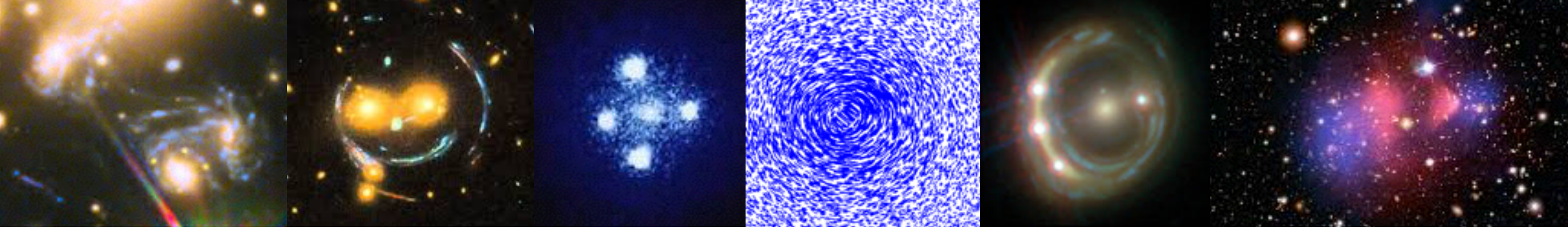


Weak Gravitational Lensing

Kasper B. Schmidt
Leibniz-Institut für Astrophysik Potsdam (AIP)

Last week - what did we learn?

- Lens Models:
  - Goal of a model is to minimize disagreement with observations

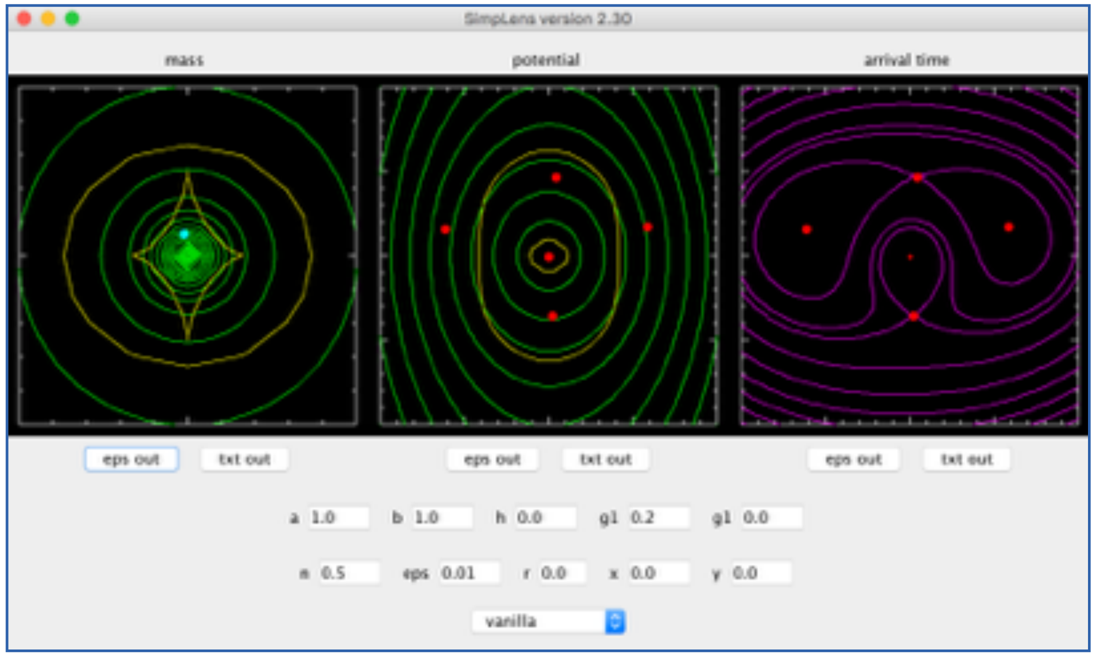
Parametric

$$\chi^2_{\mathrm{img}} = \sum_i \left(\frac{\boldsymbol{\theta}_i(\boldsymbol{\beta}) - \boldsymbol{\theta}_i}{\sigma_i} \right)^2$$

Non-Parametric

$$\chi^2 = \frac{\left| \mathbf{S}_I - P(\mathrm{PSF},\ \mathrm{lens\ model})\ \mathbf{S}_{\mathrm{source\ plane}} \right|^2}{\sigma^2}$$

- Iterative modeling "by hand" with SimpLens



- The Mass Sheet Degeneracy:
  - 'classes of models' provide equally good fit
  - broken with multiple z-systems or indepen- mass estimate (e.g. velocity dispersion)

- Recent attempts at comparing and sharing lens modeling codes
  - SN Refsdal re-appearance predictions and comparisons; Treu et al. (2016)
  - Comparison of predictions on mock clusters; Meneghetti et al (2017)

The aim of today

- Describe weak gravitational lensing
- The ideal spherical case of shearing images
- Relation between ellipticity and the gravitational potential (κ & γ)
- A few examples of effects from imaging to account for
- The Bullet Cluster - a spectacular DM proof from strong and weak lensing

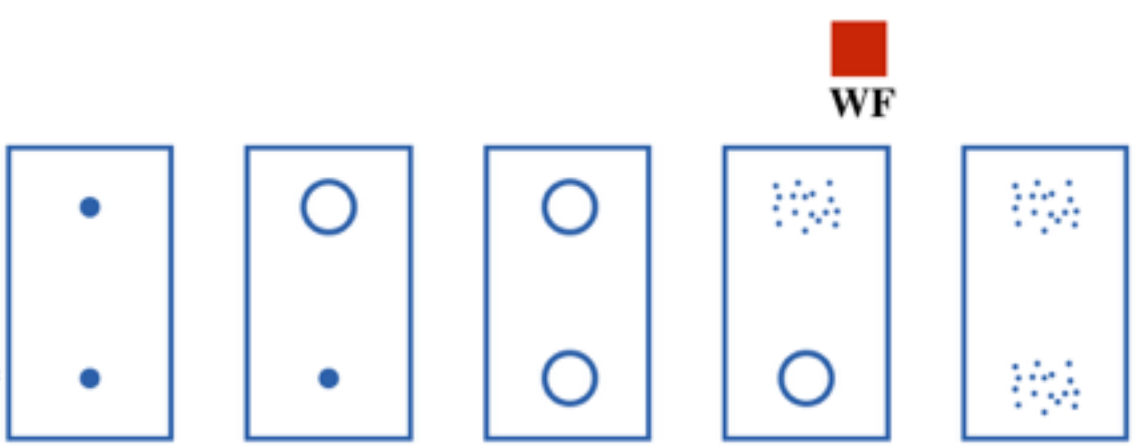


Image Distortions - κ and γ

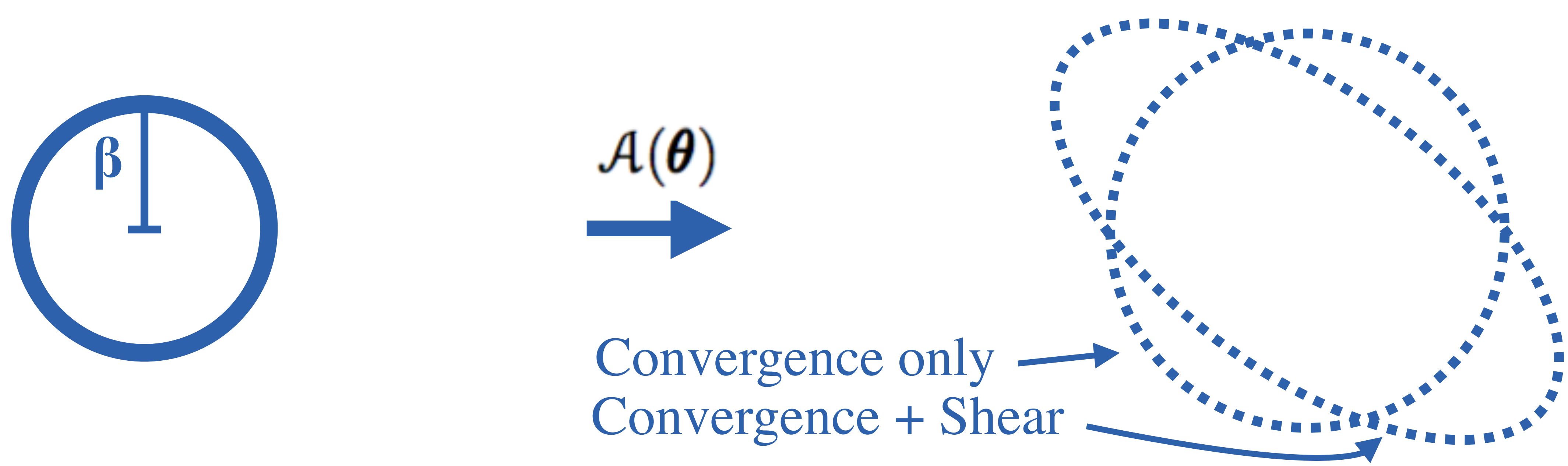
- Image distortions can be described by the Jacobian matrix (week 7)

$$\mathcal{A}(\boldsymbol{\theta}) = \frac{\partial \boldsymbol{\beta}}{\partial \boldsymbol{\theta}} = \begin{pmatrix} \frac{\partial \beta_i}{\partial \theta_i} & \frac{\partial \beta_i}{\partial \theta_j} \\ \frac{\partial \beta_j}{\partial \theta_i} & \frac{\partial \beta_j}{\partial \theta_j} \end{pmatrix} = \begin{pmatrix} 1 - \frac{\partial \alpha_i}{\partial \theta_i} & -\frac{\partial \alpha_i}{\partial \theta_j} \\ -\frac{\partial \alpha_j}{\partial \theta_i} & 1 - \frac{\partial \alpha_j}{\partial \theta_j} \end{pmatrix} = (\delta_{ij} - \boldsymbol{\Psi}_{ij})$$

$$\boldsymbol{\Psi}_{ij} \equiv \begin{pmatrix} \kappa + \gamma_1 & \gamma_2 \\ \gamma_2 & \kappa - \gamma_1 \end{pmatrix} \qquad \mu = \frac{1}{(1-\kappa)^2 - \gamma^2} \qquad \gamma^2 \equiv \gamma_1^2 + \gamma_2^2$$

