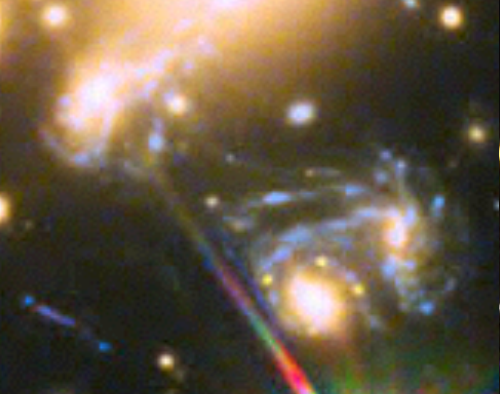
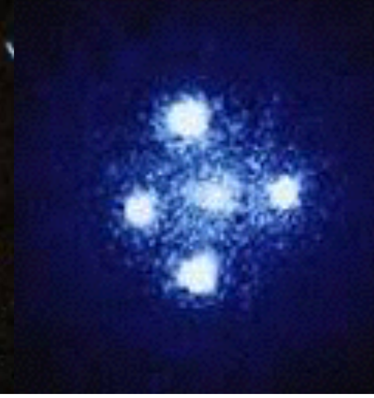
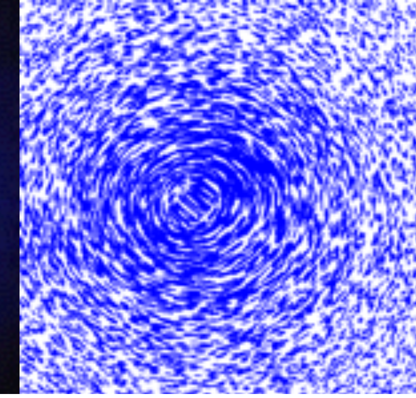
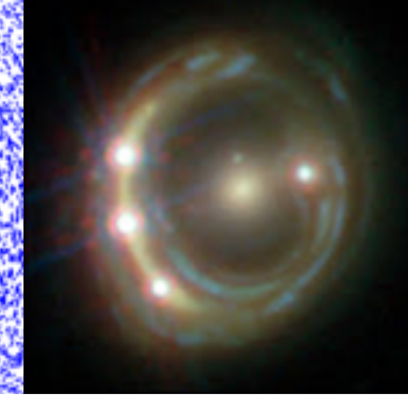



PHY-765 SS19 Gravitational Lensing Week 11

Modeling Gravitational Lenses

Kasper B. Schmidt
Leibniz-Institut für Astrophysik Potsdam (AIP)

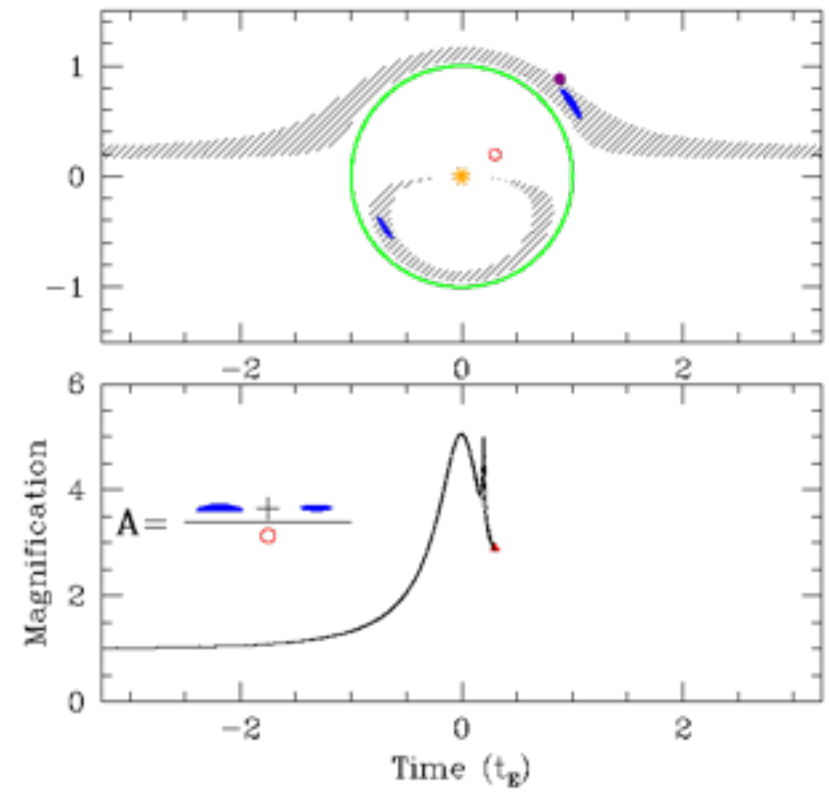
Last week - what did we learn?

