$t_1 - t_2 = (1 + z_{\mathrm{L}}) \frac{D_{\mathrm{L}} D_{\mathrm{S}}}{c D_{\mathrm{LS}}} \left(\left[\frac{(\boldsymbol{\theta}_1 - \boldsymbol{\beta})^2}{2} - \frac{\Phi(\boldsymbol{\theta}_1)}{c^2} \right] - \left[\frac{(\boldsymbol{\theta}_2 - \boldsymbol{\beta})^2}{2} - \frac{\Phi(\boldsymbol{\theta}_2)}{c^2} \right] \right)$$

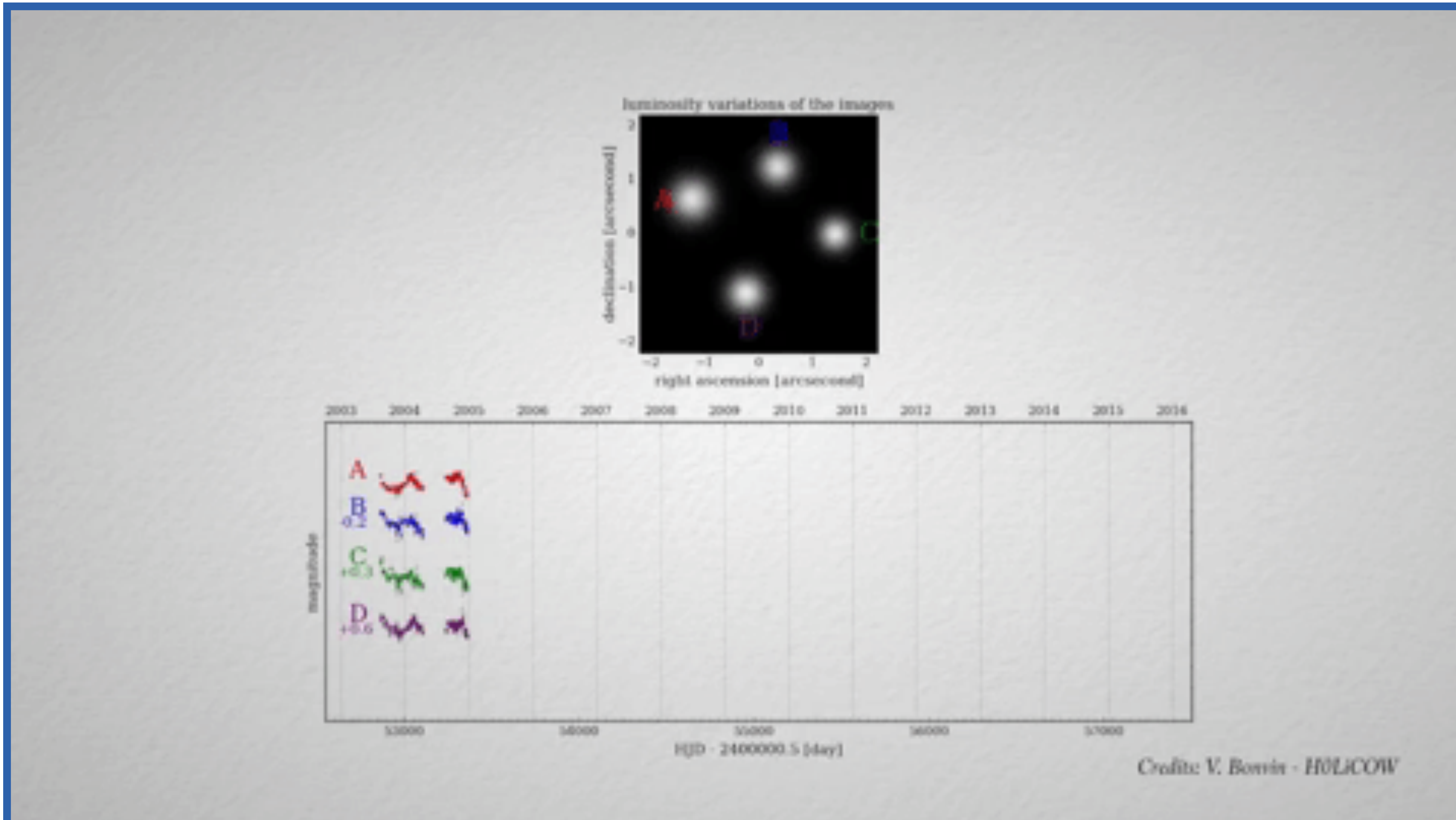
- Lens models provide $\boldsymbol{\beta}$, $\boldsymbol{\theta}$ and $\boldsymbol{\Phi}$ (mass of lens) and predict $H_0(t_1 - t_2)$
- This is compared to measurements of $(t_1 - t_2)$ from light curve monitoring