Source Plane

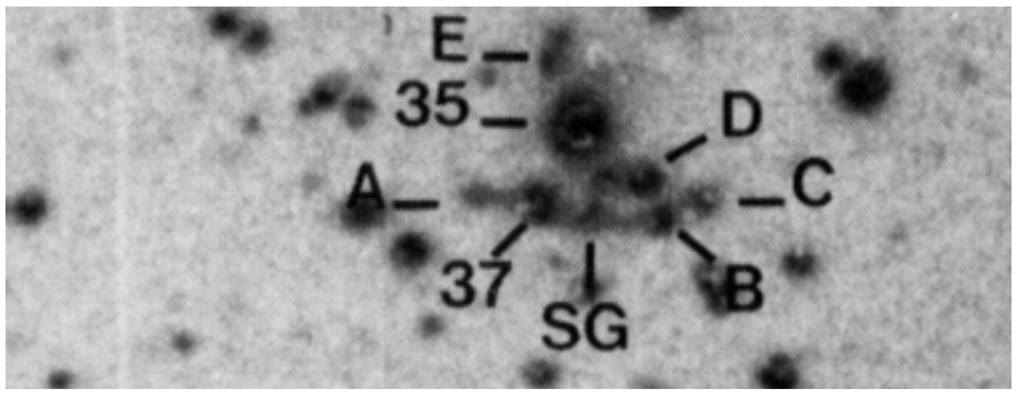
Lens (Image) Plane



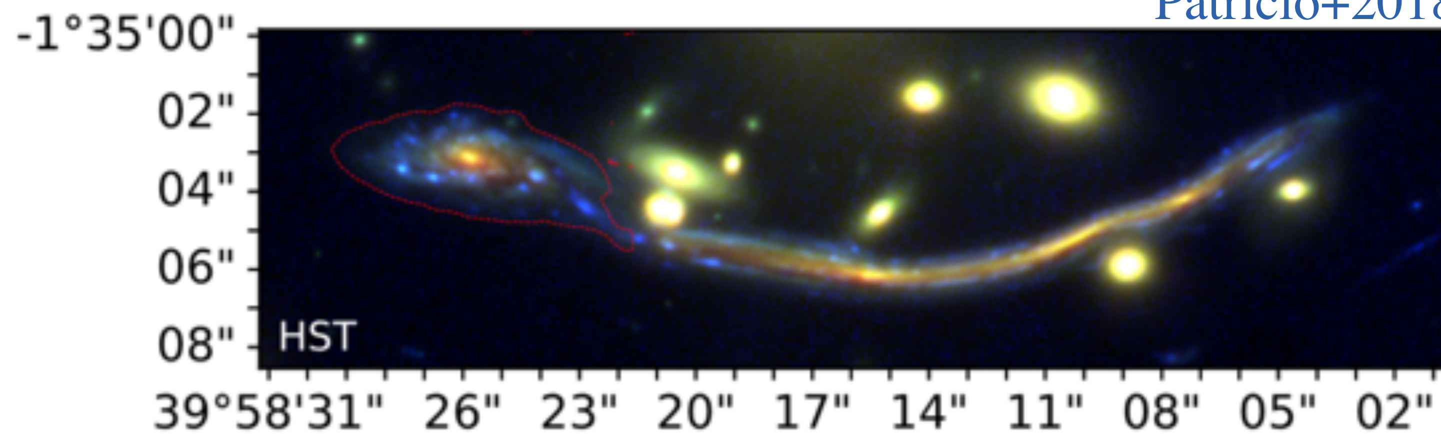
Weak Lensing

- Already in the early days of lensing, arcs were observed, e.g. in A370

Soucail+1987



Patricio+2018



- Clear that this effect, i.e. shearing of images must happen on all scales
- Weak lensing is the regime where the distortions become small

$$\mathcal{A}(\boldsymbol{\theta}) = (\delta_{ij} - \boldsymbol{\Psi}_{ij}) \qquad\qquad \mathcal{A}(\boldsymbol{\theta}) \rightarrow \delta_{ij}$$

- To measure the effect, statistical approaches need to be applied
  - Collect as many galaxies as possible → go faint (dig into the noise)
  - Digging into the noise requires good handle of telescope and obs. effects
- Photographic plates not linear, so not possible before the CCD (early 90s)

Idealized Weak Lensing

- Consider an un-lensed spherical object with radius β_0
- If $\kappa = 0$, $\gamma_2 = 0$ and $\gamma_1 \neq 0$ (but small) then $\mathcal{A}(\boldsymbol{\theta}) = \begin{pmatrix} 1-\gamma_1 & 0 \\ 0 & 1+\gamma_1 \end{pmatrix}$
- So the projection from source (β) to image (θ) plane is

$$\frac{d\beta_1}{d\theta_1} = 1 - \gamma_1 \qquad \frac{d\beta_2}{d\theta_2} = 1 + \gamma_1$$

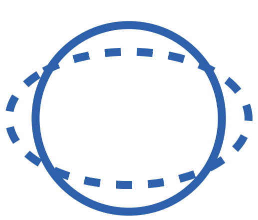
- Solving and adding in quadrature gives an ellipse on the form $1 = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

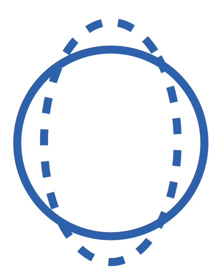
$$1 = \frac{(1-\gamma_1)^2}{\beta_0^2}\theta_1^2 + \frac{(1+\gamma_1)^2}{\beta_0^2}\theta_2^2$$

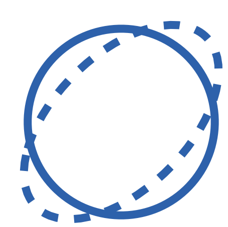
- If γ_2 is the only non-zero component the distortion is given by

$$1 = \frac{(\theta_1 - \gamma_2\theta_2)^2}{\beta_0^2} + \frac{(\theta_2 - \gamma_2\theta_1)^2}{\beta_0^2}$$

(Exercise 2)

$\gamma_1 > 0$ 

$\gamma_1 < 0$ 

$\gamma_2 > 0$ 

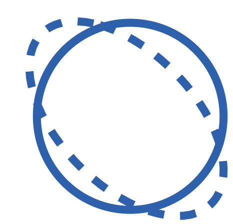
$\gamma_2 < 0$ 

Image Moments and Ellipticity

- A convenient method to obtain shape from images is via “moments”
  - standard image processing tool to recognize/describe shapes in images
- The moment of an image is the (intensity) weighted sum of image pixels

$$M_{pq} = \sum_i \sum_j i^p j^q I(i,j)$$

- The 0th order moment is:

$$M_{00} = \sum_i \sum_j i^0 j^0 = N_{\text{pixels}}$$

- The first order moments are:

$$M_{10} = \sum_i \sum_j i^1 j^0 \quad \Rightarrow \quad \bar{x} = M_{10}/M_{00}$$

$$M_{01} = \sum_i \sum_j i^0 j^1 \quad \Rightarrow \quad \bar{y} = M_{01}/M_{00}$$

- 2nd order moments M_{20}, M_{02} and M_{11} provides covariance matrixes
  - Shape information (read: ellipticity)

Image Moments and Ellipticity

- This image moment corresponds to integrating over the surface brightness

$$q_{ij} \equiv \int d^2\theta \, \mathcal{S}^{\mathrm{obs}}(\boldsymbol{\theta})\theta_i\theta_j$$

- Then ellipticity can be defined in terms of surface brightness moments as

$$\epsilon_1 \equiv \frac{q_{11} - q_{22}}{q_{11} + q_{22}} \qquad \epsilon_2 \equiv \frac{2q_{12}}{q_{11} + q_{22}}$$

  - For example, elongation along the x-axis: $q_{11} > q_{22}$ so $\varepsilon_1 > 0$

- We are assuming that the objects are spherical: $\mathcal{S}(\boldsymbol{\beta}) = \mathcal{S}(|\boldsymbol{\beta}|)$
- Preferable to integrate in the source plane (no distortions)
- Transforming coordinates is done with the Jacobian elements $\mathcal{A}_{ij} = \dfrac{\partial \beta_i}{\partial \theta_j}$

$$d^2\theta \rightarrow \left| \frac{\partial \theta_j}{\partial \beta_i} \right| d^2\beta \qquad \beta_i = \mathcal{A}_{ij}\theta_j$$

  - have used that the Jacobian is roughly constant over the galaxy size (~arcsec)

Image Moments and Ellipticity

- Hence, the moments can be calculated in the source plane with

$$q_{ij} = \mathcal{N}(\mathcal{A}^{-1})_{ii'}(\mathcal{A}^{-1})_{jj'} \int d^2\beta\, \mathcal{S}(\beta)\beta_{i'}\beta_{j'}$$

- This is independent of direction for spherically symmetric sources
- Inserting into definition of ellipticity components gives (integral drops out)

$$\epsilon_1 \equiv \frac{(\mathcal{A}^{-1})_{1i}(\mathcal{A}^{-1})_{1i} - (\mathcal{A}^{-1})_{2i}(\mathcal{A}^{-1})_{2i}}{(\mathcal{A}^{-1})_{1i}(\mathcal{A}^{-1})_{1i} + (\mathcal{A}^{-1})_{2i}(\mathcal{A}^{-1})_{2i}}$$

$$\epsilon_2 \equiv \frac{2(\mathcal{A}^{-1})_{1i}(\mathcal{A}^{-1})_{2i}}{(\mathcal{A}^{-1})_{1i}(\mathcal{A}^{-1})_{1i} + (\mathcal{A}^{-1})_{2i}(\mathcal{A}^{-1})_{2i}}$$

Using Einstein summation:

$(A^{-1})_{1i}\,(A^{-1})_{1i} = \Sigma_i(A^{-1})_{1i}(A^{-1})_{1i}$

- Using the inverse of the Jacobian matrix we get (Exercise 3)

$$\epsilon_1 \equiv 2\gamma_1 \frac{1-\kappa}{(1-\kappa)^2 + \gamma_1^2 + \gamma_2^2}$$

$$\epsilon_2 \equiv 2\gamma_2 \frac{1-\kappa}{(1-\kappa)^2 + \gamma_1^2 + \gamma_2^2}$$

$$\epsilon_i = \frac{2\gamma_i}{1-\kappa}\left[1 - \frac{\gamma^2}{(1-\kappa)^2}\right]^{-1}$$

The reduced shear

(1-κ) independent of shape

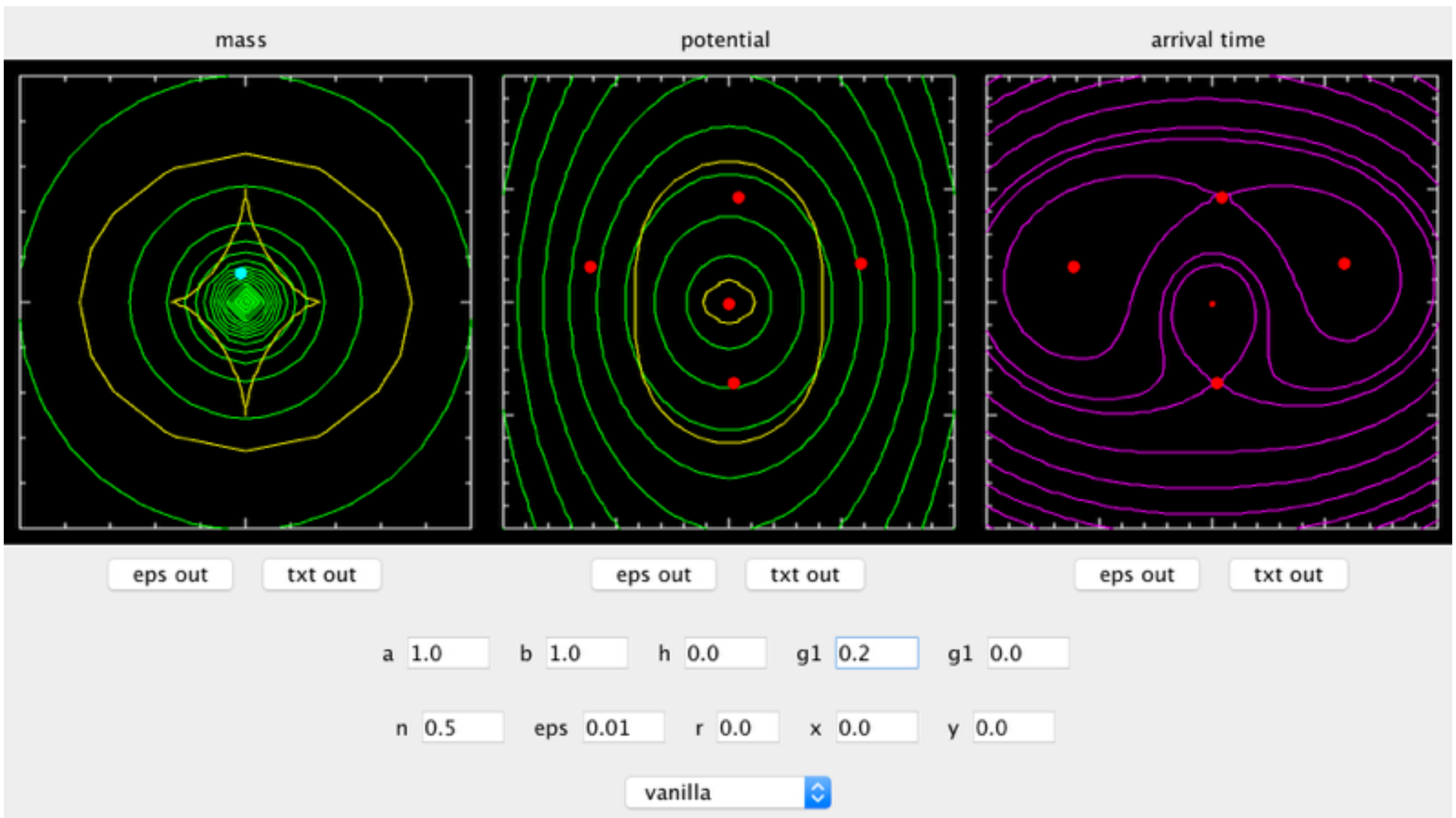
- So the ellipticity relates to γ and κ, and hence the gravitational potential

Accounting for Intrinsic Ellipticity

- Determining the ellipticity is straight forward in the ideal case
- Which of these is an elliptical and which one is spherical but sheared?