- Defined lens equation for multiple point mass lenses and star+planet lens

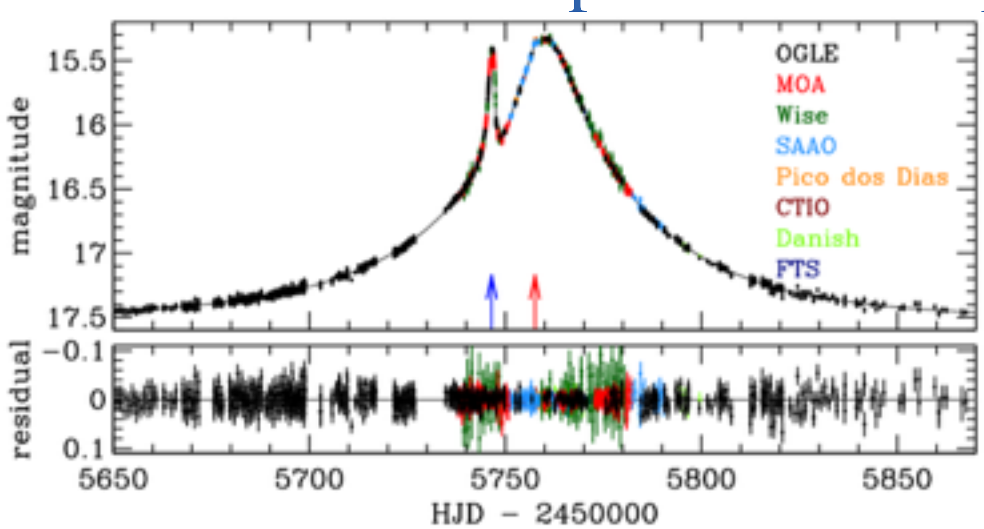
$$\boldsymbol{y} = \boldsymbol{x} - \sum_i \frac{m_i}{M} \frac{\boldsymbol{x} - \boldsymbol{x}_i}{|\boldsymbol{x} - \boldsymbol{x}_i|^2} \qquad \boldsymbol{y} \simeq \boldsymbol{x} - \frac{\boldsymbol{x}}{|\boldsymbol{x}|^2} - q \frac{\boldsymbol{x} - \boldsymbol{x}_\mathrm{p}}{|\boldsymbol{x} - \boldsymbol{x}_\mathrm{p}|^2}$$

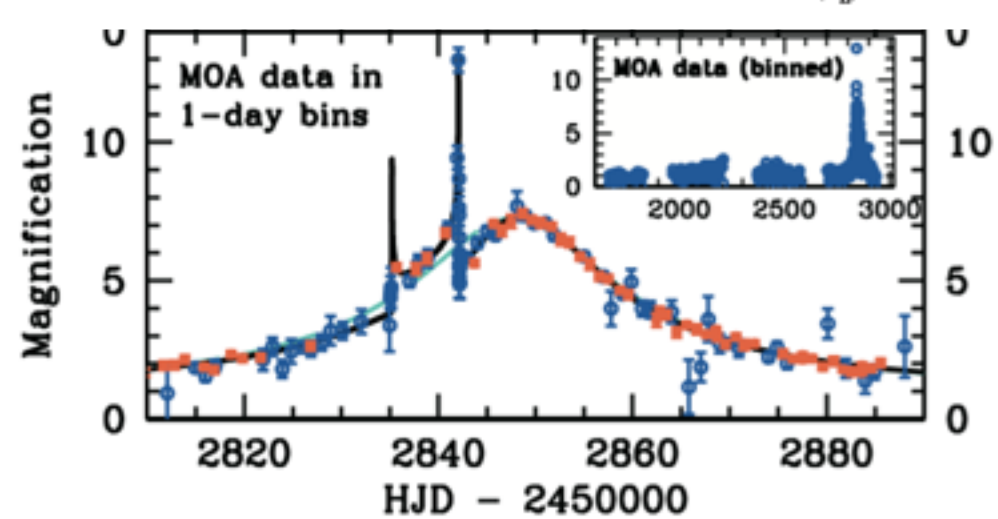
- Effects on source magnification "light curves"

