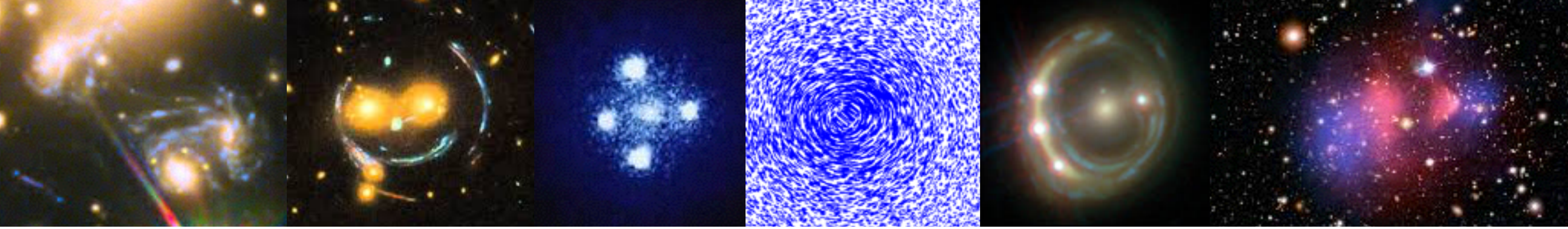


PHY-765 SS19 Gravitational Lensing Week 13

Cosmic Shear & the CMB

Kasper B. Schmidt
Leibniz-Institut für Astrophysik Potsdam (AIP)

Last week - what did we learn?

- Talked about the weak lensing shearing of objects
- Used Jacobian Matrix and assumptions about sphericity to see that
  - (simple) shearing corresponds to ellipticity

$$1 = \frac{(1-\gamma_1)^2}{\beta_0^2}\theta_1^2 + \frac{(1+\gamma_1)^2}{\beta_0^2}\theta_2^2$$

- Described the surface brightness moments ellipticity

$$q_{ij} \equiv \int d^2\theta \, S^{\rm obs}(\boldsymbol{\theta})\theta_i\theta_j \qquad \epsilon_1 \equiv \frac{q_{11} - q_{22}}{q_{11} + q_{22}} \qquad \epsilon_2 \equiv \frac{2q_{12}}{q_{11} + q_{22}}$$

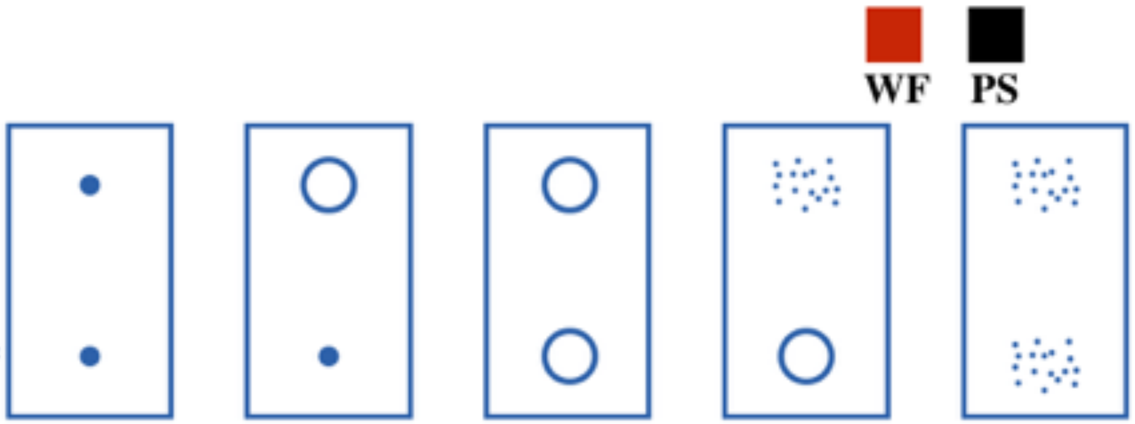
- Using the Jacobian Matrix this can be expressed in terms of κ and γ

$$\epsilon_i = \frac{2\gamma_i}{1-\kappa}\left[1 - \frac{\gamma^2}{(1-\kappa)^2}\right]^{-1}$$

- Considered challenges with determining weak lensing
  - Intrinsic ε, weighing of images, accounting for PSF, etc.
- The Bullet Cluster as a proof of the existence of dark matter

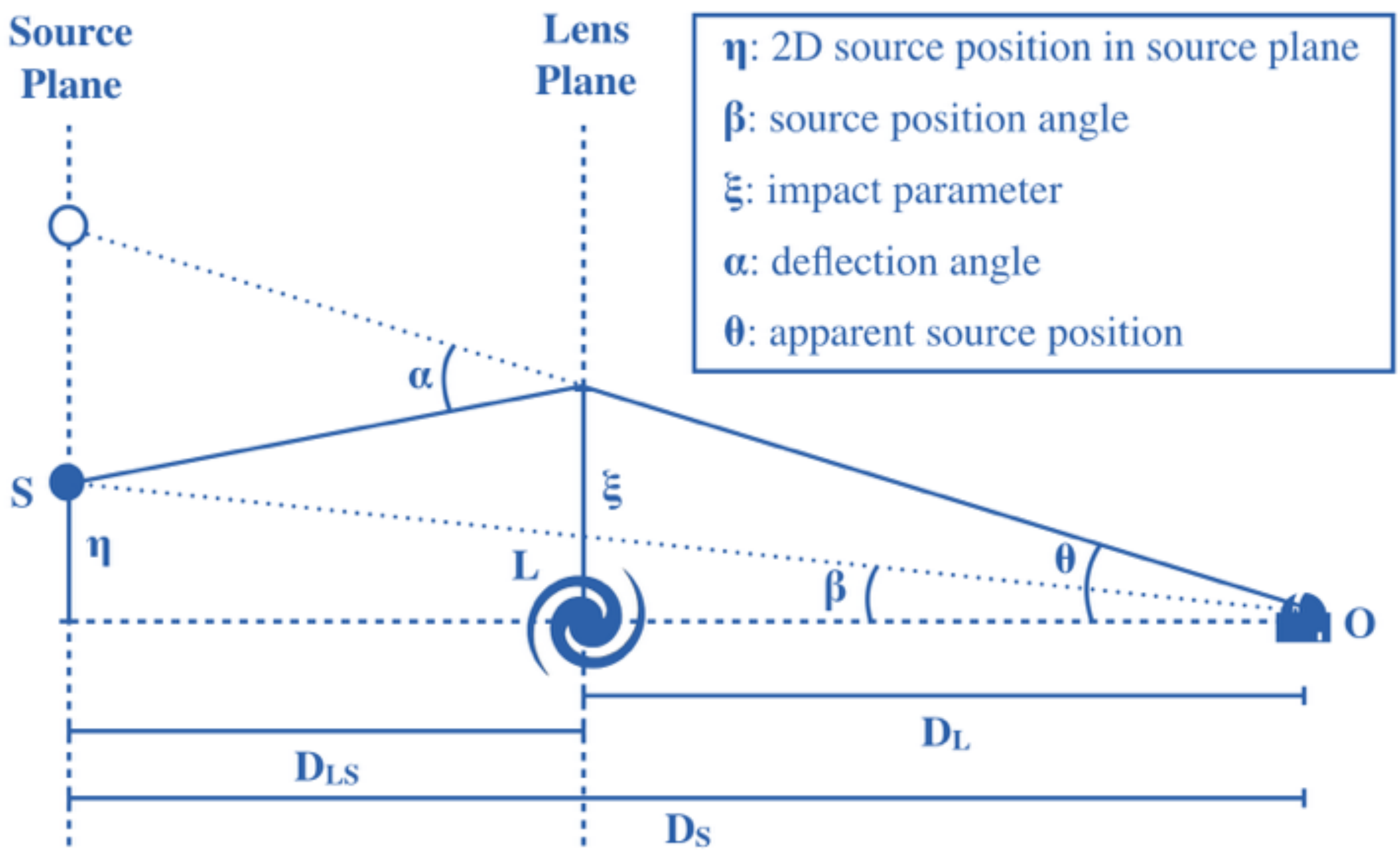
The aim of today

- Deflection of ‘diffuse mass’ by ‘diffuse mass’
- The concept of cosmic shear
- Fourier space description of cosmic shear lensing effects
- The power spectrum as a tool
- Lensing of the Cosmic Microwave Background



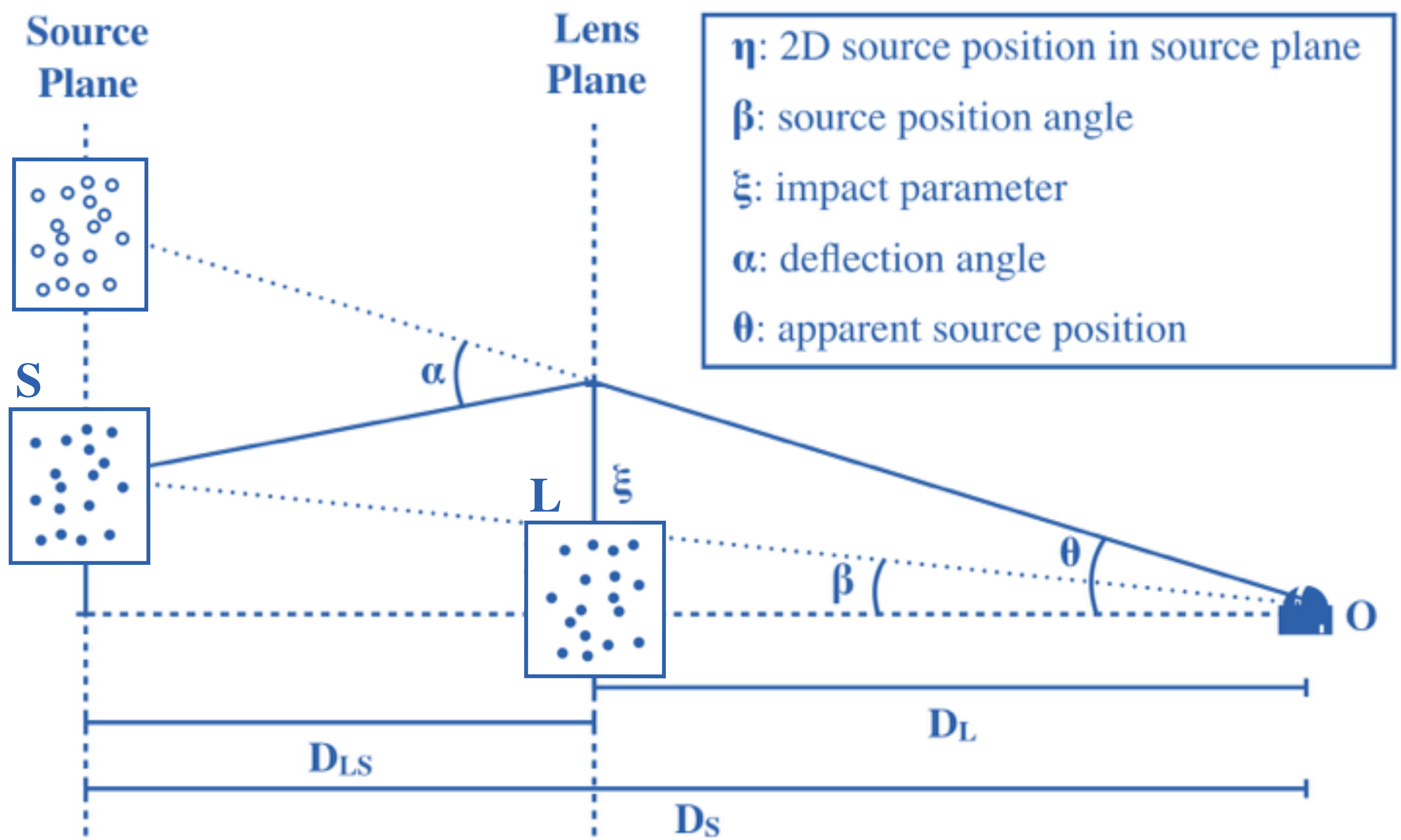
Light Deflection (Lensing) Regime

- Strong lensing: Concentrated source deflected by concentrated lens



Light Deflection (Lensing) Regime

- Strong lensing: Concentrated source deflected by concentrated lens
- Weak lensing: Concentrated source deflected by diffuse lens
- ‘Cosmic weak lensing’: Diffuse source deflected by diffuse lens