?

?

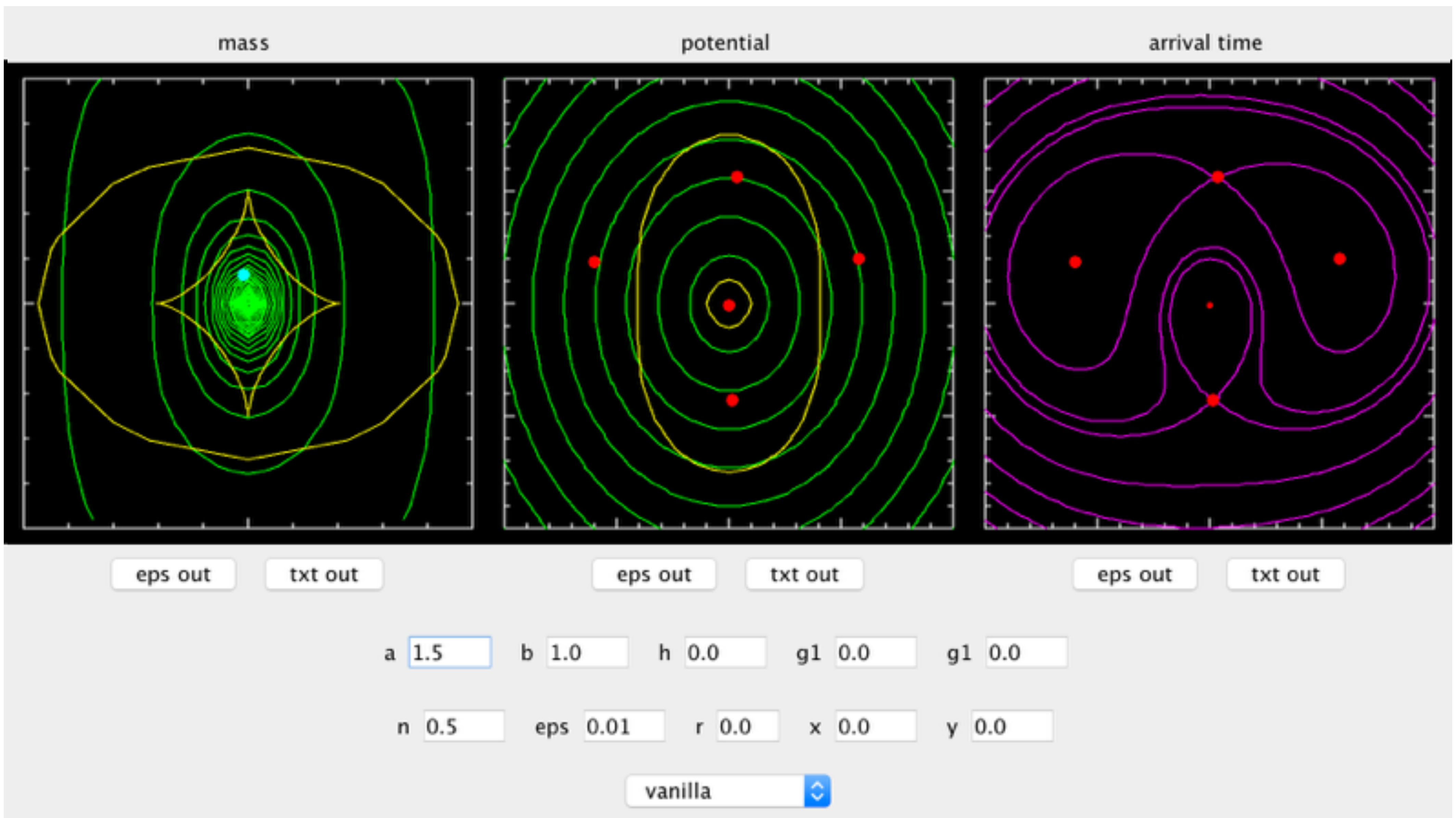


Accounting for Intrinsic Ellipticity

- Determining the ellipticity is straight forward in the ideal case
- Which of these is an elliptical and which one is spherical but sheared?

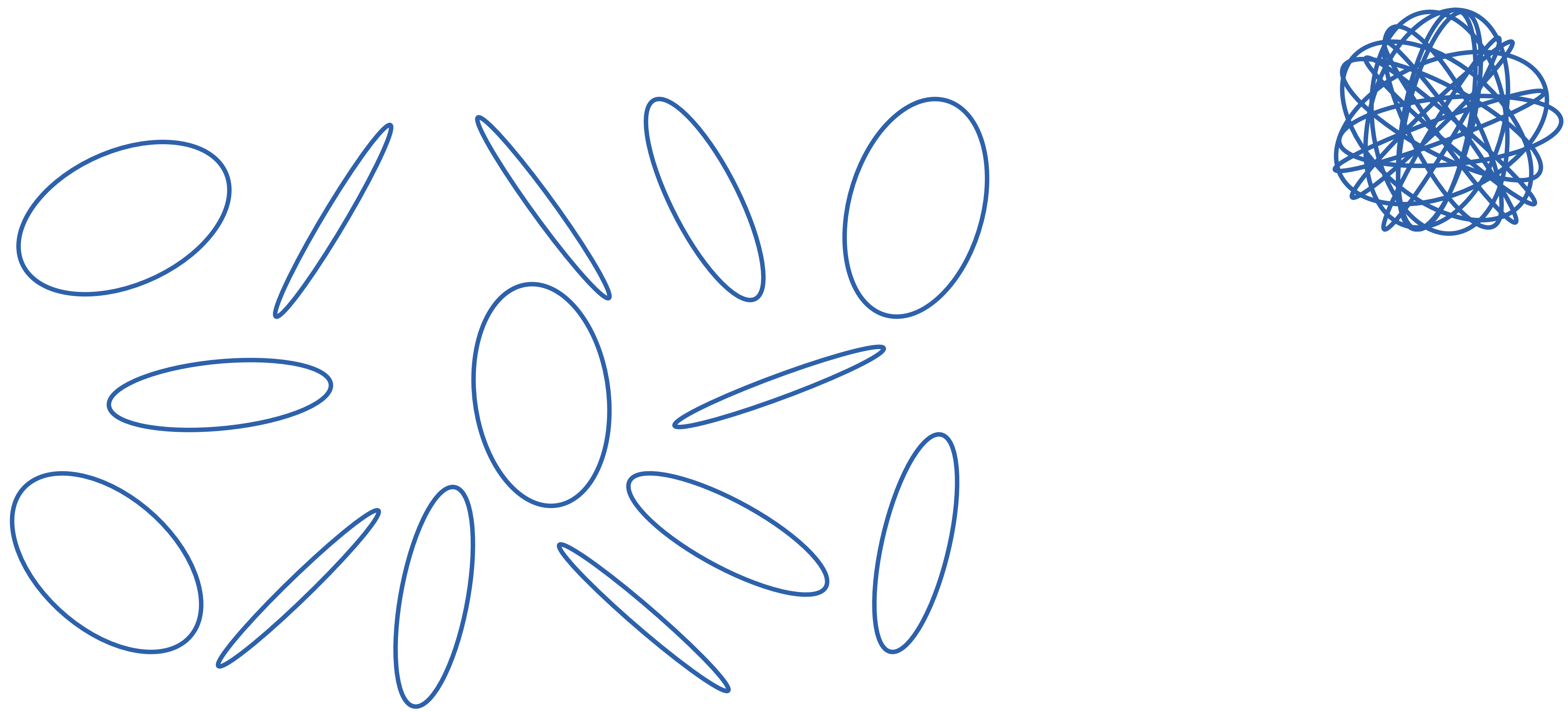
?

?