$$\Delta\mu_\mathrm{p} \simeq \frac{2\mu_0^2 q}{x^2 (x - x_\mathrm{p})^2}$$



- Discussed strengths of lensing planet search
  - No pre-selection on planet host star
  - No mass bias
  - Sensitive to planets in 'habitable zone'
- Discussed a few examples of found planets





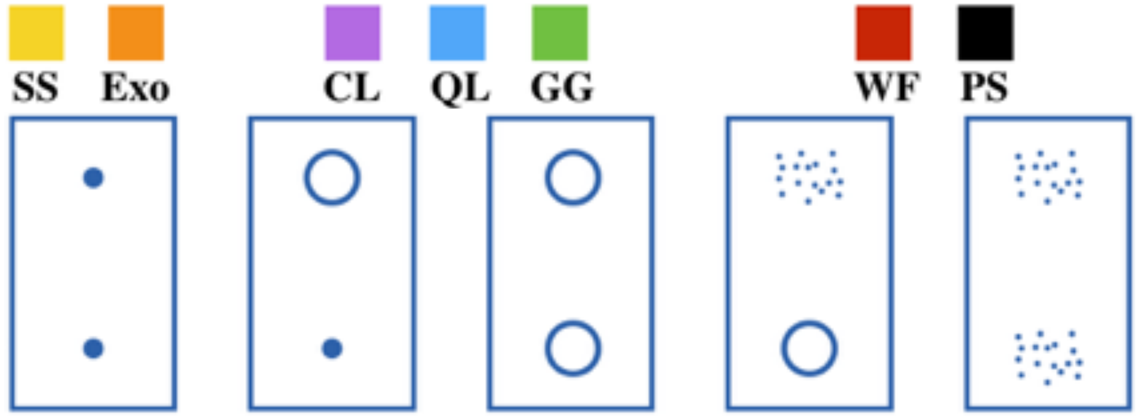
The aim of today

- What is relevant for the lens models
  - constraints and assumptions

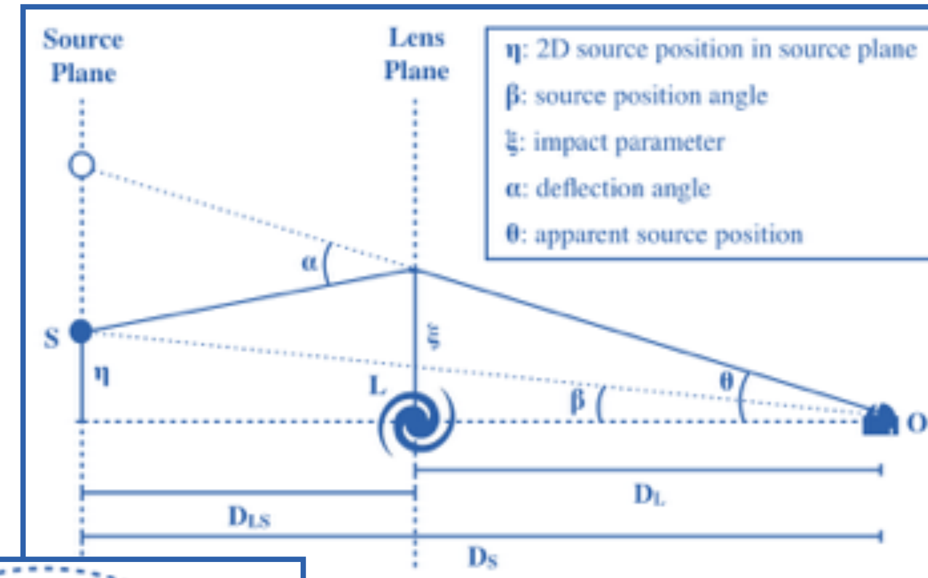
- Parametric vs. Non-Parametric modeling