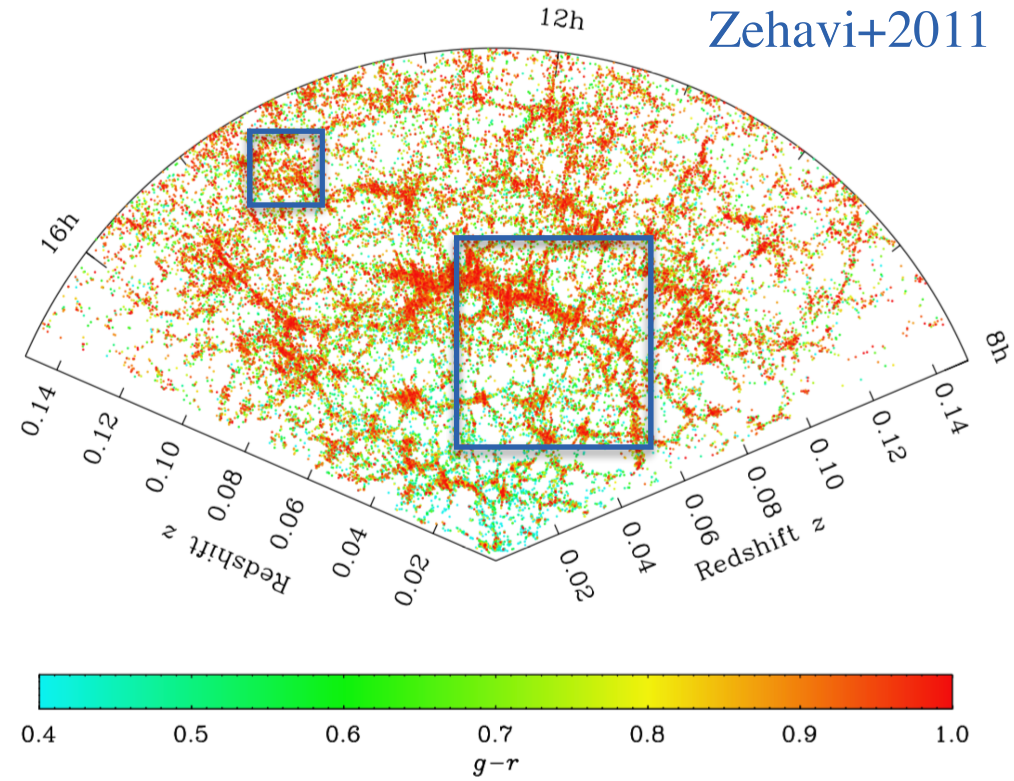
Cosmic Shear

- The large scale structure of the Universe
- Web of mass affecting background sources
- Lensing (shearing) from cosmic structure
- Assessment done in a statistical manner
- LSS predicted by cosmological models

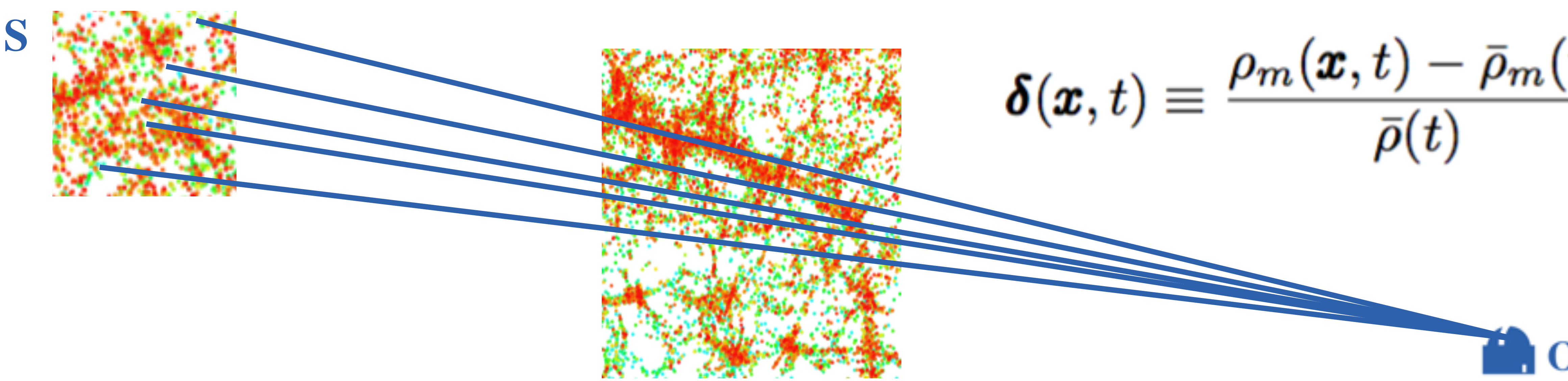
$$p(t) = \sum w \times \rho_w(t) \quad ; \quad w = m, r, \Lambda$$

$$\rho(t) = \rho_m(t) + \rho_r(t) + \rho_\Lambda(t)$$

Zehavi+2011



- Cosmic shear probes cosmology via the density contrast defined as



$$\boldsymbol{\delta}(\boldsymbol{x}, t) \equiv \frac{\rho_m(\boldsymbol{x}, t) - \bar{\rho}_m(t)}{\bar{\rho}(t)}$$

Shear and the Gravitational Potential

- The Jacobian relates κ and γ to the gravitational potential (week 6)

$$\mathcal{A}(\boldsymbol{\theta}) = \begin{pmatrix} 1 - \frac{\partial \alpha_i}{\partial \theta_i} & -\frac{\partial \alpha_i}{\partial \theta_j} \\ -\frac{\partial \alpha_j}{\partial \theta_i} & 1 - \frac{\partial \alpha_j}{\partial \theta_j} \end{pmatrix} = \begin{pmatrix} 1 - \frac{\partial^2 \psi}{\partial \theta_i^2} & -\frac{\partial^2 \psi}{\partial \theta_i \partial \theta_j} \\ -\frac{\partial^2 \psi}{\partial \theta_j \partial \theta_i} & 1 - \frac{\partial^2 \psi}{\partial \theta_j^2} \end{pmatrix}$$

$$\kappa \equiv \frac{1}{2}\left(\frac{\partial^2 \psi}{\partial \theta_i^2} + \frac{\partial^2 \psi}{\partial \theta_j^2}\right) \qquad \gamma_1 \equiv \frac{1}{2}\left(\frac{\partial^2 \psi}{\partial \theta_i^2} - \frac{\partial^2 \psi}{\partial \theta_j^2}\right)$$

$$\gamma_2 \equiv \frac{\partial^2 \psi}{\partial \theta_i \partial \theta_j}$$

- The projected gravitational potential along the line of sight is defined as

$$\Phi(\boldsymbol{\theta}) \equiv \frac{2}{D_\mathrm{S}} \int_0^{D_\mathrm{S}} dD_\mathrm{L}\, \phi\left(x^i = D_\mathrm{L}\theta^i, D_\mathrm{L}; t = t_0 - \frac{D_\mathrm{L}}{c}\right) \frac{D_\mathrm{LS}}{D_\mathrm{L}}$$

- Introduced in week 6 as $c^2\psi(\boldsymbol{\theta}) = \Phi(\boldsymbol{\theta})$

The Projected Gravitational Potential

Gives deflection angle as

$$\alpha^i(\boldsymbol{\theta}) = \frac{\partial \psi(\boldsymbol{\theta})}{\partial \theta^i}$$

Integrate along line of sight to source

Grav. potential evaluated at time t when photons pass the lens plane D_L

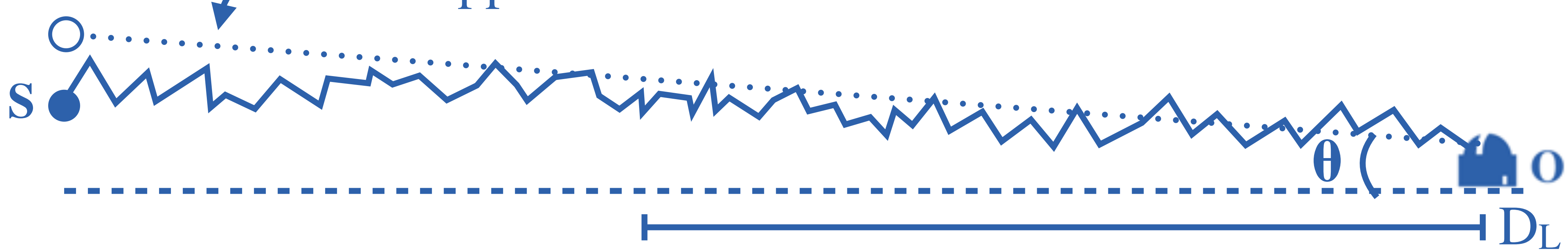
$$\Phi(\boldsymbol{\theta}) \equiv \frac{2}{D_\mathrm{S}} \int_0^{D_\mathrm{S}} dD_\mathrm{L}\, \phi\left(x^i = D_\mathrm{L}\theta^i, D_\mathrm{L}; t = t_0 - \frac{D_\mathrm{L}}{c}\right) \frac{D_\mathrm{LS}}{D_\mathrm{L}}$$

Newtonian grav. potential

x^i is the (2D) spatial component of four vector $x^\alpha = (t, x, y, z)$ (week 2)

Grav. potential sampled at transverse distance $D_\mathrm{L}\theta^i$

Born Approximation



- Gravitational potential directly relates to the density contrast via the Poisson equation:

$$\nabla^2 \Phi(\boldsymbol{x}, t) = \frac{4\pi G}{c^2} \bar{\rho}_m(t)\, a^2(t)\, \delta(\boldsymbol{x}, t)$$

$$\frac{1}{a(t)} = \frac{a(t_\mathrm{obs})}{a(t_\mathrm{emit})} = (1 + z)$$

$$H(t) \equiv \dot{a}(t)/a(t)$$

Matter Distribution Described in Fourier Space

- As γ relates to the gravitational potential it relates to the density contrast
- We are interested in the statistics of these density fluctuations
- Convenient to perform such investigations in Fourier space
- Any field can be expressed in terms of its Fourier transform:

$$f(x) = \int \frac{dk}{2\pi} \, \tilde{f}(k) e^{ikx}$$

- In 2D taking κ from Fourier ($\boldsymbol{l}$) space to real ($\boldsymbol{\theta}$) space would then be

$$\kappa(\boldsymbol{\theta}) = \int \frac{d^2 l}{(2\pi)^2} \, \tilde{\kappa}(\boldsymbol{l}) e^{i\boldsymbol{l}\cdot\boldsymbol{\theta}}$$

- and you can go back with

$$\tilde{\kappa}(\boldsymbol{l}) = \int d^2\theta \, \kappa(\boldsymbol{\theta}) e^{-i\boldsymbol{l}\cdot\boldsymbol{\theta}}$$

- Using the relations between κ, γ and Φ, you get Fourier space expressions:

$$\tilde{\kappa}(\boldsymbol{l}) = \frac{-l^2}{2c^2} \, \tilde{\Phi}(\boldsymbol{l}) \qquad \tilde{\gamma}_1(\boldsymbol{l}) = \frac{-l_x^2 + l_y^2}{2c^2} \, \tilde{\Phi}(\boldsymbol{l}) \qquad \tilde{\gamma}_2(\boldsymbol{l}) = \frac{-l_x l_y}{c^2} \, \tilde{\Phi}(\boldsymbol{l})$$

Matter Distribution Described in Fourier Space

- As γ relates to the gravitational potential it relates to the density contrast
- We are interested in the statistics of these density fluctuations
- Convenient to perform such investigations in Fourier space
- Any field can be expressed in terms of its Fo

$$f(x) = \int \frac{dk}{2\pi}\, \tilde{f}(k)$$

- In 2D taking κ from Fourier ($\boldsymbol{l}$) space to real

$$\kappa(\boldsymbol{\theta}) = \int \frac{d^2 l}{(2\pi)^2}\, \tilde{\kappa}($$

- and you can go back with

$$\tilde{\kappa}(\boldsymbol{l}) = \int d^2\theta\, \kappa(\boldsymbol{\theta})$$

$$\kappa \equiv \frac{1}{2}\left(\frac{\partial^2\psi}{\partial\theta_i^2} + \frac{\partial^2\psi}{\partial\theta_j^2}\right)$$

$$\gamma_1 \equiv \frac{1}{2}\left(\frac{\partial^2\psi}{\partial\theta_i^2} - \frac{\partial^2\psi}{\partial\theta_j^2}\right)$$

$$\gamma_2 \equiv \frac{\partial^2\psi}{\partial\theta_i\partial\theta_j}$$

- Using the relations between κ, γ and Φ, you get Fourier space expressions:

$$\tilde{\kappa}(\boldsymbol{l}) = \frac{-l^2}{2c^2}\,\tilde{\Phi}(\boldsymbol{l}) \qquad \tilde{\gamma}_1(\boldsymbol{l}) = \frac{-l_x^2 + l_y^2}{2c^2}\,\tilde{\Phi}(\boldsymbol{l}) \qquad \tilde{\gamma}_2(\boldsymbol{l}) = \frac{-l_x l_y}{c^2}\,\tilde{\Phi}(\boldsymbol{l})$$

- In Fourier space, real-space derivatives appear as powers of conjugate, $\boldsymbol{l}$

The Two-Point Correlation Function

- Combining these three equations gives expression of Fourier convergence

$$\tilde{\boldsymbol{\kappa}}(\boldsymbol{l}) = \frac{(l_x^2 - l_y^2)\tilde{\gamma}_1(\boldsymbol{l}) + 2l_x l_y \tilde{\gamma}_2(\boldsymbol{l})}{l^2}$$ (Exercise 2)