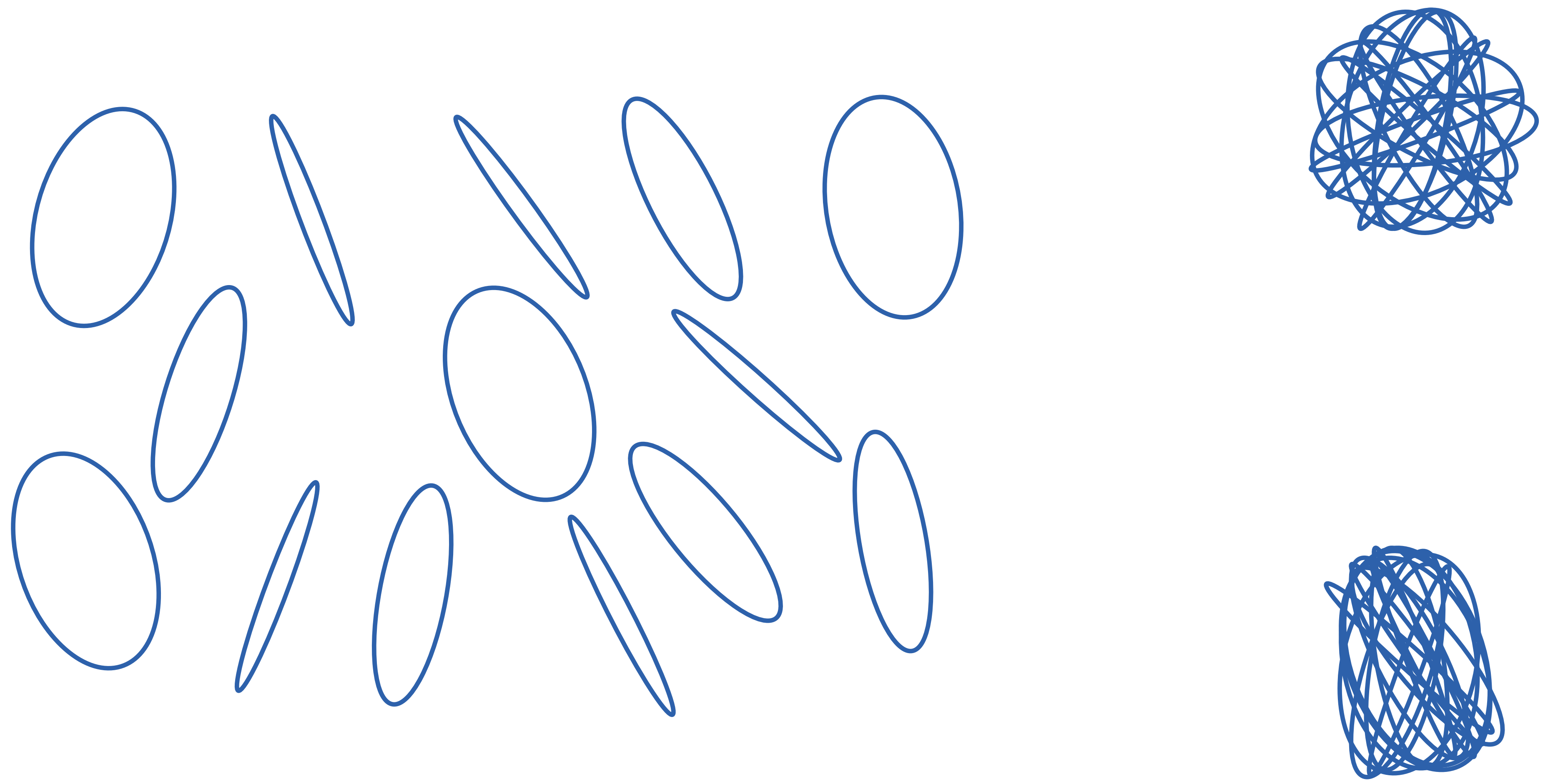
Accounting for Intrinsic Ellipticity

- Intrinsic ε averages to 0 as they are random



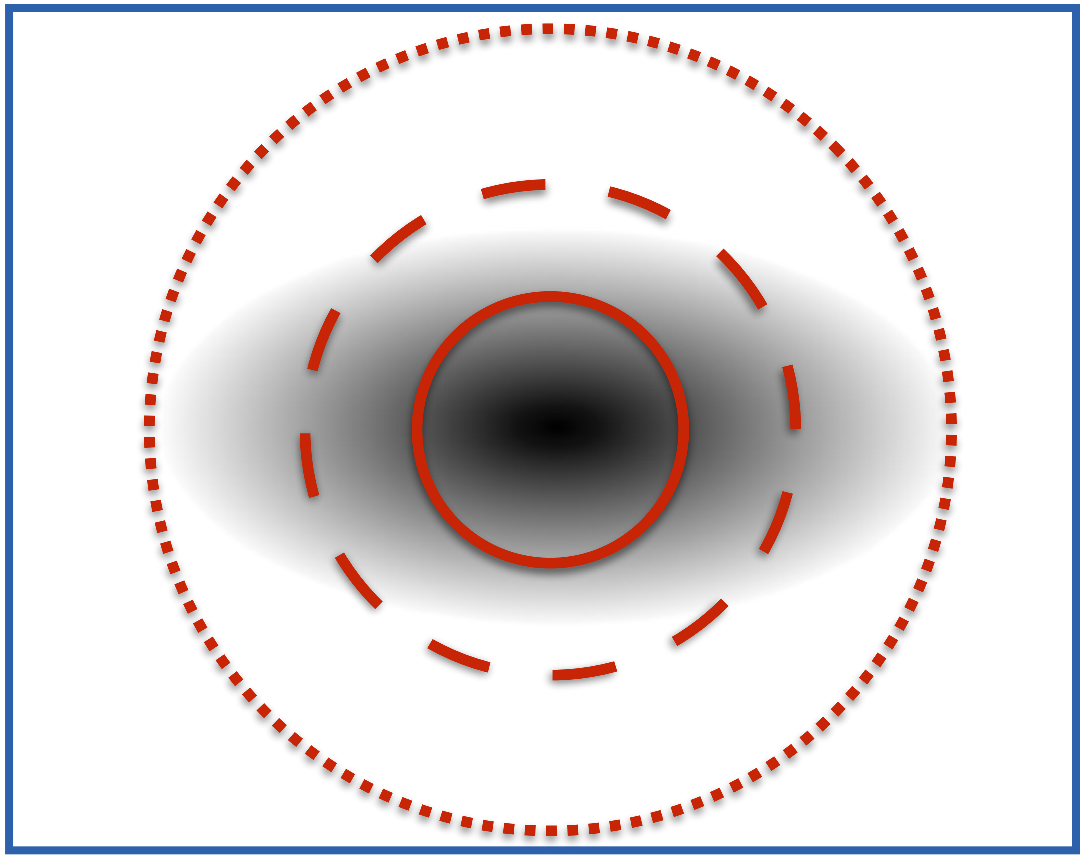
Accounting for Intrinsic Ellipticity

- Intrinsic ε averages to 0 as they are random
- Shear is correlated with the mass, i.e., not random.
  - At a given position the same for all galaxies



Weighting of pixels

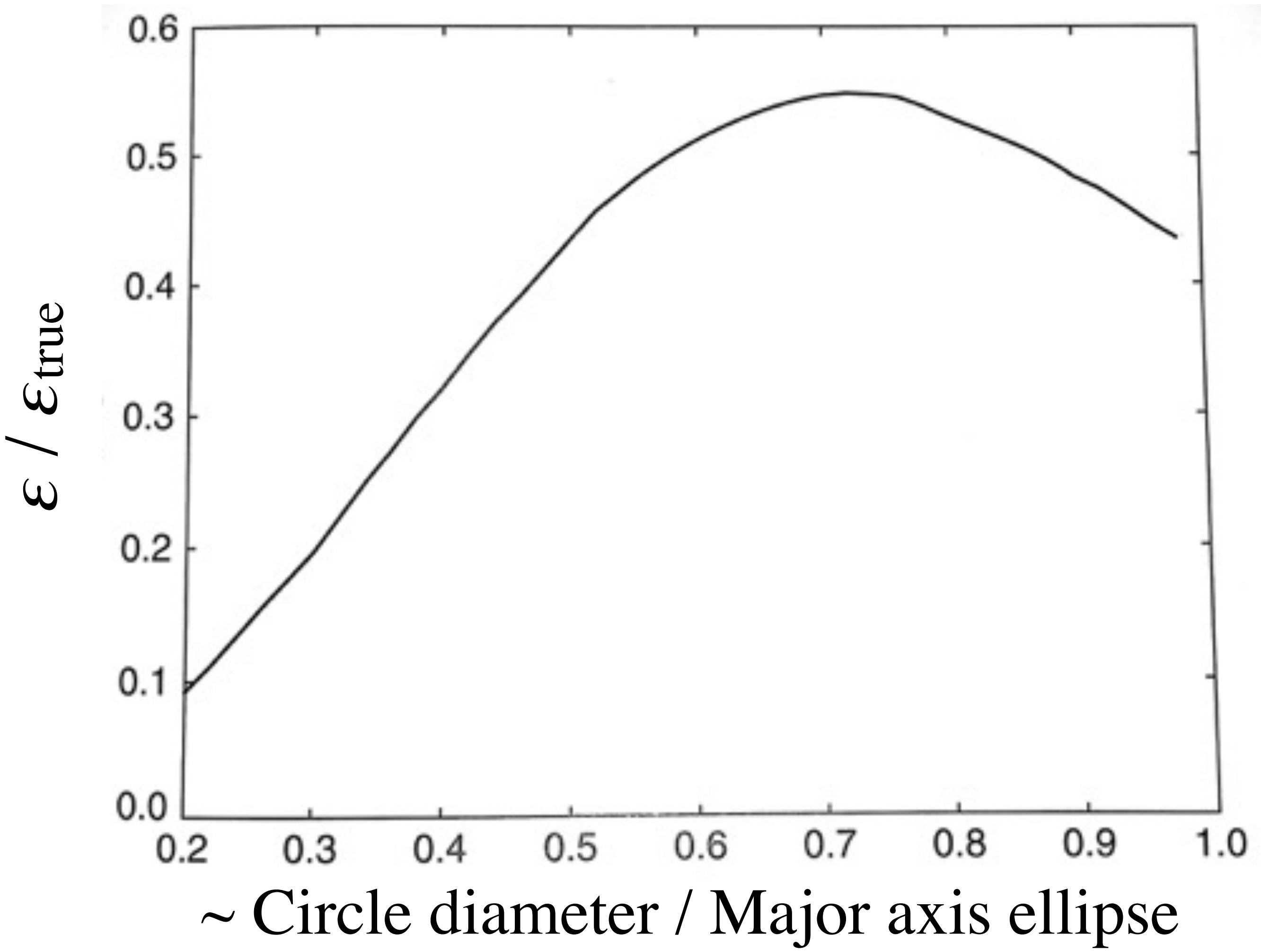
- Noise on observations also affects our ability to determine ellipticity
- Which pixels should be used for calculating moments?
  - Want to get as reliable an estimate of ε as possible
  - Want to optimize the amount of signal compared to noise, i.e., S/N

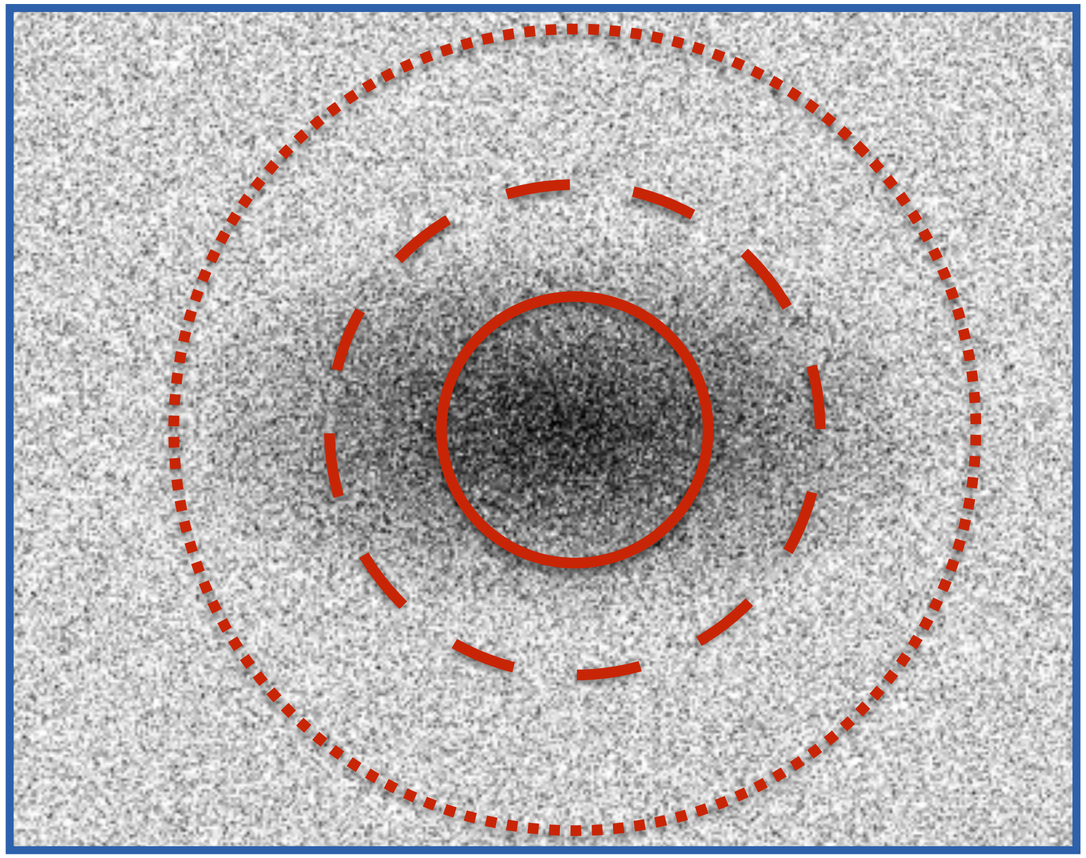


Adapted from Dodelson (2017)

Weighting of pixels

- Noise on observations also affects our ability to determine ellipticity
- Which pixels should be used for calculating moments?
  - Want to get as reliable an estimate of ε as possible
  - Want to optimize the amount of signal compared to noise, i.e., S/N





Adapted from Dodelson (2017)