- (Mass-Sheet Degeneracy in lens modeling)

- Cluster lens modeling comparison efforts

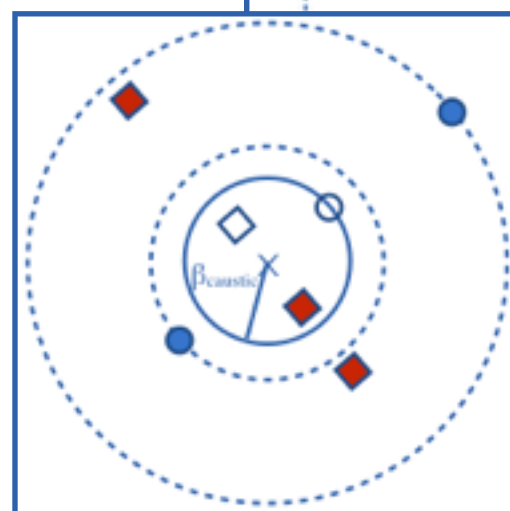


Aspects Relevant for Modeling Covered So Far



- Lens Geometry & Light Deflection
- Lens Equation $\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta})$

$$\kappa(\boldsymbol{\theta}) \equiv \frac{\Sigma(D_{\rm L}\boldsymbol{\theta})}{\Sigma_{\rm cr}}$$



- Multiple images $\boldsymbol{\beta} = \boldsymbol{\theta} - \langle\kappa(\theta)\rangle\boldsymbol{\theta}$

$$\rho(r) = \frac{\sigma^2}{2\pi G(r^2 + r_{\rm core}^2)}$$

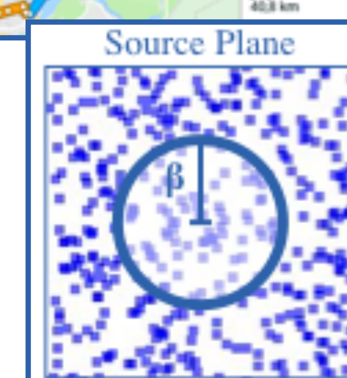
- Time Delays

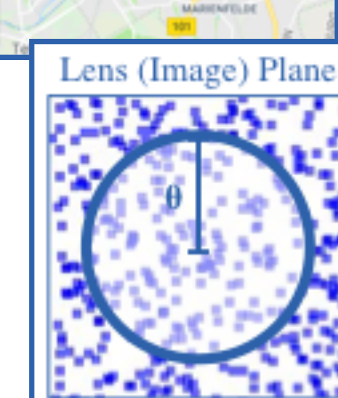
Distance & Mass

$$\Delta t = \frac{D_{\rm L} D_{\rm S}}{c D_{\rm LS}} \left[\frac{(\boldsymbol{\theta} - \boldsymbol{\beta})^2}{2} - \frac{\Phi(\boldsymbol{\theta})}{c^2} \right]$$

- Magnification

$$\frac{\mathcal{S}(\boldsymbol{\theta}) d\Omega_{\rm lens\ plane}}{\mathcal{S}(\boldsymbol{\beta}) d\Omega_{\rm source\ plane}} = \frac{\mathcal{S}(\boldsymbol{\theta}) d\theta^2}{\mathcal{S}(\boldsymbol{\beta}) d\beta^2} = \frac{F(\boldsymbol{\theta})}{F(\boldsymbol{\beta})} = \frac{d\theta^2}{d\beta^2} = \mu$$





Why Model Gravitational Lenses?