COSMOGRAIL & H0LiCOW

- **COS**mological **MO**nitoring of **GRA**v**I**tational **L**enses (www.cosmograil.org)
  - Imaging campaign to sample lensed QSO light curves
  - Time delay measurements [e.g., WFI J2033-4723 (Vuissoz+08), RXJ1132 (Suyu+13)]
- **H_0 L**enses **in COS**MOGRAIL's **W**ellspring (www.h0licow.org)
  - Extending work from COSMOGRAIL with focus on estimating H_0
- H0LiCOW is focusing on 5 lensed QSOs
- First set of papers from 2017 focused on HE0435-1223
  - Bonvin+17, Wang+17, Rusu+17, Sluse+17

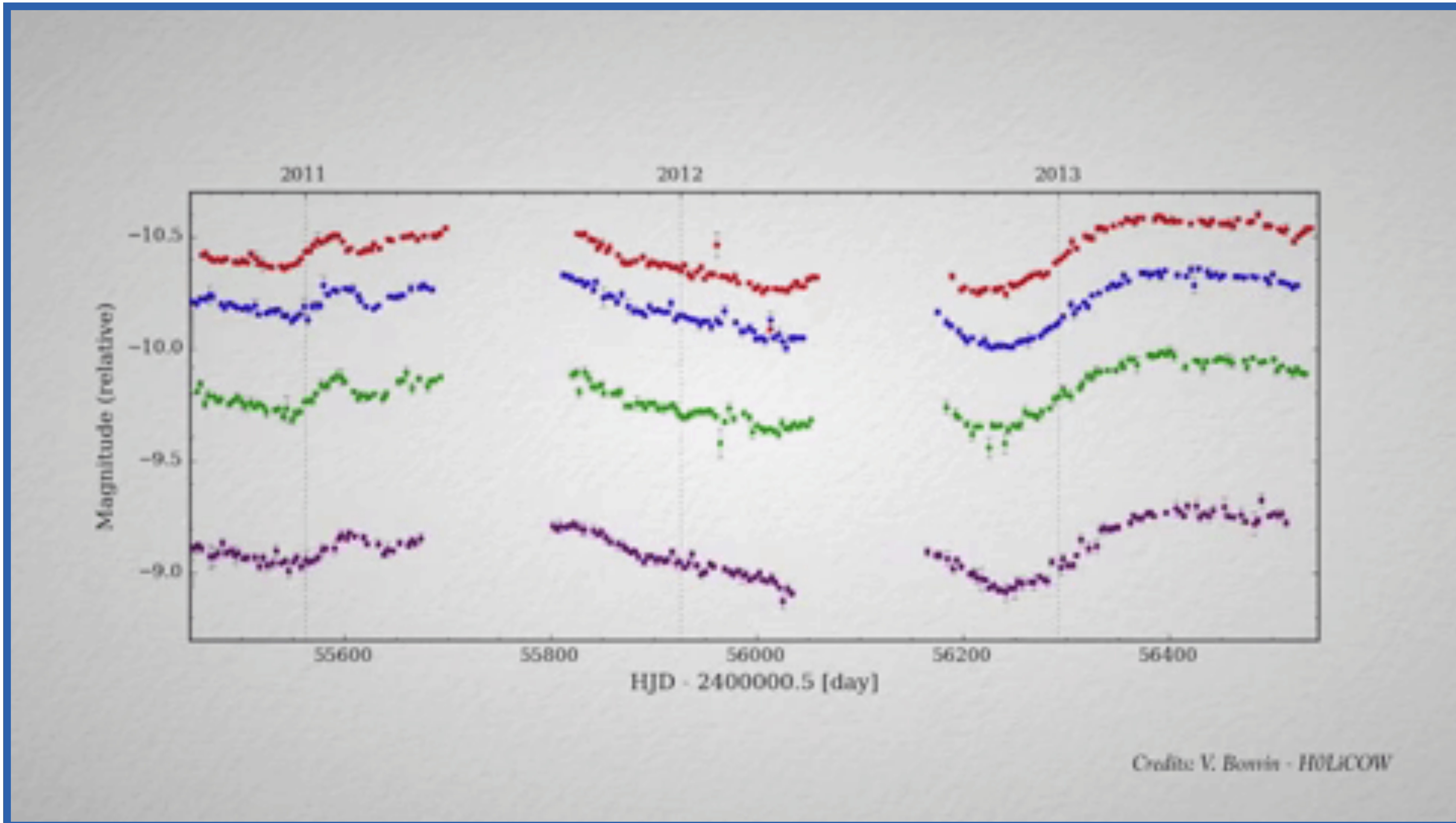


Light Cruve for HE0435-1223 images



https://www.youtube.com/watch?v=qoVQ8f5nVOw

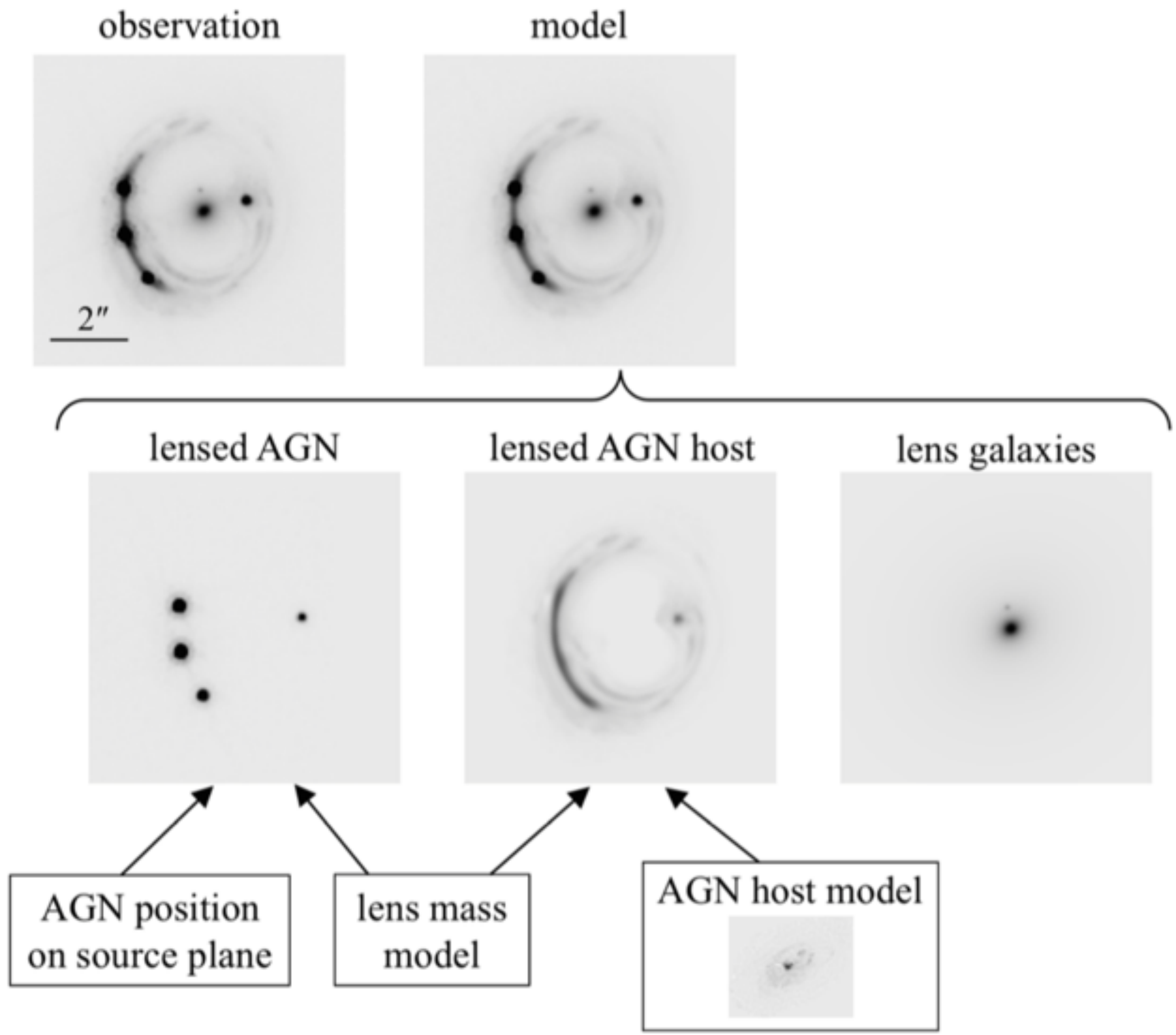