Ellipticities accounting for W and $\varepsilon_{\text{intrinsic}}$

- Introducing a weighting of the pixels (not just by surface brightness)

$$Q_{ij} \equiv \int d^2\theta \, \mathcal{S}^{\text{obs}}(\boldsymbol{\theta}) W(\boldsymbol{\theta}) \theta_i \theta_j$$

- So for the circular source, W($\boldsymbol{\theta}$) is a circular step function
- Defining ε_1 and ε_2 in terms of Q_{11} and Q_{22} one can express these including the weight function in a way similar to the simplified spherical case
- The observed moments can be expressed as

$$Q_{ij}^{\text{obs}} \simeq Q_{\text{ij}} - \Psi_{lm} \int d^2\theta \frac{\partial \mathcal{S}(\boldsymbol{\theta})}{\partial \theta_l} W(\boldsymbol{\theta}) \theta_i \theta_j \theta_m$$

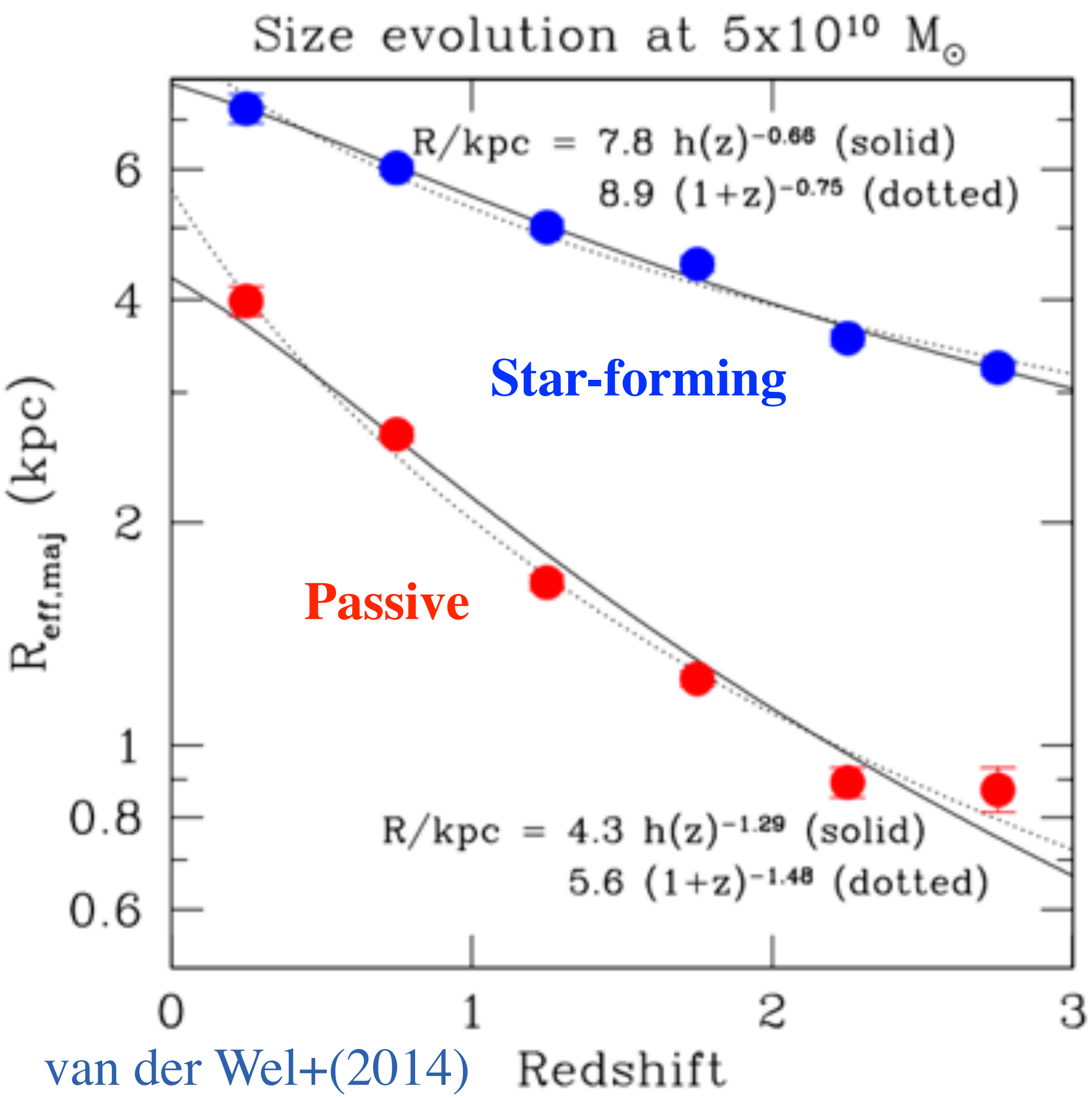
Gives the observed ellipticity

expresses the intrinsic ellipticity

Accounts for the distortion from shearing

Assembling large samples of galaxies

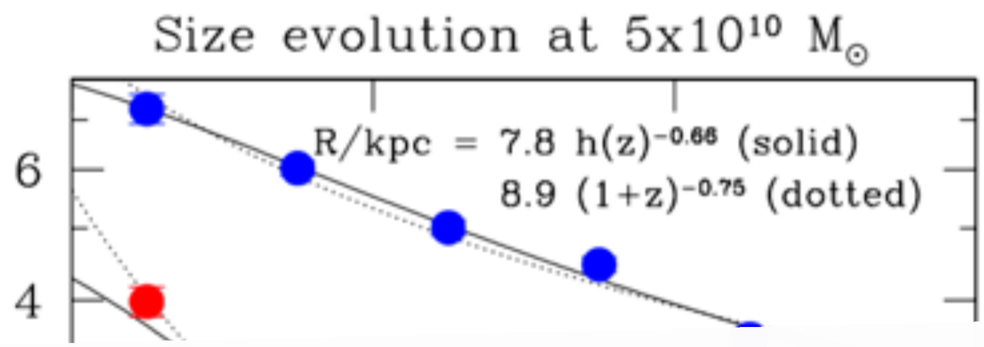
- Decrease the noise of the weak lensing signal by large galaxy samples
  - Wide-field and deep observations
- Distant faint galaxies are small (Exercise 4)
  - ~a few arcsec comparable to seeing/PSF



van der Wel+(2014)

Assembling large samples of galaxies

- Decrease the noise of the weak lensing signal by large galaxy samples
  - Wide-field and deep observations
- Distant faint galaxies are small (Exercise 4)
  - ~a few arcsec comparable to seeing/PSF
- Results dependent on PSF