  - *l*=0 corresponds to a Fourier mode with no variation, i.e., a constant
  - So up to some constant value of *κ* this expression holds (MSD week 11)

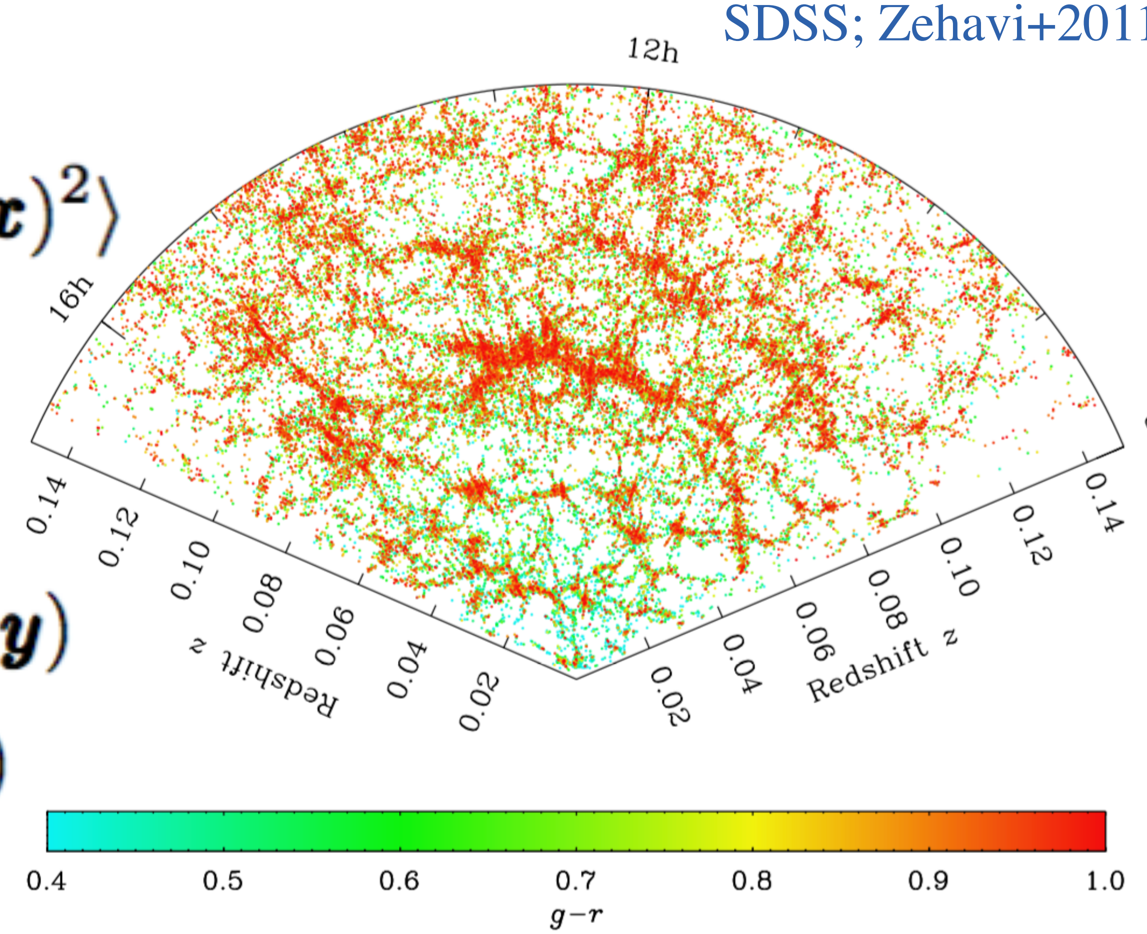
- We want to describe the matter density field statistically:
  - 1st order statistic (mean): $\langle \delta(\boldsymbol{x}) \rangle = 0$
  - 2nd order statistic (variance) : $\sigma^2 \equiv \langle \delta(\boldsymbol{x})^2 \rangle$

- Define the two point correlation as

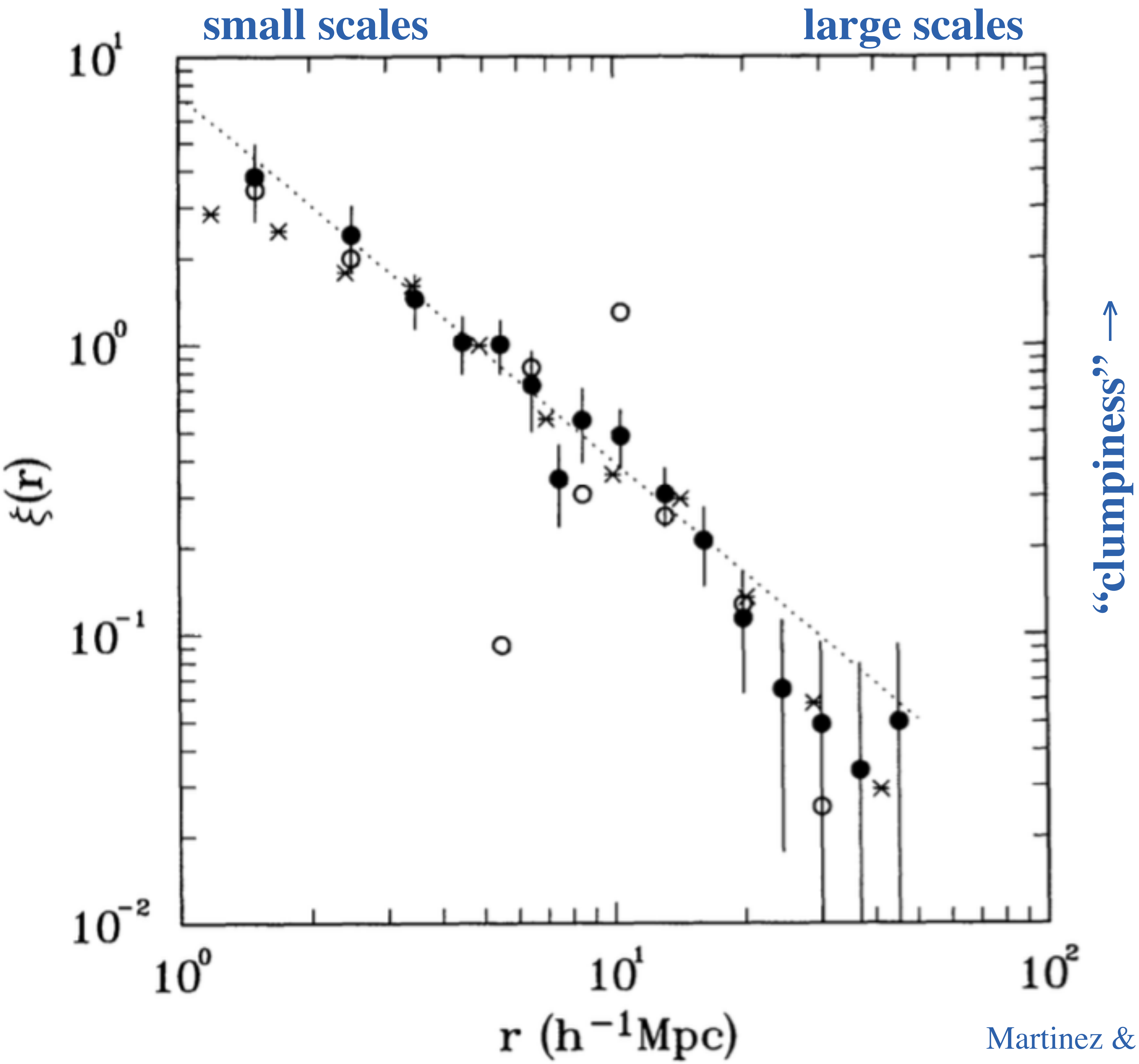
$$\xi(\boldsymbol{x}, \boldsymbol{y}) \equiv \langle \delta(\boldsymbol{x}) \delta(\boldsymbol{y}) \rangle$$

- Then from homogeneity: $\xi(\boldsymbol{x}, \boldsymbol{y}) = \xi(\boldsymbol{x} - \boldsymbol{y})$
- And from isotropy: $\xi(\boldsymbol{x} - \boldsymbol{y}) = \xi(|\boldsymbol{x} - \boldsymbol{y}|)$

SDSS; Zehavi+2011



The Two-Point Correlation Function



Martinez & Coles (1994)

The Power Spectrum

- The Fourier transform of the correlation function, is the Power Spectrum

$$\langle\tilde{\delta}(\boldsymbol{k})\tilde{\delta}(\boldsymbol{k'})\rangle = \int d^3x e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \int d^3y e^{-i\boldsymbol{k'}\cdot\boldsymbol{y}} \xi(\boldsymbol{x}-\boldsymbol{y})$$

$$\ldots = (2\pi)^3\delta^3_{\rm Dirac}(k-k')P(k) \quad \text{for} \quad \boldsymbol{x}_- = \boldsymbol{x}-\boldsymbol{y}$$

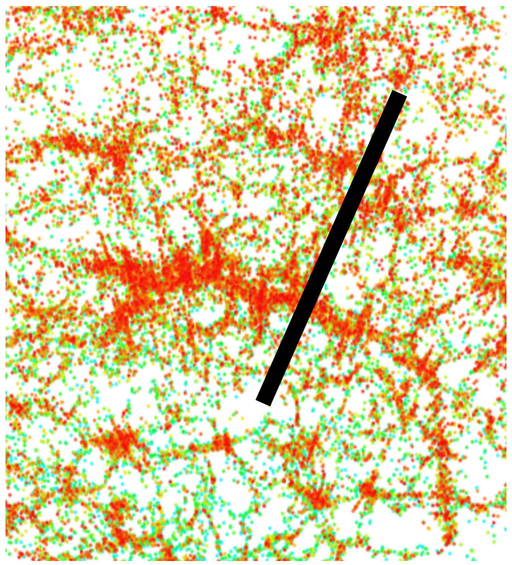
with

$$P(|\boldsymbol{k}|) \equiv \int d^3x_- e^{i\boldsymbol{k}\cdot\boldsymbol{x}_-}\xi(\boldsymbol{x}_-)$$

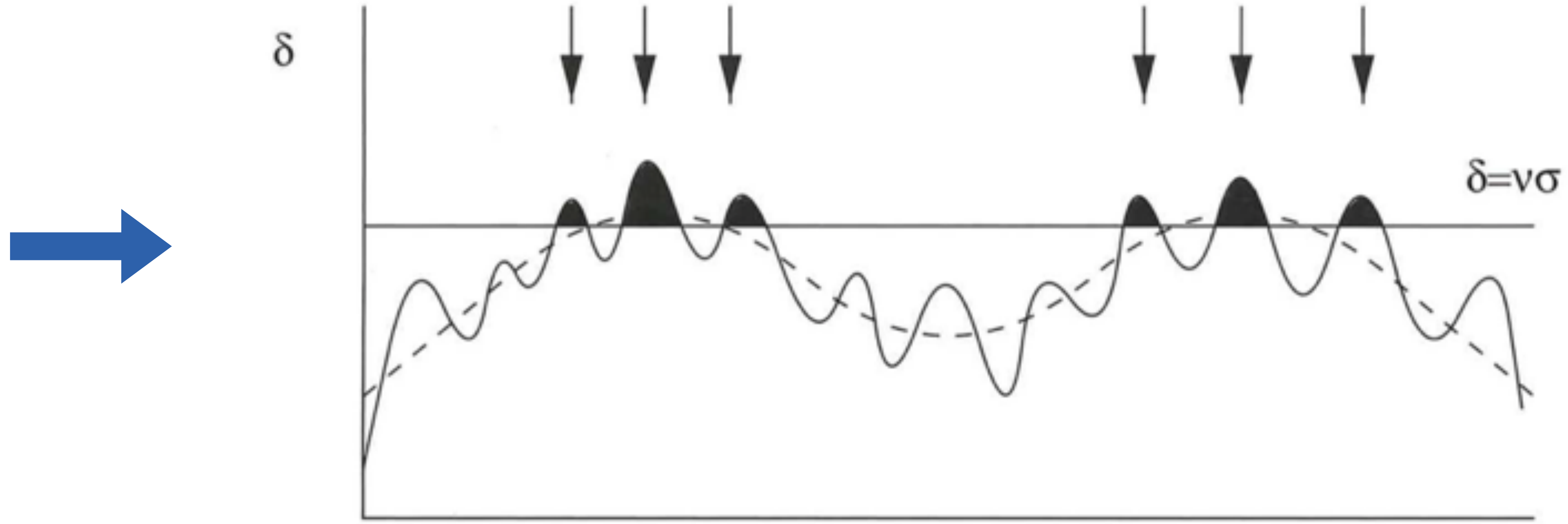
(Exercise 3)