- Determine mass *distribution* of lenses
  - Individual galaxy (mass) studies
  - Test of gravity models
  - Infer size of cosmic over densities
  - Constrain dark matter nature
- Constrain time-delays
  - Determine cosmological parameters (H_0)
  - Predict astronomical events (SN Refsdal)
- Reconstruct lensed sources in source plane
  - Resolved studies impossible without lens magnification
  - Combine data from multiple images to increase S/N

Modeling Gravitational Lenses

- Lens modeling has been considered a "black art"/"black box"
- Partially due to lack of community-wide naming conventions and secrecy

  In short, the problem with lens modeling is not that it is a "black art", but that the practitioners try to make it seem to be a "black art" presumably so that people will believe they need wizards […] any idiot can model a lens and interpret it properly with a little thinking about what it is that lenses constrain. - C. S. Kochanek, 2006

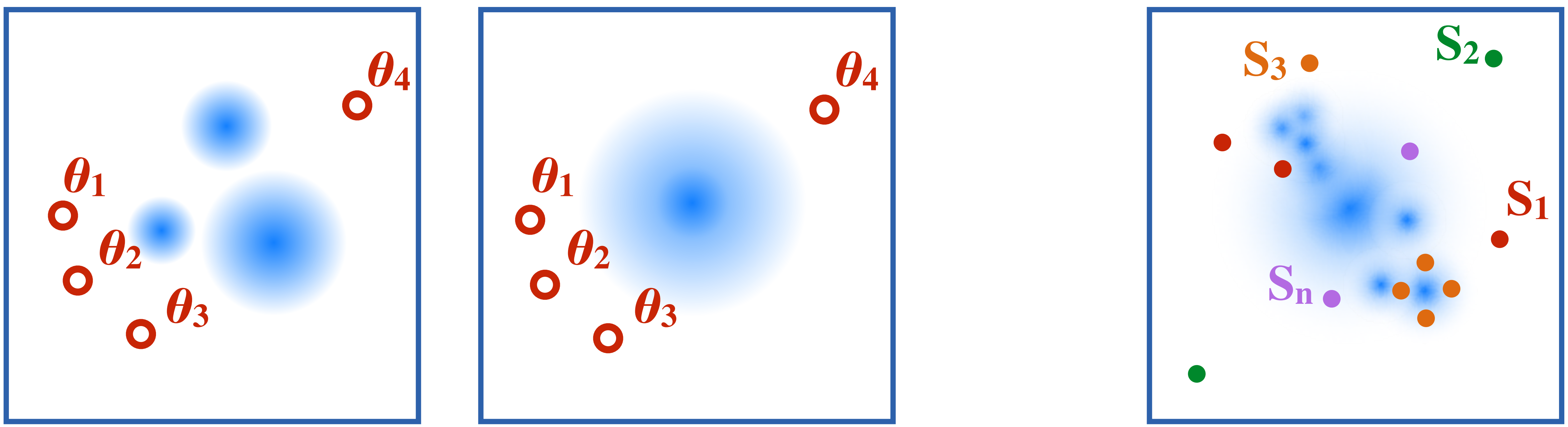
- More efforts in recent years to mitigate this
  - Public availability of modeling codes
  - Modeling challenges to compare models
  - Larger campaigns involving multiple teams

Modeling Gravitational Lenses

- Constraints for the model
  - Source Redshift
  - Multiple image positions
  - Relative fluxes and surface brightnesses
  - Galaxy morphologies and (distorted) sizes - shear measurements
  - Parity measurements
  - Time-delays
  - Kinematics (stellar dynamics/cluster velocity dispersions) - independent mass
- Assumptions about the model
  - Parametric and/or non-parametric modeling
  - Mass distribution relative to light (light traces mass - LTM)
  - Smooth and/or multiple individual components
  - Single or multiple screen lens