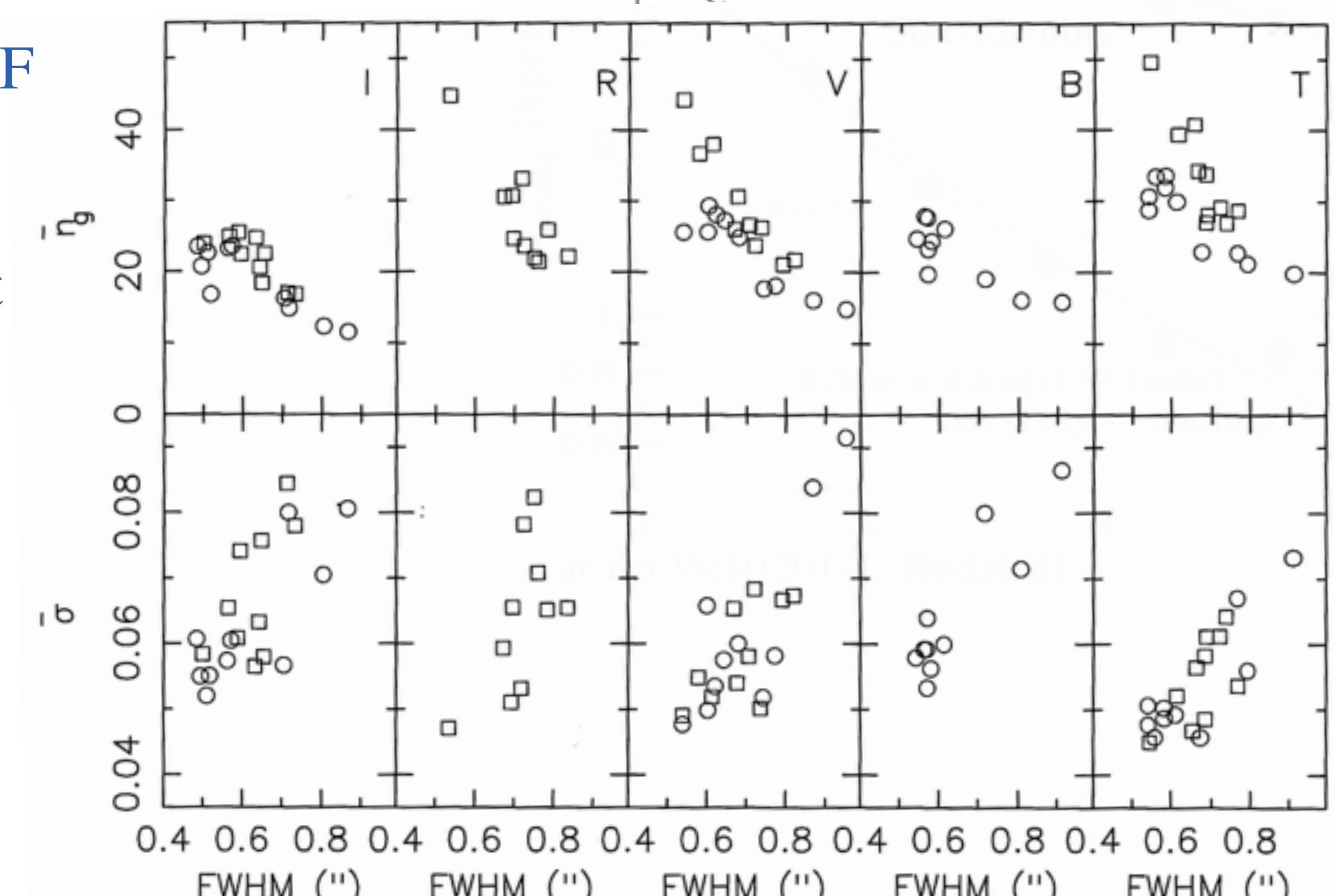


Number of galaxies with reliable shear measurement

The noise in signal from intrinsic ellipticity

□ t_{exp} ~ 45 min

○ t_{exp} ~ 2 hours



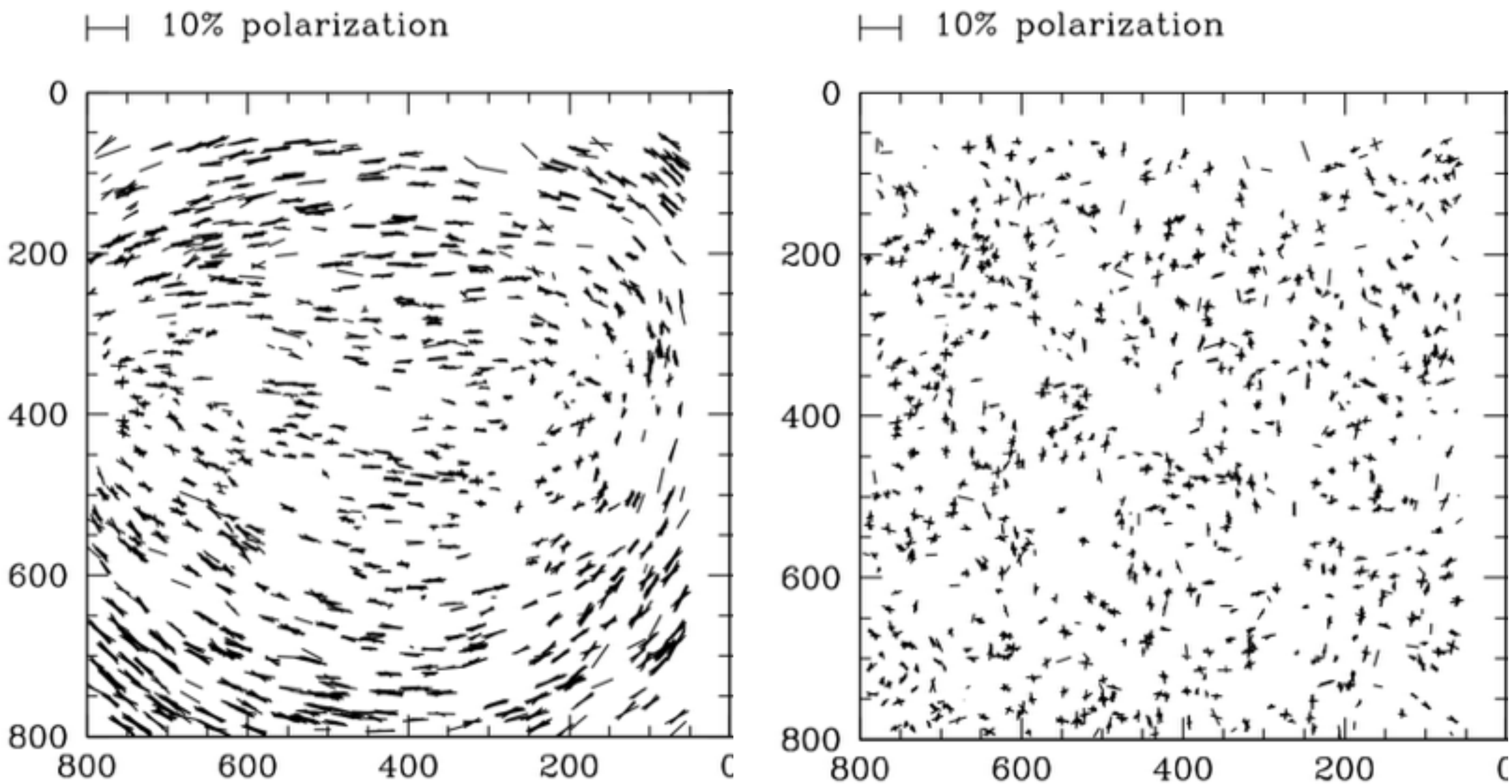
Schneider+(2006)

Correcting for the PSF

- PSF can be treated as another weight functions varying over the FoV

$$Q_{ij} \equiv \int d^2\theta \, \mathcal{S}^{\mathrm{obs}}(\boldsymbol{\theta}) P(\boldsymbol{\theta}) \theta_i \theta_j \qquad \Rightarrow \qquad Q'_{ij} = Q_{ij} + P_{lm} Y_{lmij}$$

- Iterative modeling minimizing χ^2 comparing data with model predictions

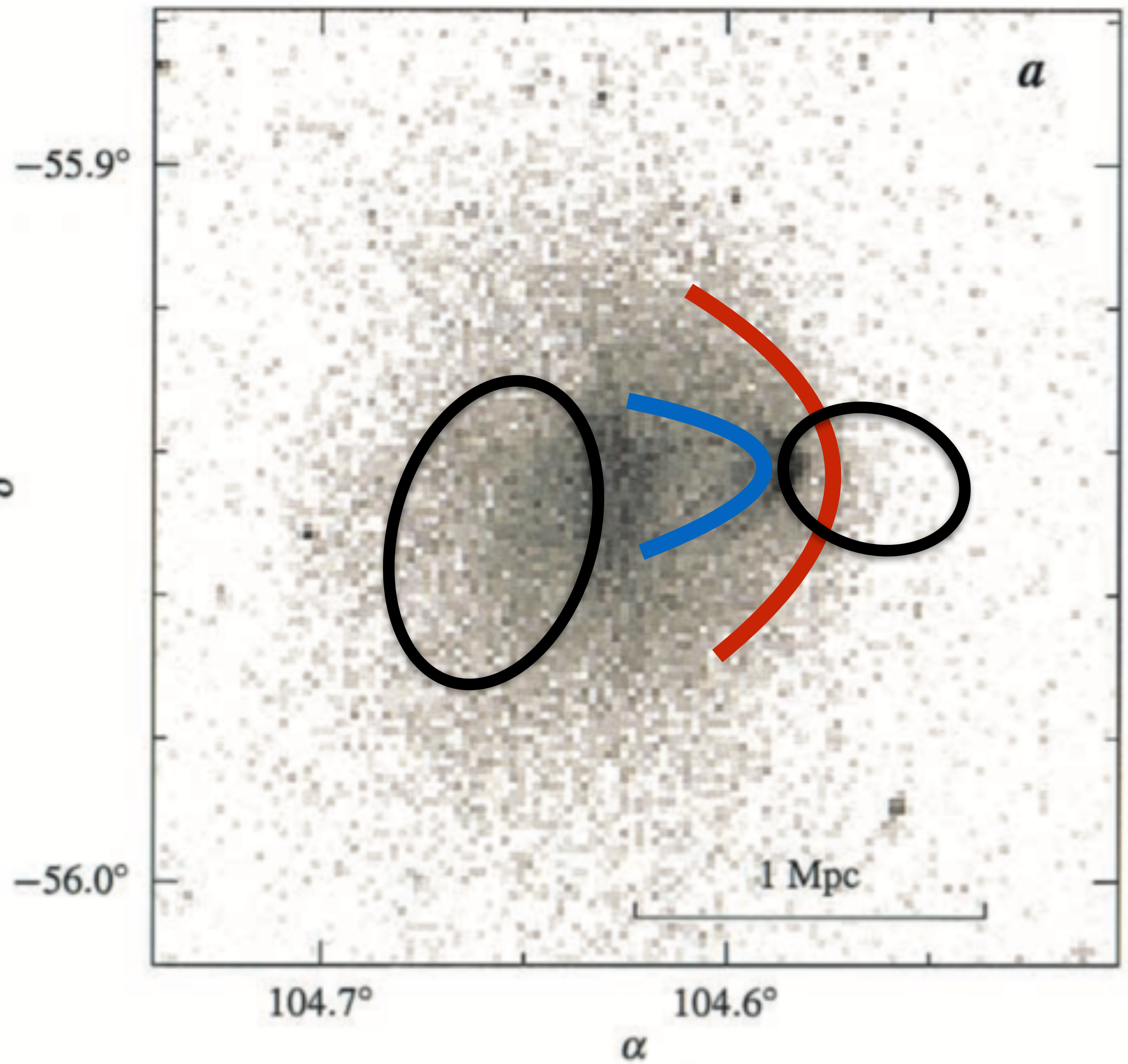


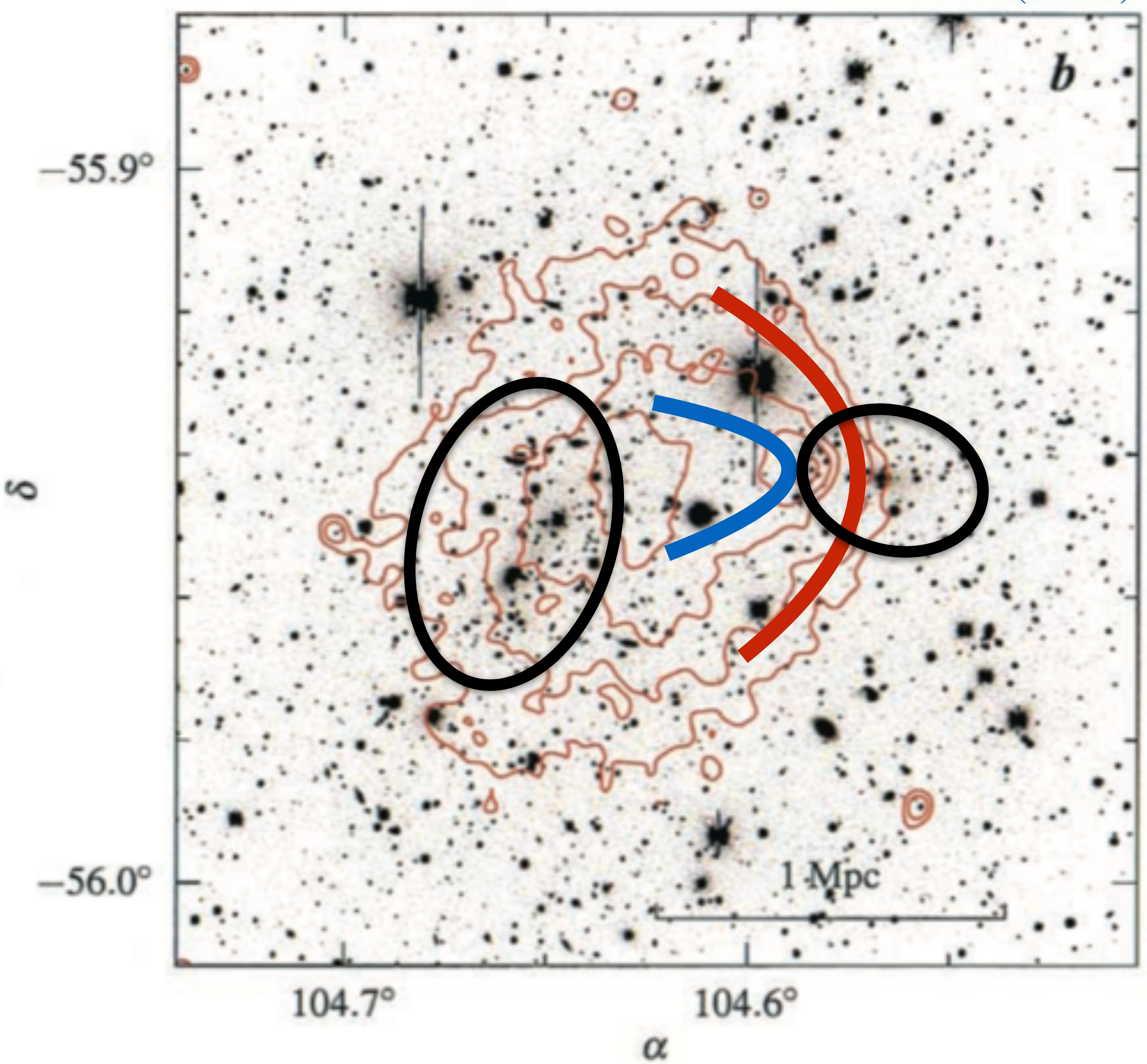
Hoekstra+(1998)

The Bullet Cluster: 1E 0657 558

- X-rays obsessed with *Chandra* probe the hot gas in the cluster
  - See a clear bow shock of hot gas in front of 'bullet' from the cluster collision
- Displacement of ~90% of the mass → galaxy clumps contain less mass