- Describes the scales ($1/k$) at which there is "power" in the density field

2D



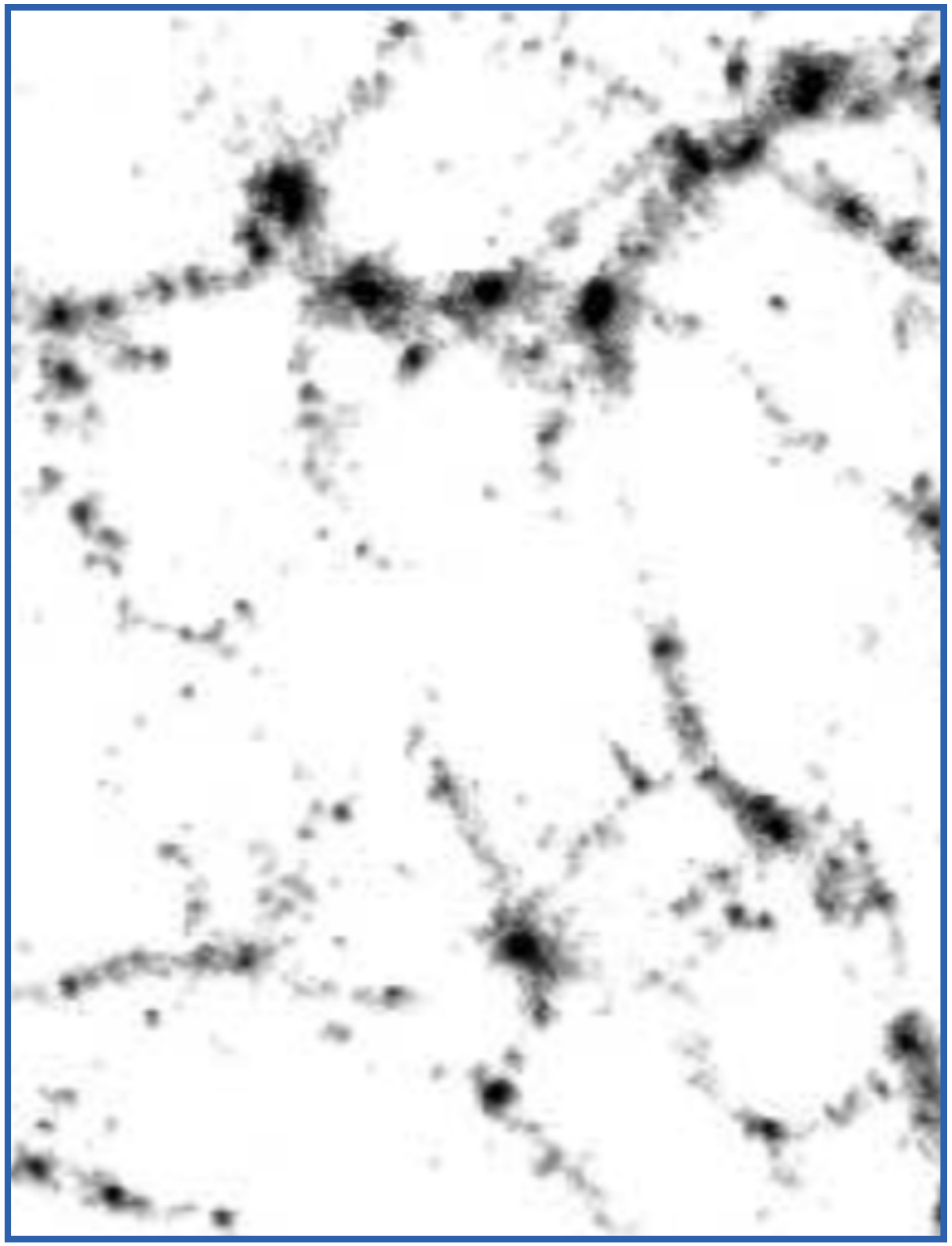
1D



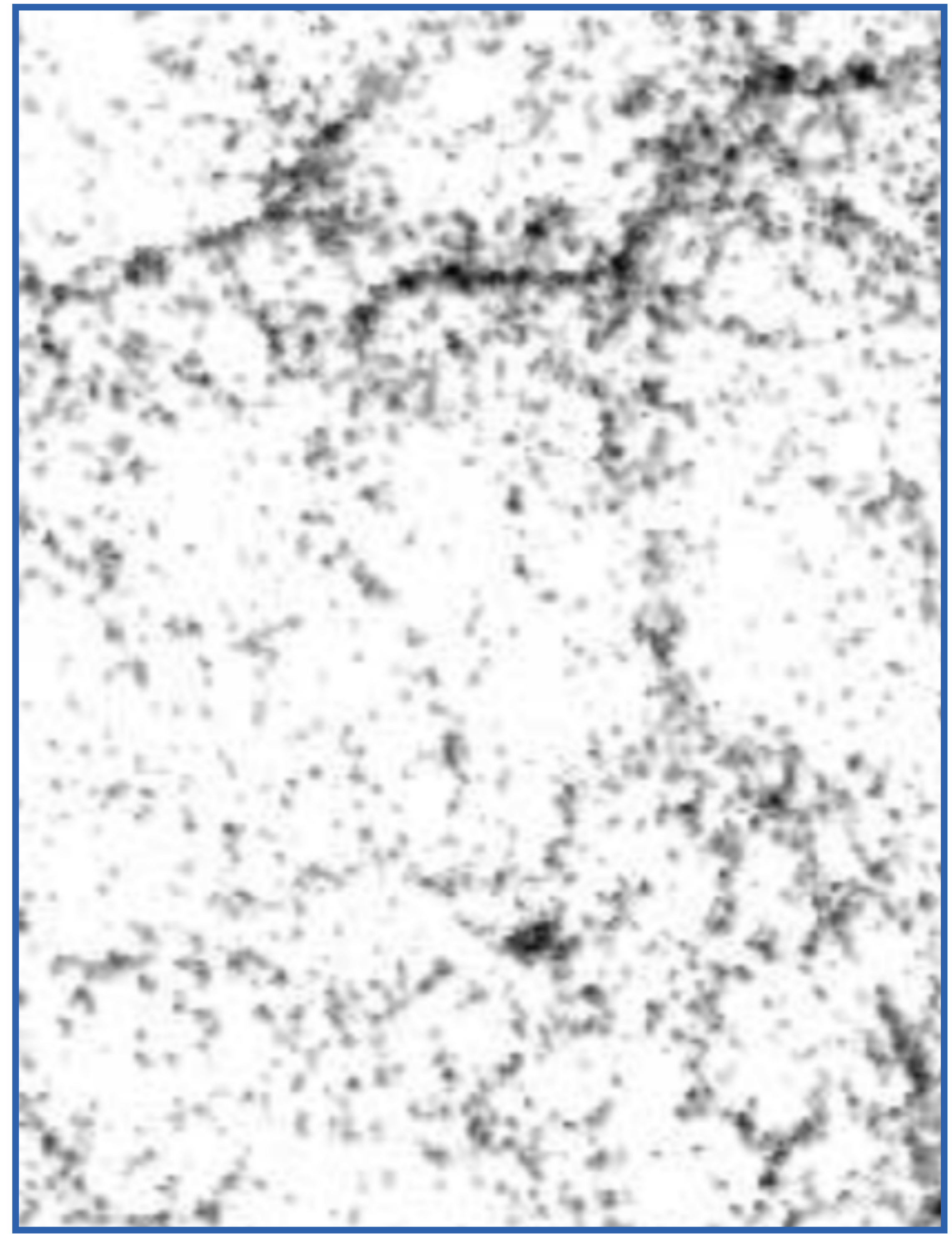
Peacock (2003)