Parametric or Non-Parametric Modeling

- Parametric: Models with parametrized assumed density profiles, e.g.,
  - Isothermal sphere (week 5): $$\rho(r) = \frac{\sigma^2}{2\pi G(r^2 + r_{\rm core}^2)}$$
  - NFW profile (Navarro+97): $$\rho(r) = \frac{\delta_c \rho_{\rm cr}(z)}{(r/r_{\rm scale})(1 + r/r_{\rm scale})^2} \quad \text{where} \quad \rho_{\rm cr}(z)\frac{3H^2(z)}{8\pi G}$$
- Populate the lens plane with such profiles to reproduce observables

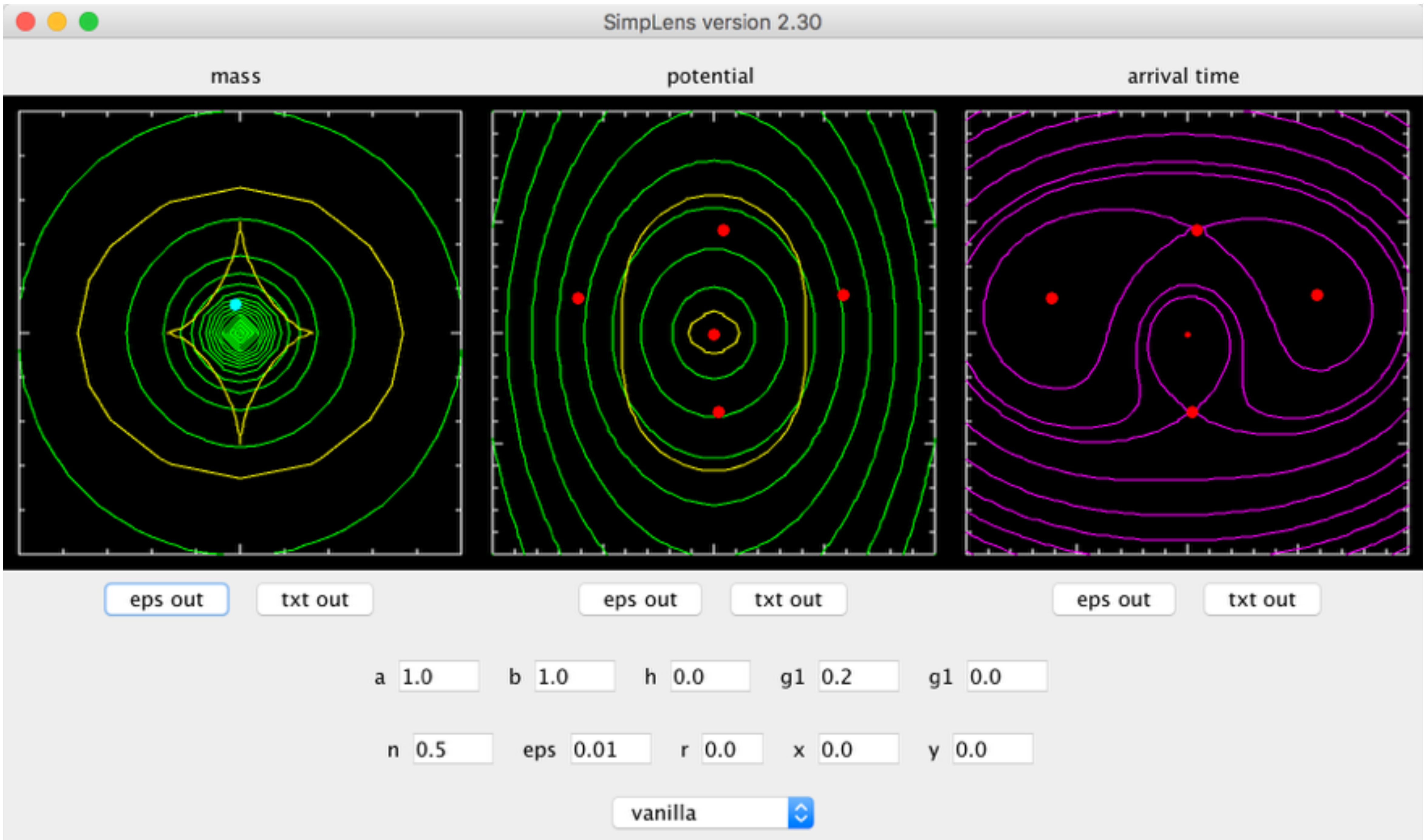


- Trace the light by solving the lens equation (transforms between β and θ)

$$\chi^2_{\rm img} = \sum_i \left(\frac{\boldsymbol{\theta}_i(\boldsymbol{\beta}) - \boldsymbol{\theta}_i}{\sigma_i} \right)^2$$