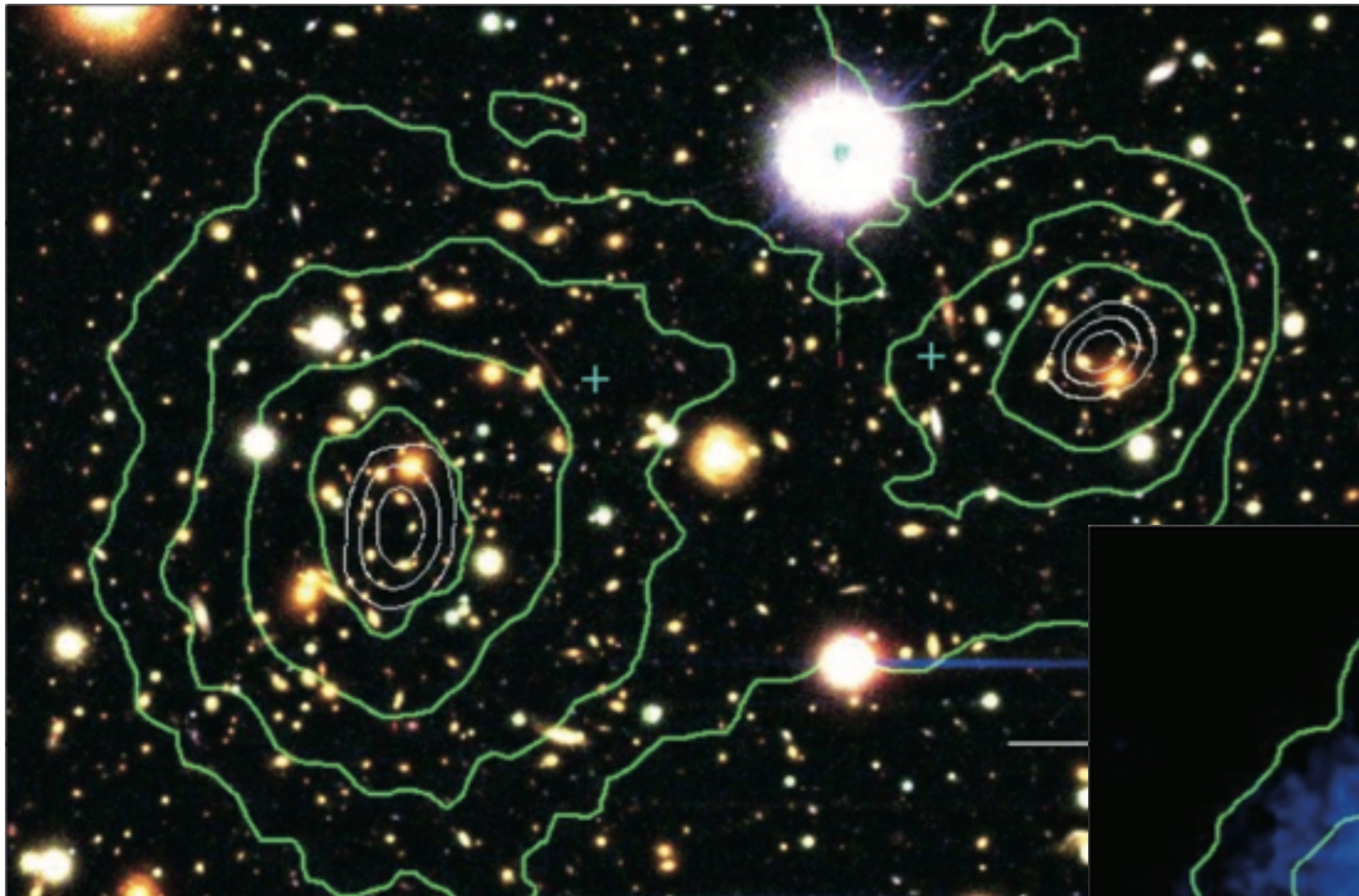
Markevitch+(2002)





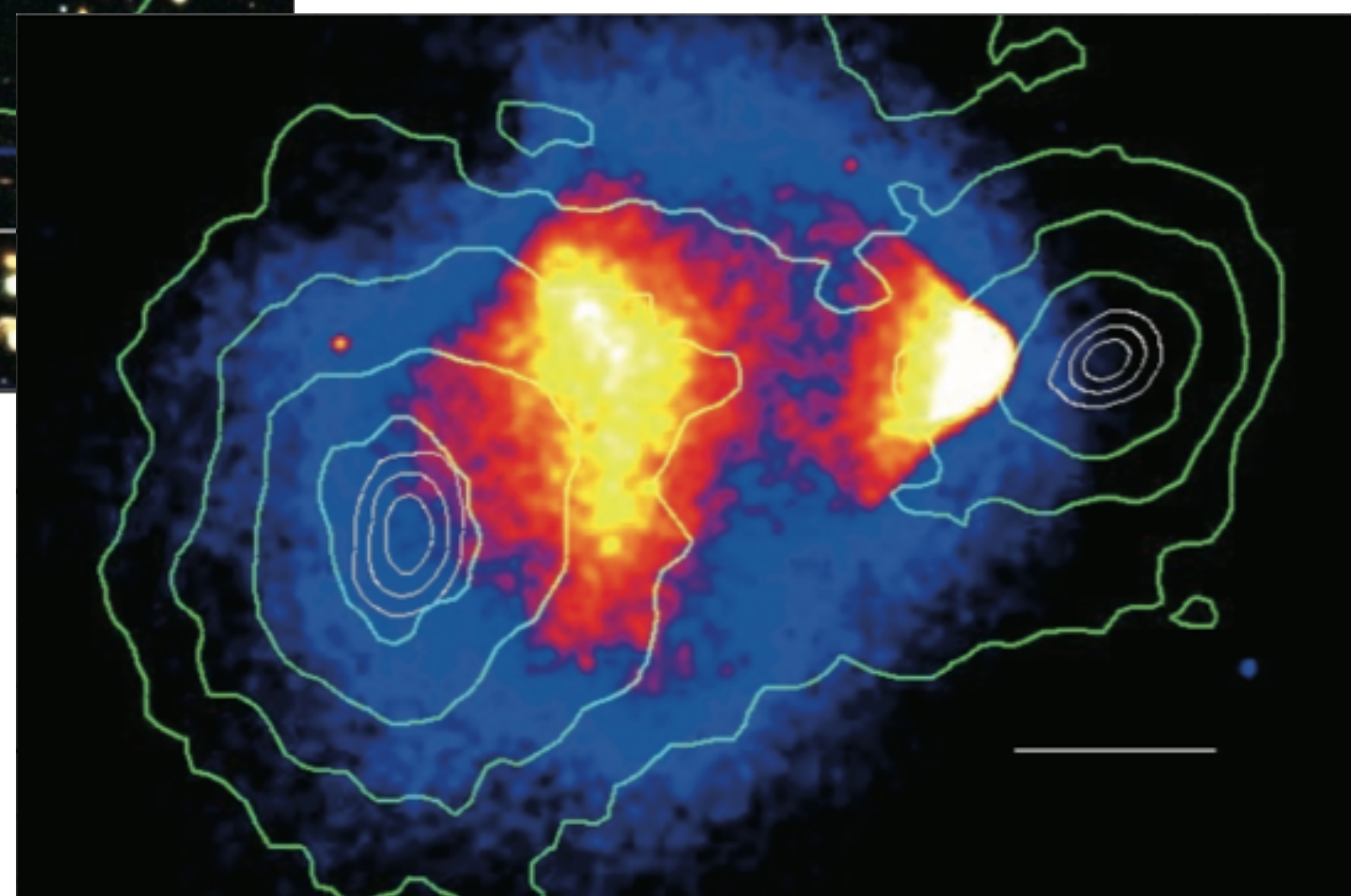
The Bullet Cluster: 1E 0657 558

- Combining the X-ray and lensing info provides a strong proof for DM



Clowe+(2006)

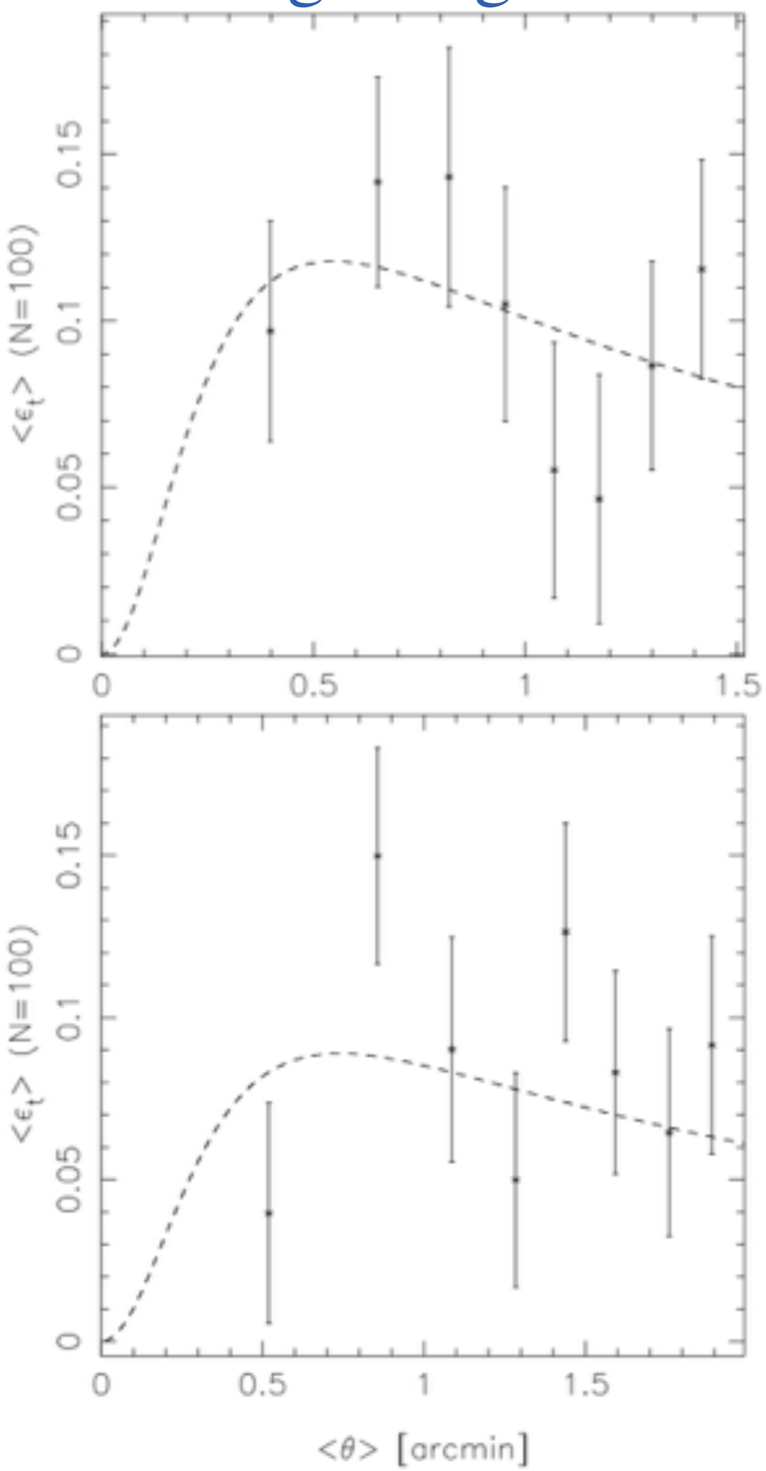
Clowe+(2006)