The Power Spectrum

Large-scale Power



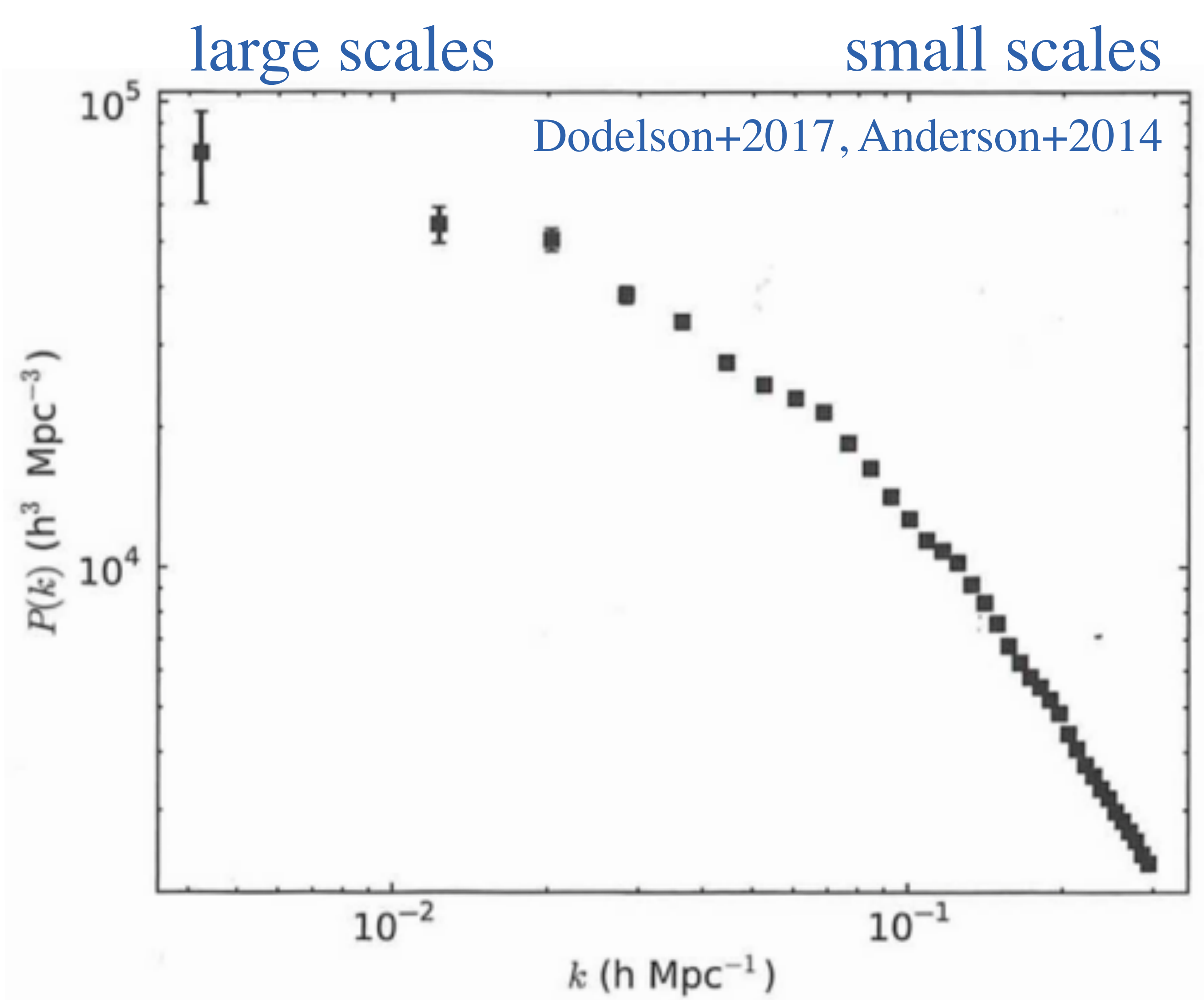
Large&Small-scale Power

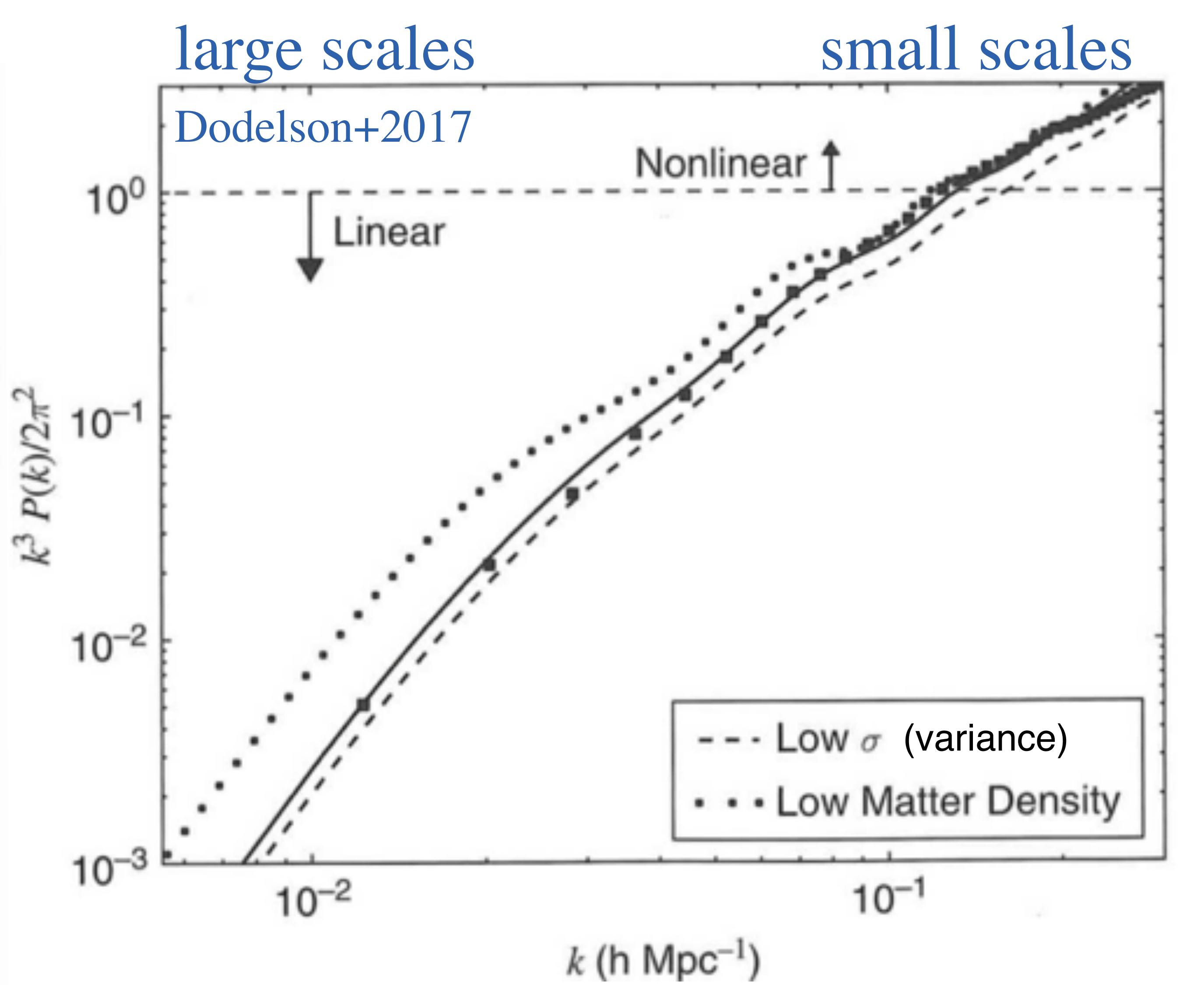


Peacock (2003)

The Power Spectrum

- Baryon Oscillation Spectroscopic Survey (BOSS) power spectrum
  - P(k) hides the expected correlation (power) at small scales
- Power spectrum needs to be unit-less to reveal correlation
  - Scaled by dimensionality; in this case k^3





Cosmic Shear Decomposition

- Considering the shear components in Fourier space

$$\tilde{\gamma}_1(\boldsymbol{l}) = \frac{-l_x^2 + l_y^2}{2c^2}\,\tilde{\Phi}(\boldsymbol{l}) \qquad \tilde{\gamma}_2(\boldsymbol{l}) = \frac{-l_x l_y}{c^2}\,\tilde{\Phi}(\boldsymbol{l})$$

- Then defining an angle ϕ that $\boldsymbol{l}$ makes with the (arbitrary) x-axis we have

$$\tilde{\gamma}_1(\boldsymbol{l}) = -\frac{l^2\tilde{\Phi}(\boldsymbol{l})}{2c^2}\cos(2\phi) \quad \text{as} \quad \cos(2\phi) = \cos^2(\phi) - \sin^2(\phi)$$

$$\tilde{\gamma}_2(\boldsymbol{l}) = -\frac{l^2\tilde{\Phi}(\boldsymbol{l})}{2c^2}\sin(2\phi) \quad \text{as} \quad \sin(2\phi) = 2\cos(\phi) + \sin(\phi)$$

- Considering linear combinations of the shear components we arrive at

$$-\tilde{\gamma}_1(\boldsymbol{l})\cos(2\phi) - \tilde{\gamma}_2(\boldsymbol{l})\sin(2\phi) = \frac{-l^2}{2c^2}\,\tilde{\Phi}(\boldsymbol{l}) = \tilde{\kappa}(\boldsymbol{l})$$

$$\tilde{\gamma}_1(\boldsymbol{l})\sin(2\phi) - \tilde{\gamma}_2(\boldsymbol{l})\cos(2\phi) = 0$$

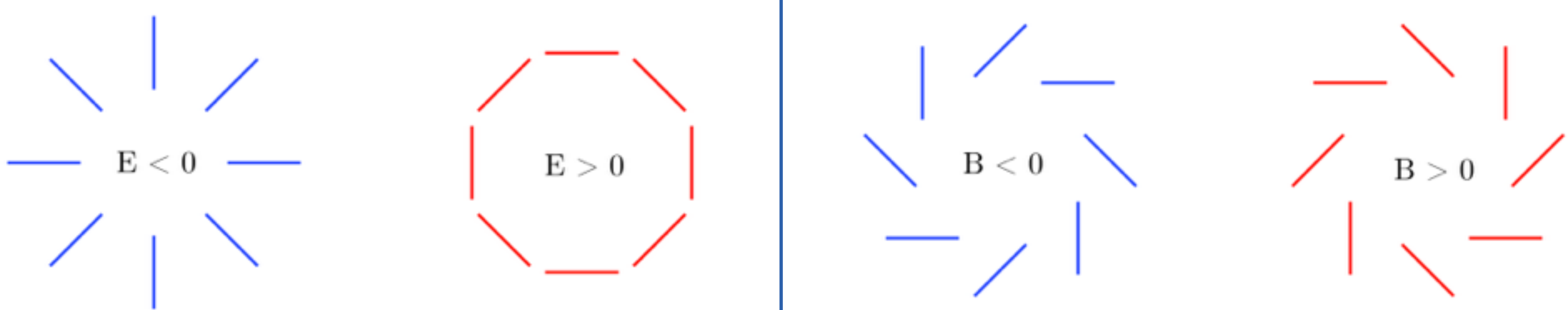
Polarization of the Lensing Shear Field

$$\tilde{E} \equiv -\tilde{\gamma}_1(\boldsymbol{l})\cos(2\phi) - \tilde{\gamma}_2(\boldsymbol{l})\sin(2\phi) = \frac{-l^2}{2c^2}\,\tilde{\Phi}(\boldsymbol{l})$$

$$\tilde{B} \equiv \tilde{\gamma}_1(\boldsymbol{l})\sin(2\phi) - \tilde{\gamma}_2(\boldsymbol{l})\cos(2\phi) = 0$$

- So any lensing survey has to prove:
  - *E*-mode ≠ 0
  - *B*-mode consistent with 0 (noise and mass screen can produce B = 0)

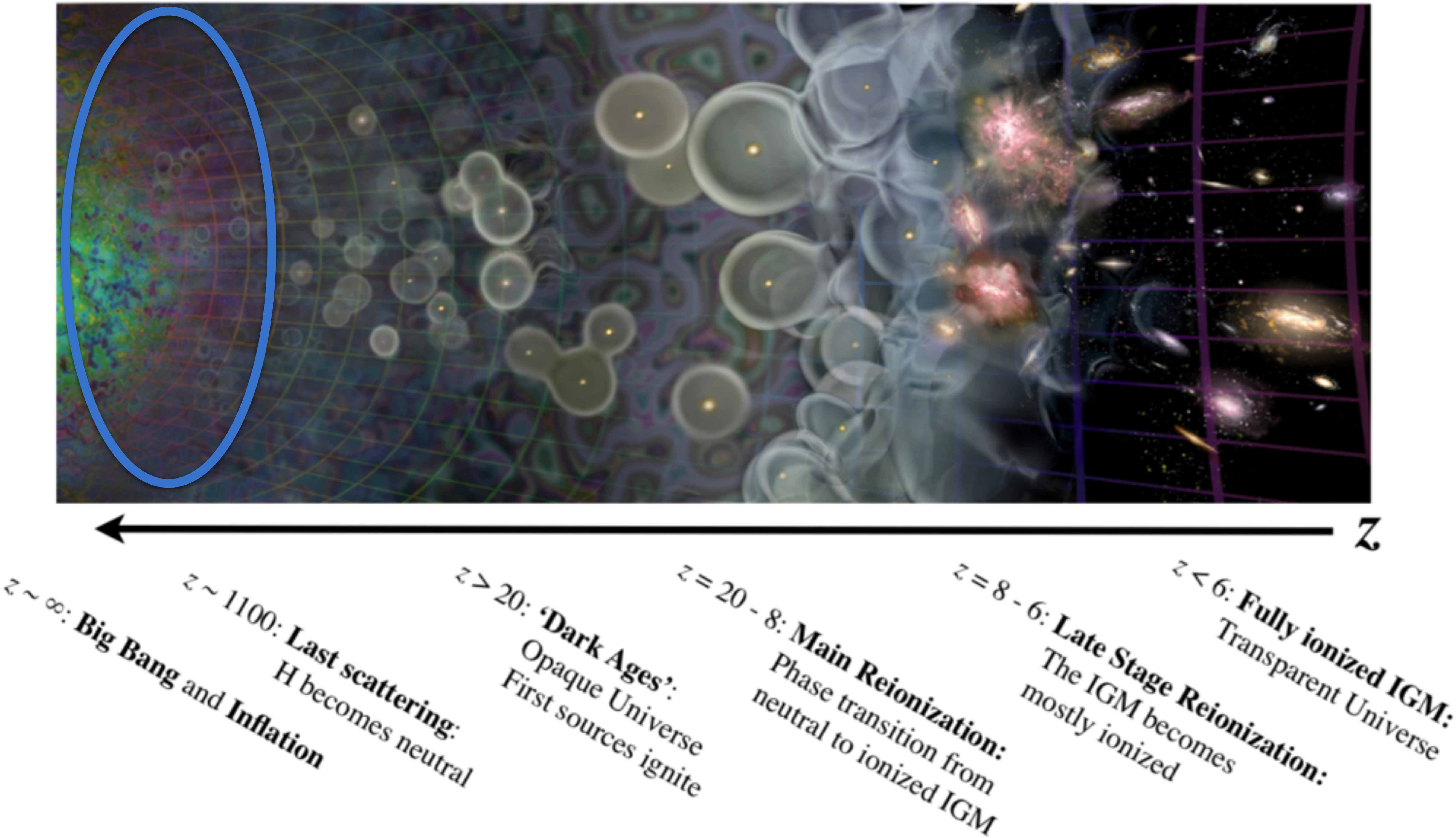
E-mode (gradient) | B-mode (curl)