Matching Fluxes and Obtain Δt



https://www.youtube.com/watch?v=qoVQ8f5nVOw

A good lens model is key

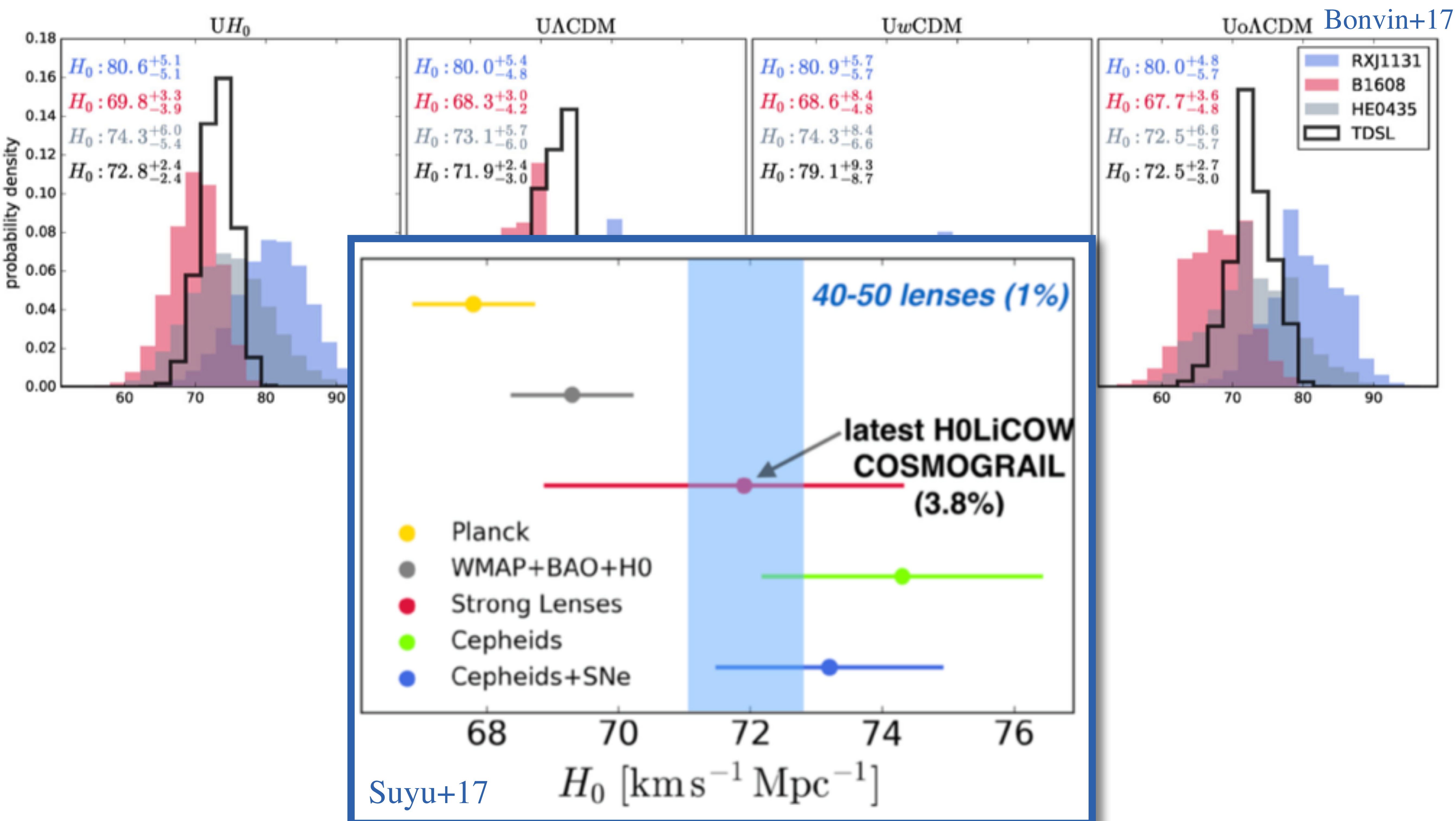
- The obtainable constraint on H_0 from comparing model-predicted $H_0(t_1\text{-}t_2)$ to observed $(t_1\text{-}t_2)$ is set by model precision/accuracy



Suyu+2018

H_0 From HE0435-1223

- Then, comparing Δt(observed) with $H_0\Delta t$(model) H_0 can be estimated



So in summary…

- Time delays are a natural consequence of appearance of multiple images

$$\Delta t = \Delta t_{\mathrm{Geometry}} + \Delta t_{\mathrm{Shapiro}}$$

- The Shapiro time delay is caused by gravitational potential ("traffic")

$$\Delta t_{\mathrm{Shapiro}} = -\frac{\Phi(\boldsymbol{\theta})}{c^2} \times \frac{D_{\mathrm{S}} D_{\mathrm{L}}}{D_{\mathrm{LS}}\, c}$$

- The Geometric time delay is caused by differences in path lengths ("route")

$$\Delta t_{\mathrm{Geometry}} = \frac{D_{\mathrm{L}} D_{\mathrm{S}}}{c D_{\mathrm{LS}}} \frac{(\boldsymbol{\theta} - \boldsymbol{\beta})^2}{2}$$

- For the point mass lens, the time delay between the two images is

$$t_+ - t_- \simeq -(1 + z_{\mathrm{L}}) \frac{D_{\mathrm{L}} D_{\mathrm{S}}}{c D_{\mathrm{LS}}} 2\theta_E \beta$$

- Time delays are useful for:
  - Confirming GR (Shapiro time delay & SN Refsdal)
  - Improving lens models (SN Refsdal)
  - Determining cosmological parameters, in particular H_0 (H0LiCOW)