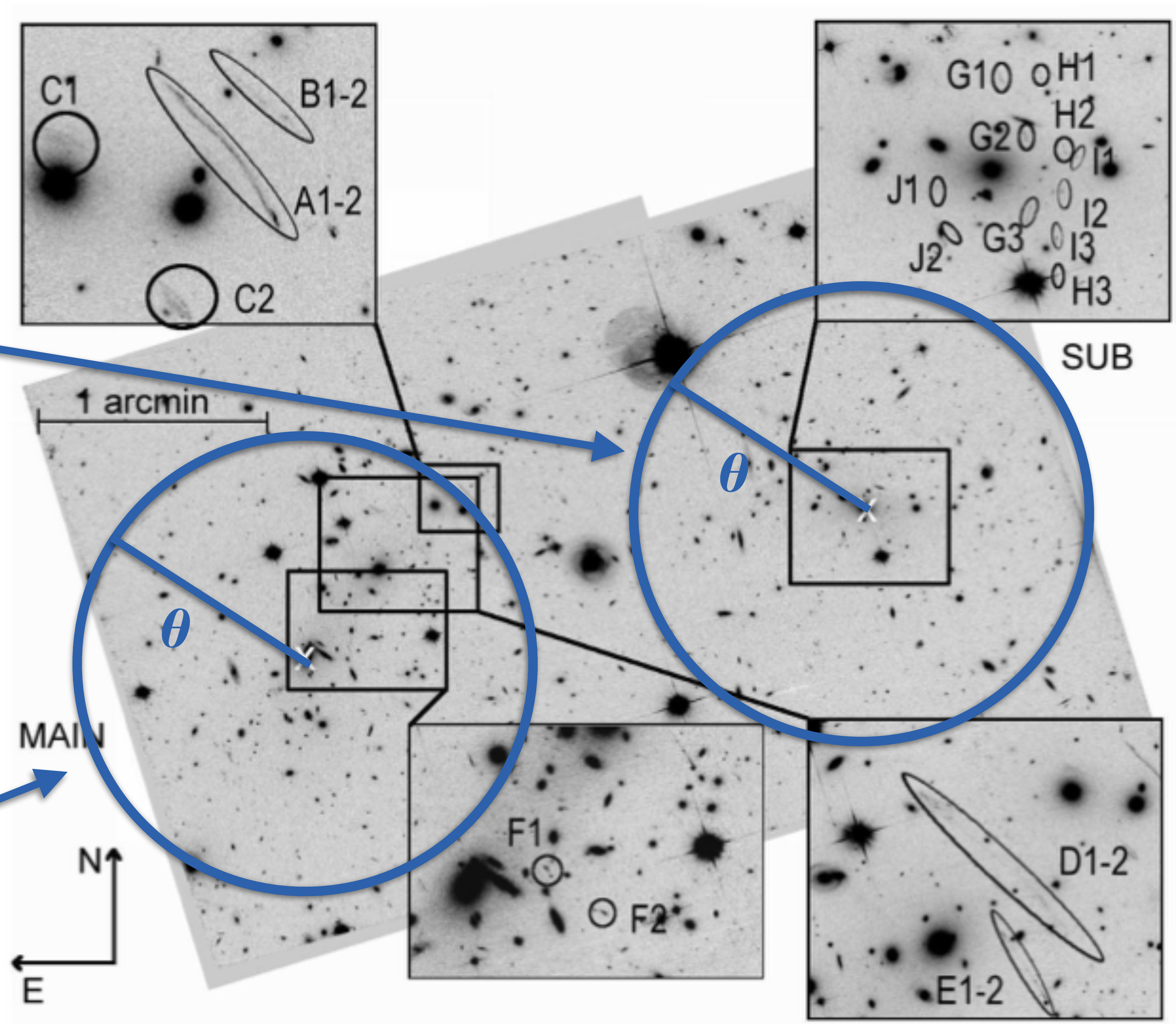


The Bullet Cluster: 1E 0657 558

- Bradac+2006 modeled mass distribution combining strong and weak lensing

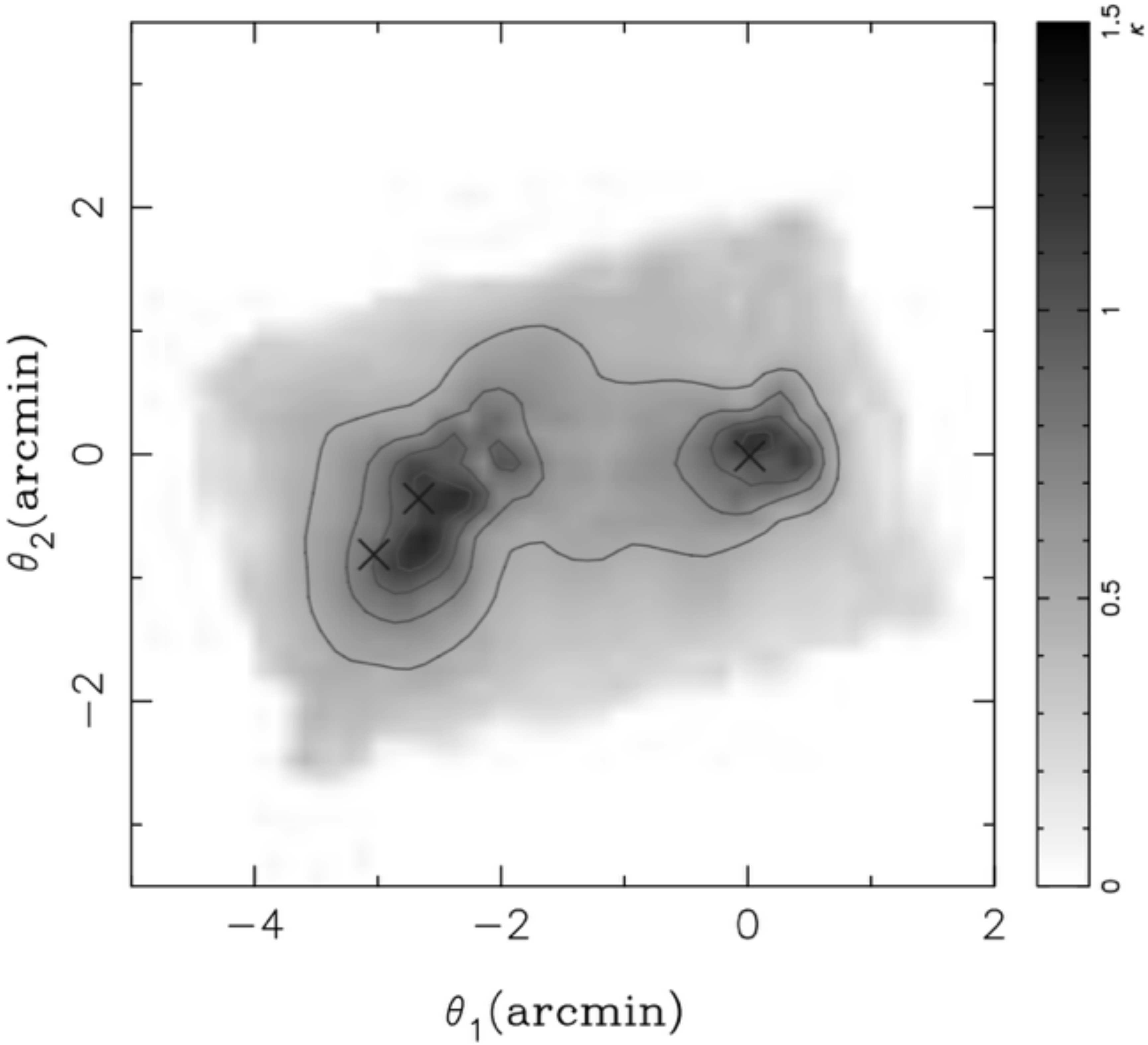
Average tangential ε



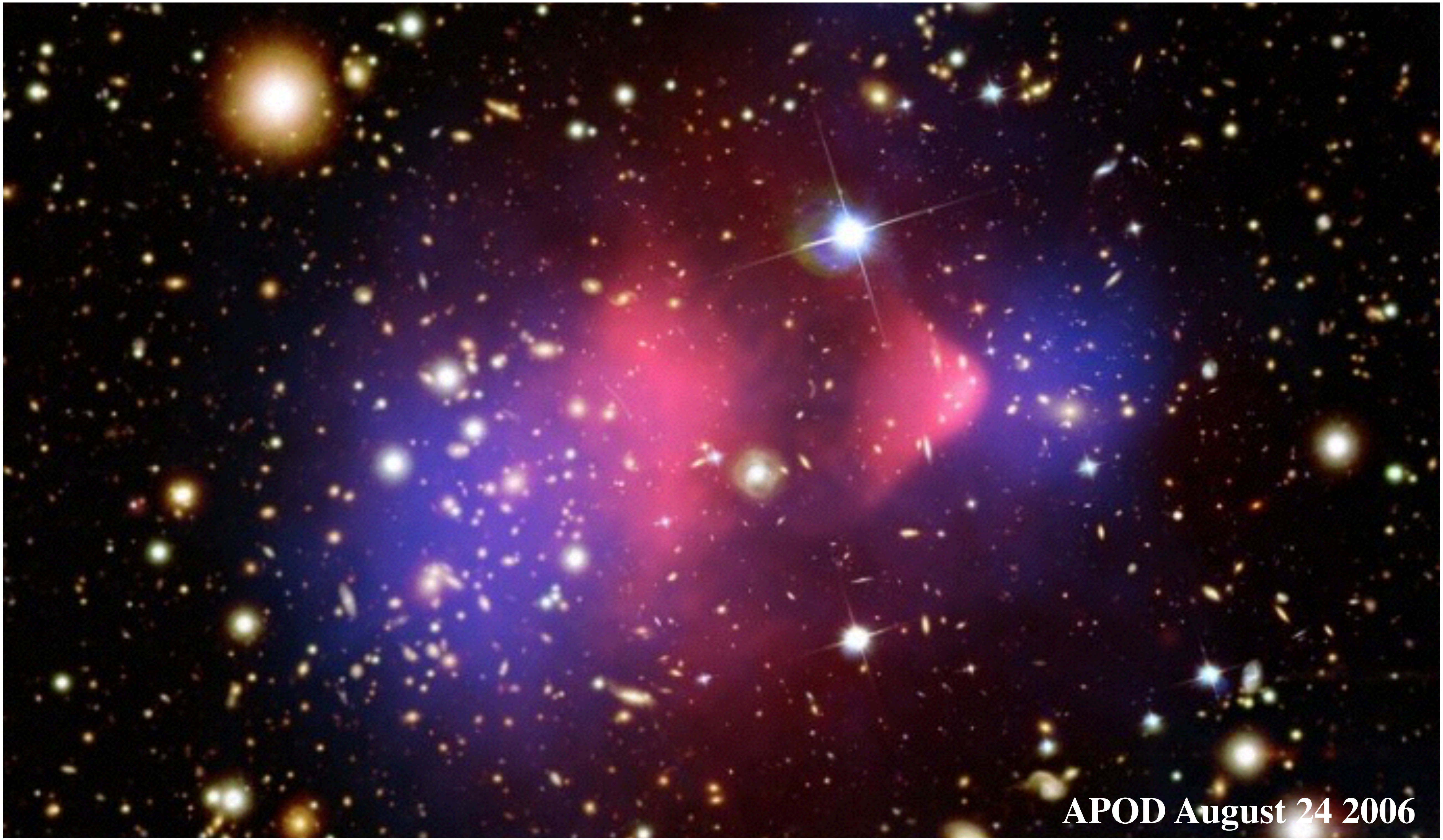


The Bullet Cluster: 1E 0657 558

- Bradac+2006 modeled mass distribution combining strong and weak lensing



The Bullet Cluster: 1E 0657 558



So in summary…

- The 2nd order surface brightness moments are defined as

$$q_{ij} \equiv \int d^2\theta \, \mathcal{S}^{\rm obs}(\boldsymbol{\theta})\theta_i\theta_j$$

- For the ideal spherical galaxy case, they can be used to express the ellipticity of an object as

$$\epsilon_i = \frac{2\gamma_i}{1-\kappa}\left[1 - \frac{\gamma^2}{(1-\kappa)^2}\right]^{-1}$$

- This depends directly on the gravitational potential through κ and γ
- In the general case also noise, PSF, intrinsic ellipticity (among other things) needs to be accounted for.

$$Q_{ij}^{\rm obs} \simeq Q_{\rm ij} - \Psi_{lm}\int d^2\theta \frac{\partial \mathcal{S}(\boldsymbol{\theta})}{\partial \theta_l} W(\boldsymbol{\theta})\theta_i\theta_j\theta_m$$

$$Q'_{ij} = Q_{ij} + P_{lm}Y_{lmij}$$

- The bullet cluster provides direct empirical proof of the existence of DM