Underdensity | Overdensity

Krauss+2010

The Cosmic Microwave Background



Schmidt 2016, Loeb 2006

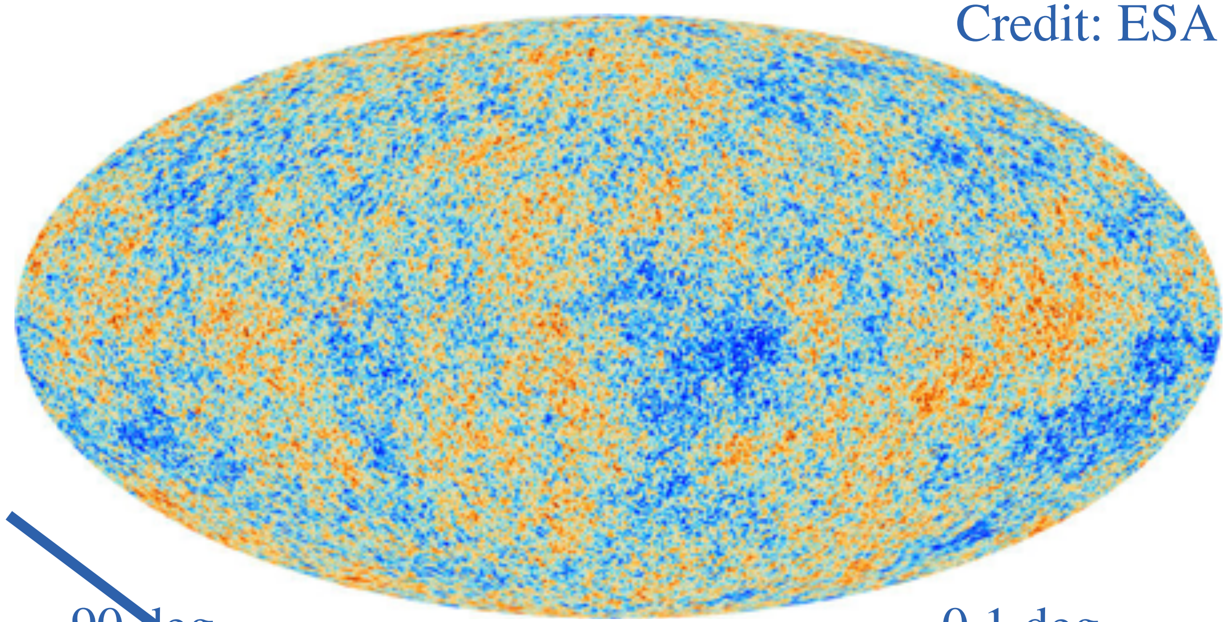
The Cosmic Microwave Background

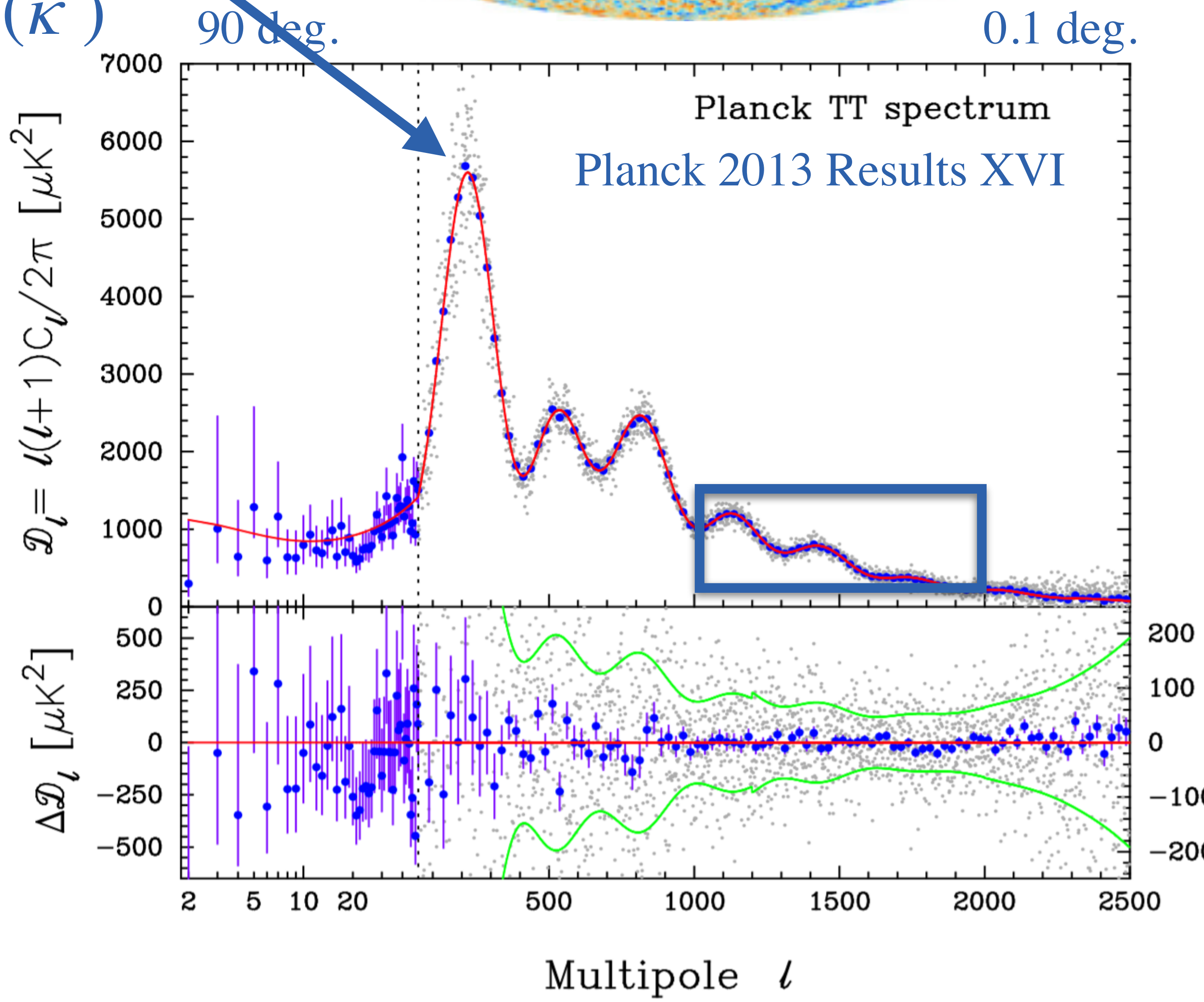
- Black body with T = 2.725K
- Peak (maximum power) at 160Ghz in the microwave today
  - But has been redshifted from universe expansion
- At recombination T~3000K ($z \sim 1100$)
  - CMB is a snapshot of Universe when the photons started traveling freely
- Surface of last scattering - the edge of the observable Universe
- Temperature contrast, i.e., the CMB temperature fluctuations are $\delta_T \sim 10^{-4}$

The Cosmic Microwave Background

- Map shows temperature fluctuations
- Power spectrum describes their oscillations around average density
- Hot and cold spots are of the order 1deg
- Shape sensitive to lensing potential (κ)

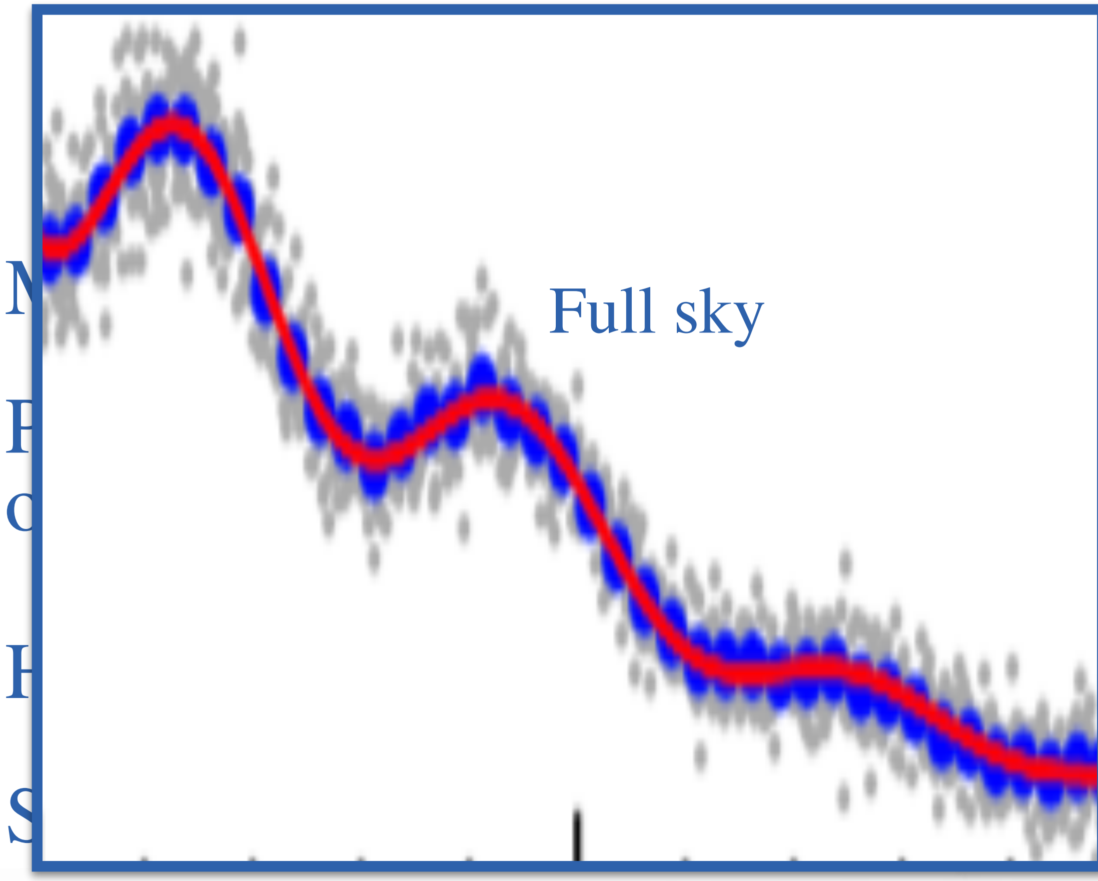
Credit: ESA

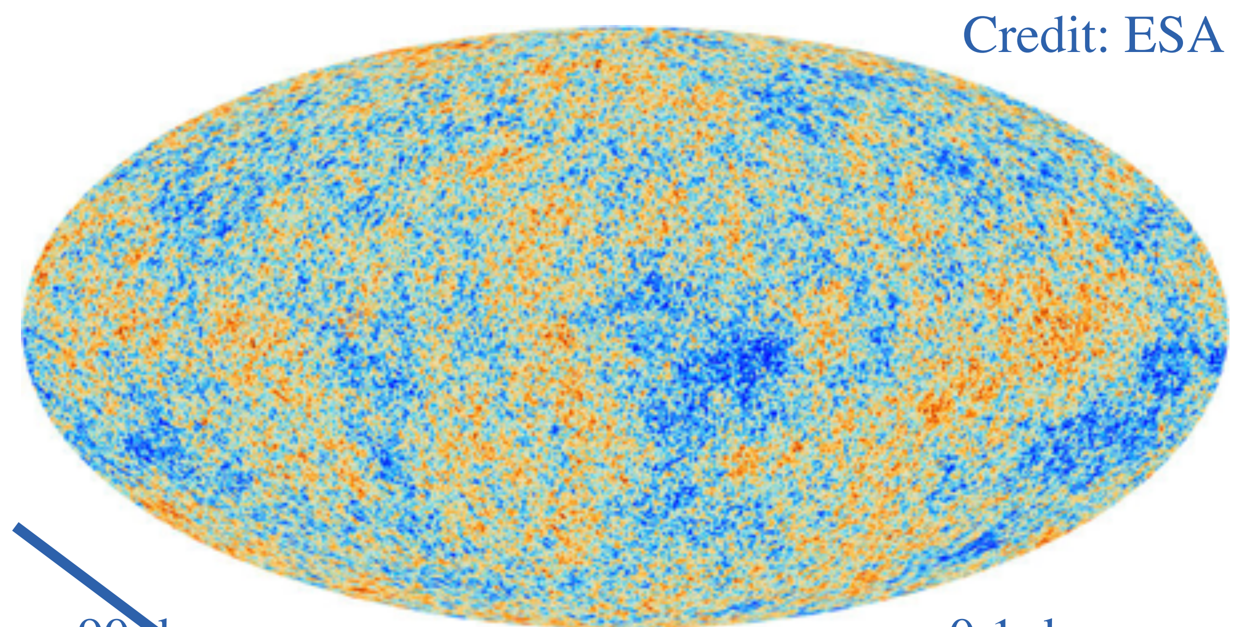


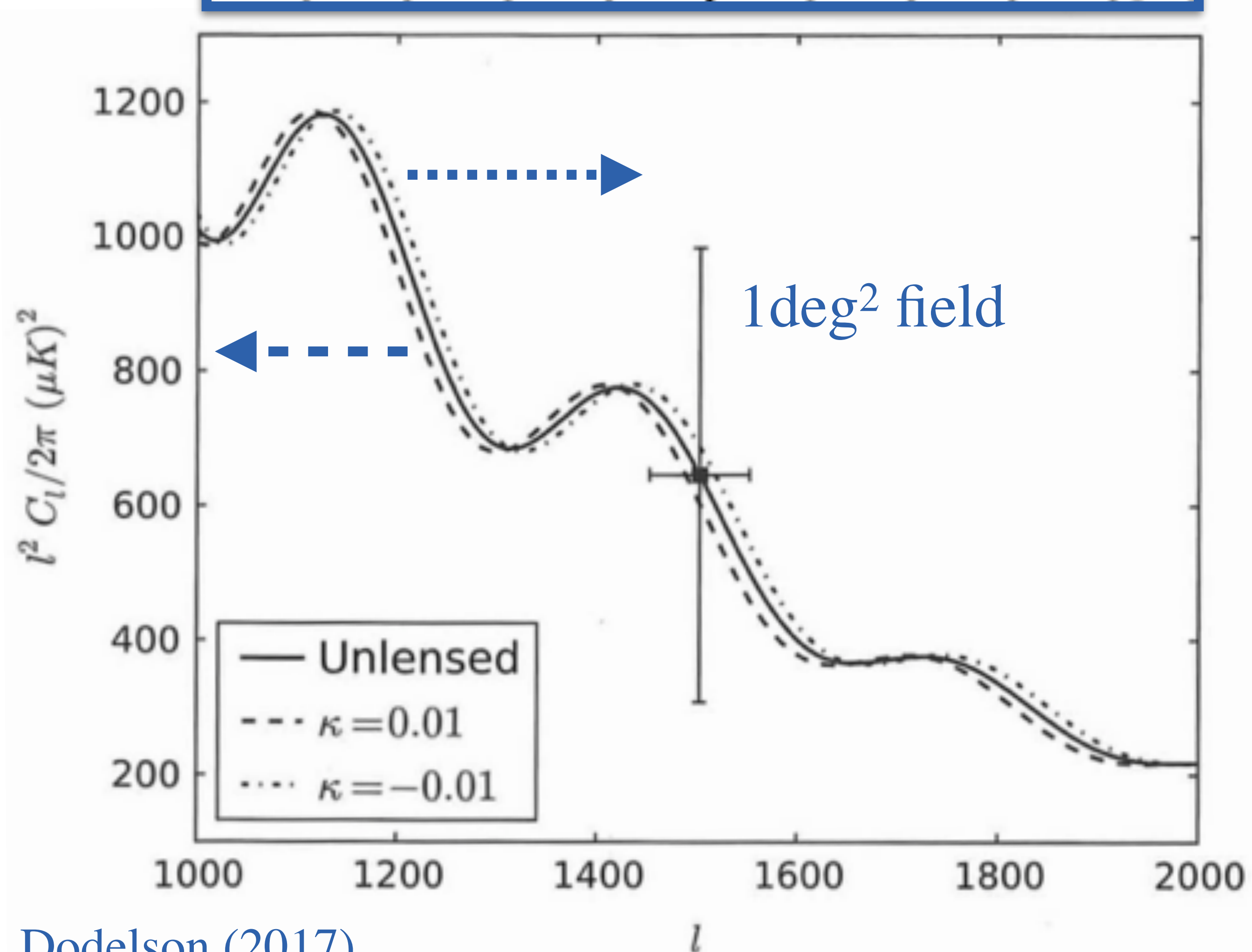


rowave Background

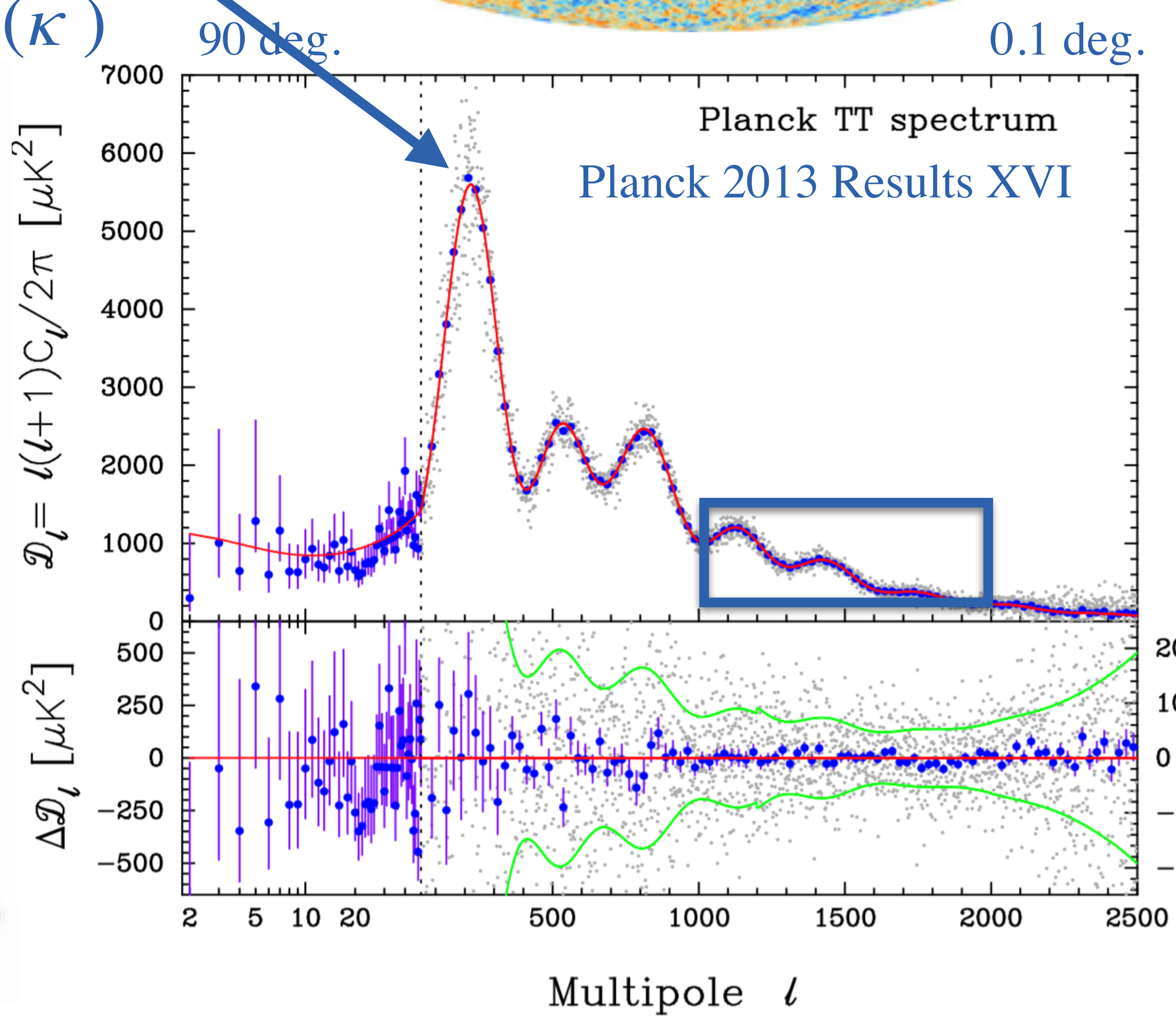
- N ns
- F
  c y
- F 1deg
- S l (κ)





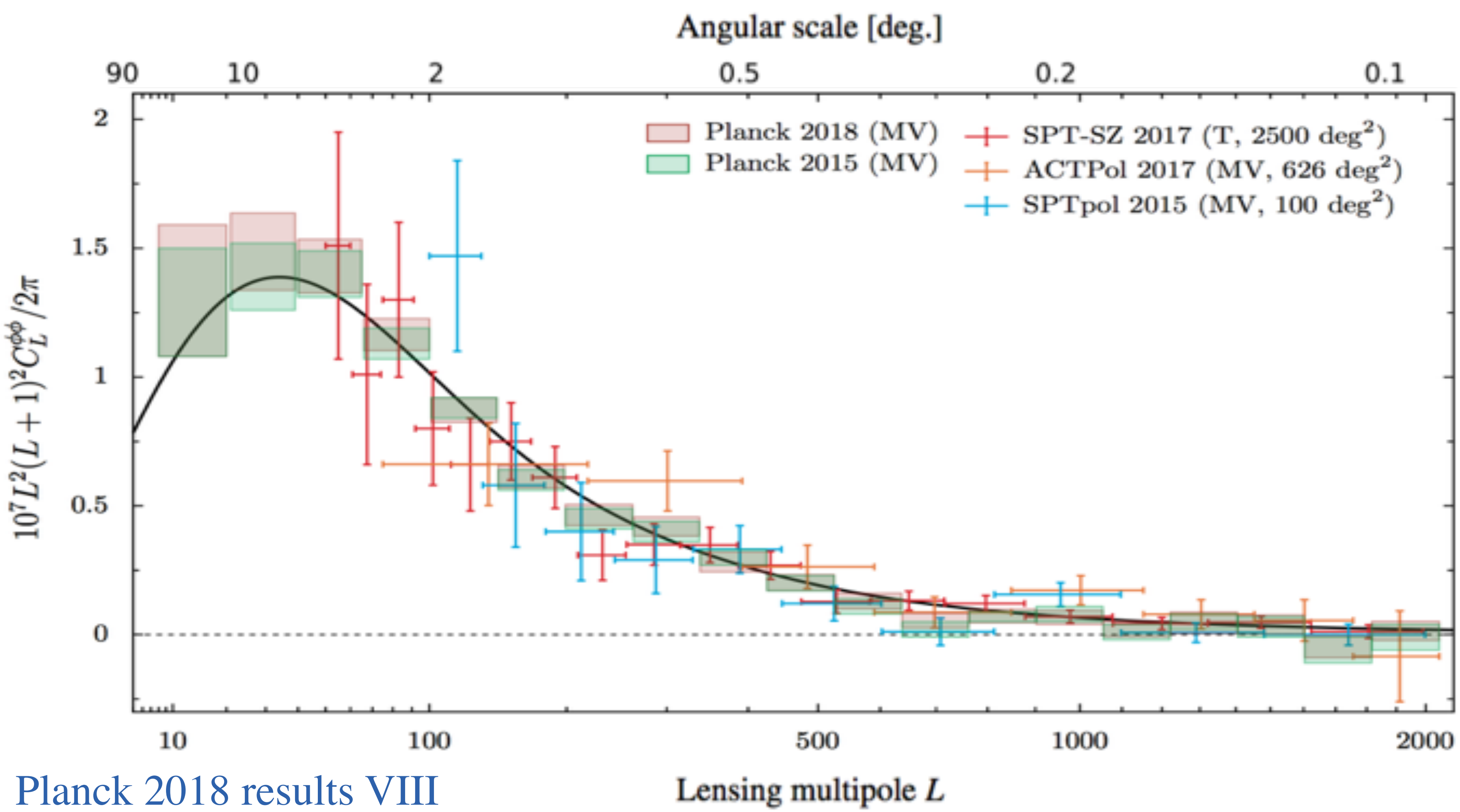


Dodelson (2017)



Lensing Potential from CMB

- Relate T-T CMB map to the gravitational potential, Φ
- As we have seen Φ is related to δ, κ, γ_1, and γ_2
  - Hence a lensing power spectrum (ϕ-ϕ instead of T-T) can be estimated
- Measured lensing fluctuations (few deg.) due to density fluctuations (~1')



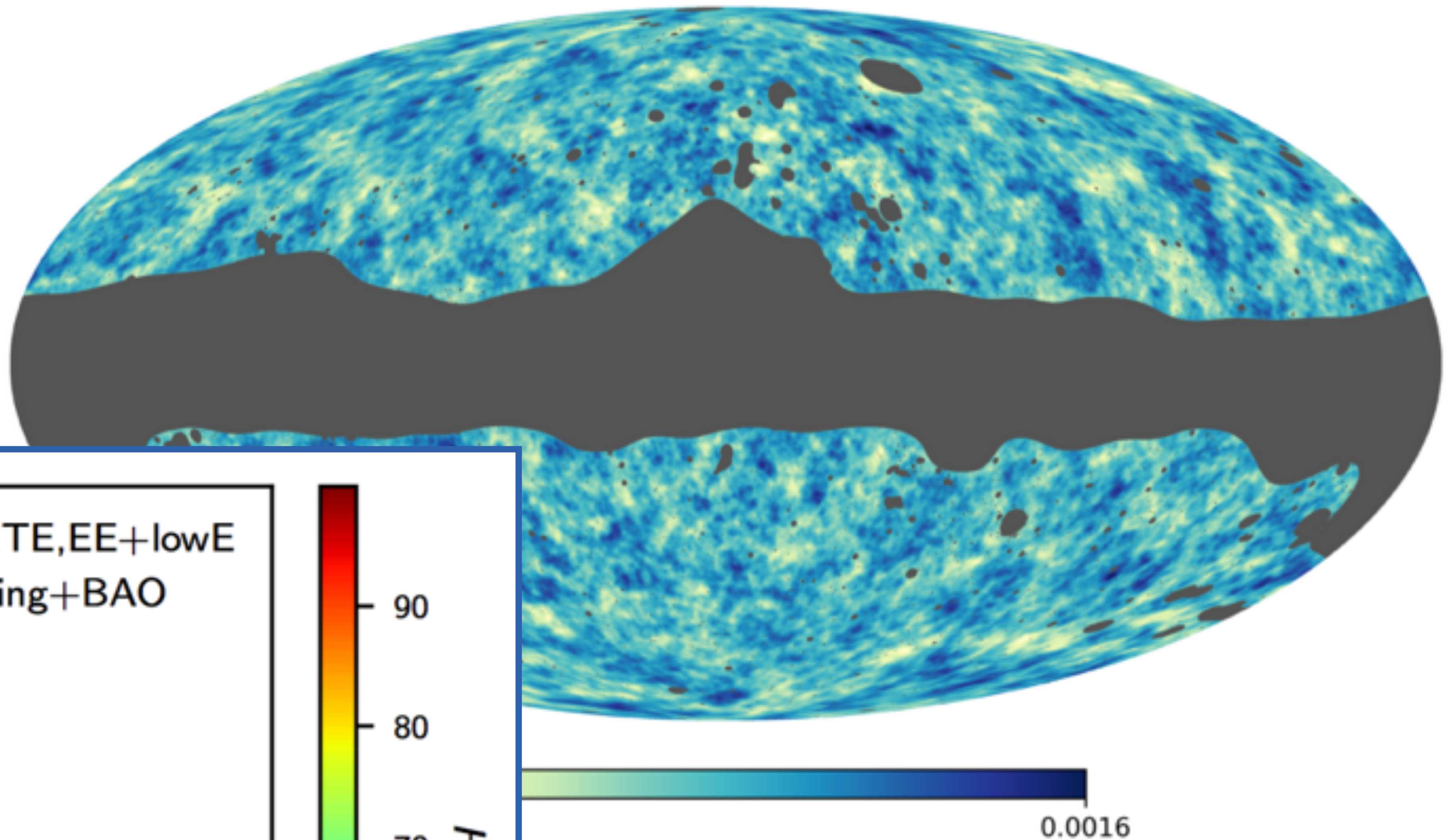
Planck 2018 results VIII

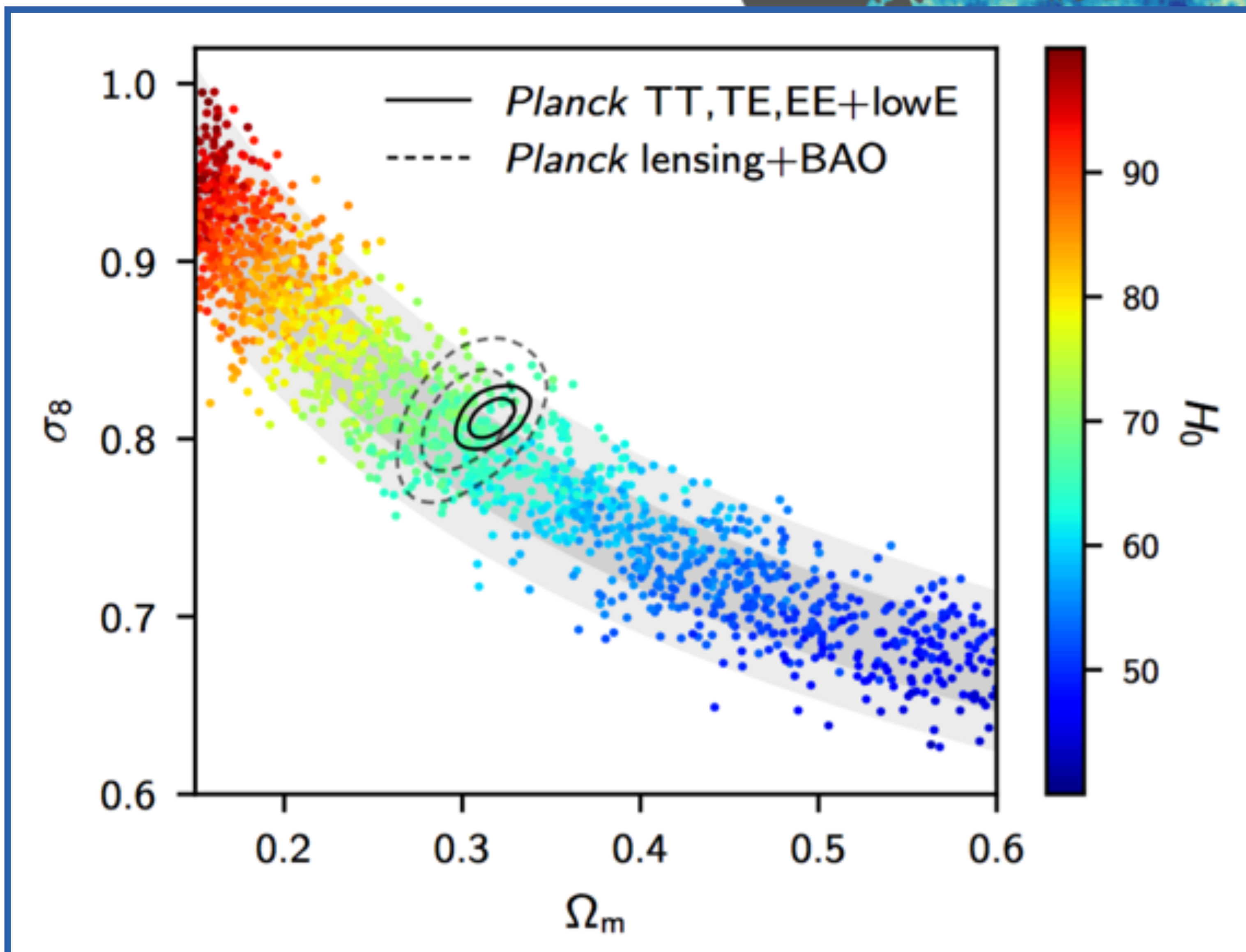
Lensing Potential from CMB

Planck 2018 results VIII:
E-mode lensing map
and constraints on
cosmological parameters

$$\rho(t) = \rho_m(t) + \rho_r(t) + \rho_\Lambda(t)$$

$$H(t) \equiv \dot{a}(t)/a(t)$$



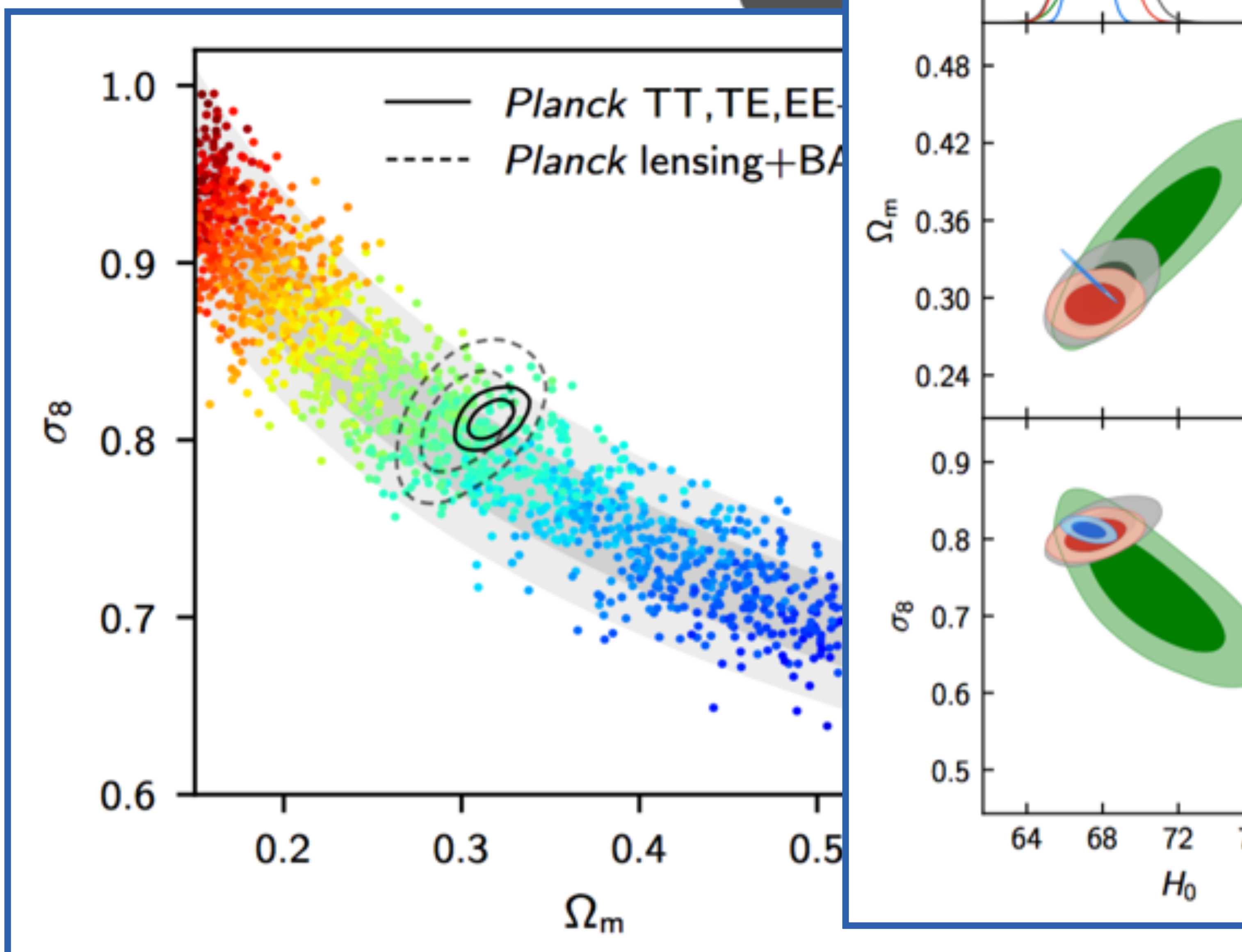


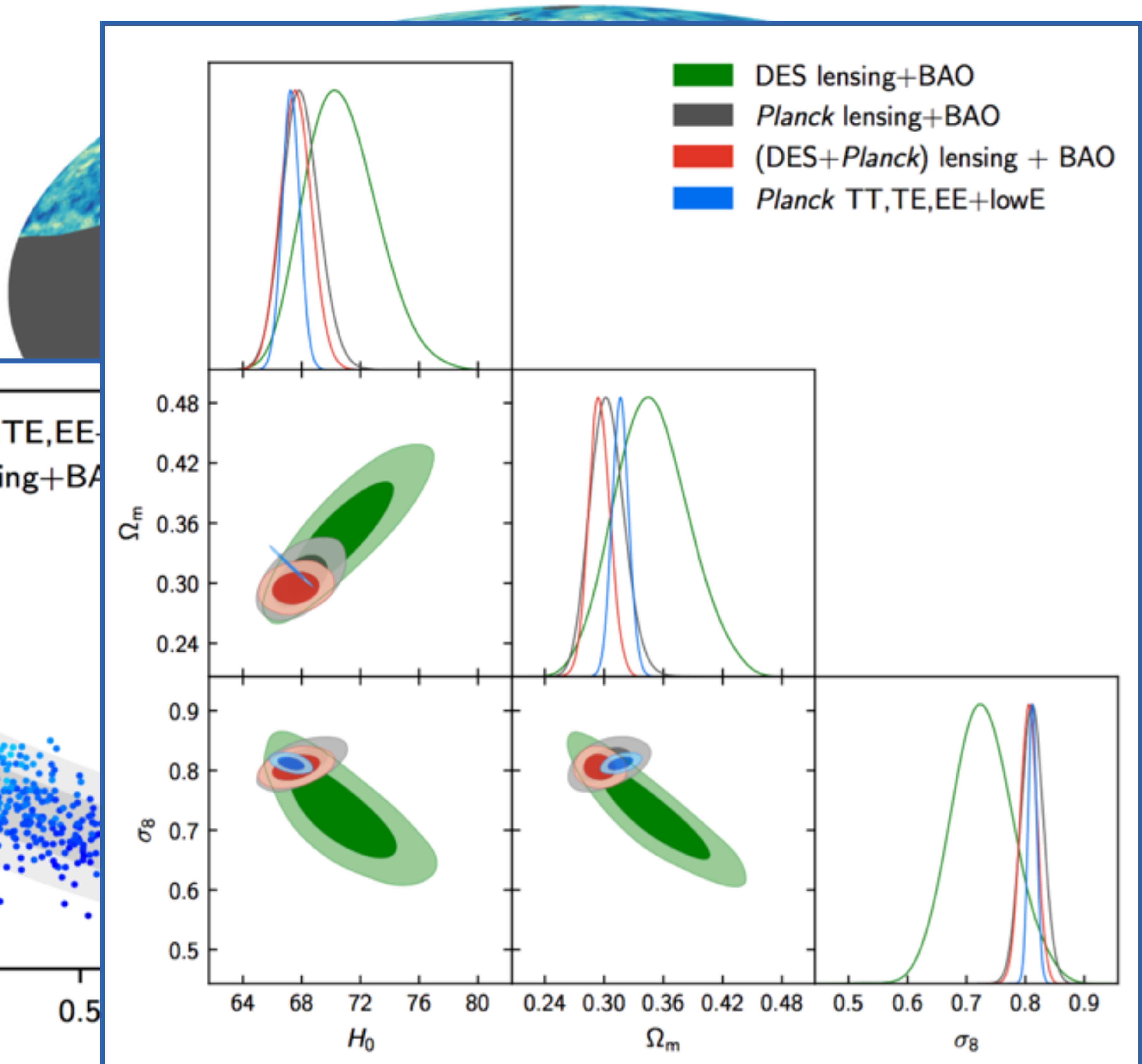
Lensing Potential from CMB

Planck 2018 results VIII:
E-mode lensing map
and constraints on
cosmological parameters

$$\rho(t) = \rho_m(t) + \rho_r(t) + \rho_\Lambda(t)$$

$$H(t) \equiv \dot{a}(t)/a(t)$$





So in summary…

- The cosmic energy density maps are lensed by matter along line of sight
  - Lensing of diffuse source by diffuse lens
- The density contrast, δ, is related to κ and γ via the gravitational potential

$$\boldsymbol{\delta}(\boldsymbol{x}, t) \longleftrightarrow \Phi(\boldsymbol{x}, t) \longleftrightarrow \psi(\boldsymbol{\theta}) \longleftrightarrow \kappa \quad \gamma_1 \quad \gamma_2$$

- The variance (2-point correlation function) of δ provides statistic on pattern
- In Fourier space, this results in the Power Spectrum
  - Describing (size) scales containing most power, i.e., correlation
- E and B mode decomposition of γ are useful sanity checks of results
- Cosmic Microwave Background T maps provide information about
  - Cosmology via T-T Power Spectrum
  - Lensing potential (matter distribution) via ϕ-ϕ Power Spectrum
- Probes the cosmological parameters of the Universe