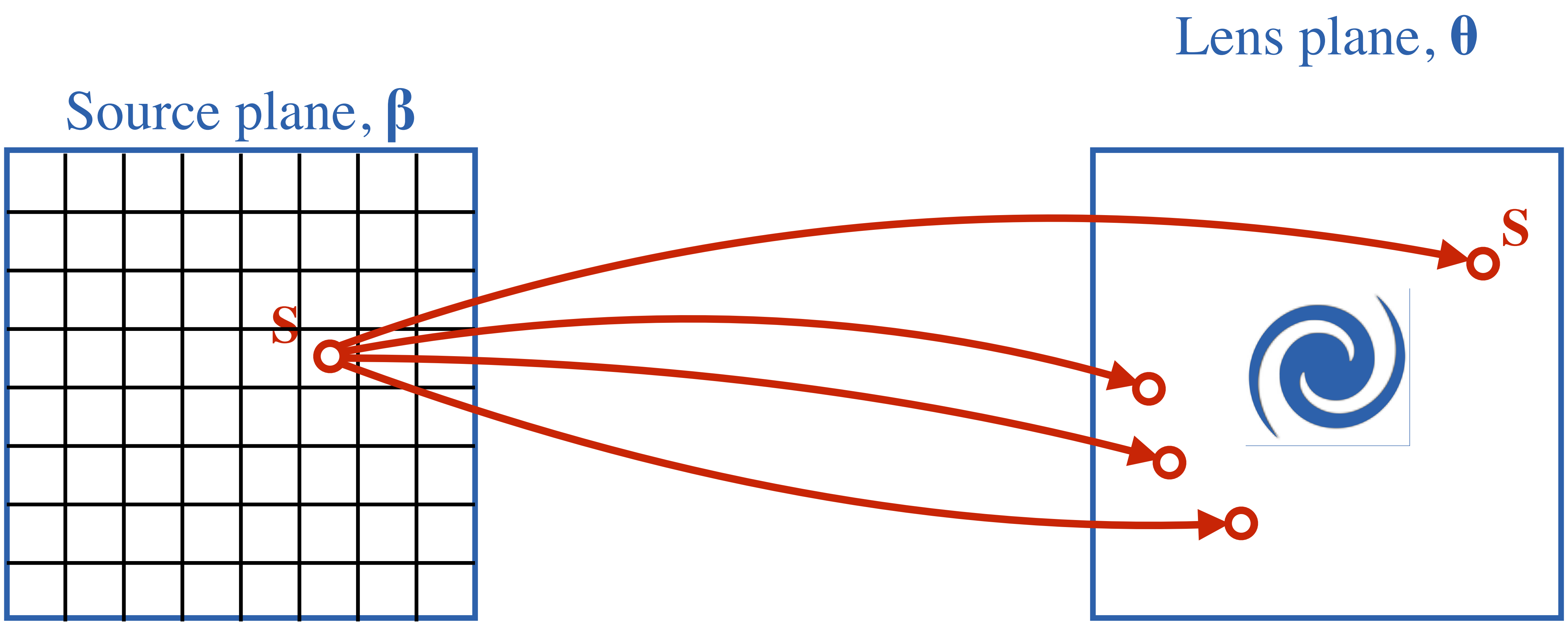
Illustration of Parametric lens modeling

- Saha & Williams (2003) presented a qualitative tool for lens modeling
  - SimpLens.jar



Parametric or **Non-Parametric** Modeling

- Basic idea: "There is an optimal estimate of source structure for any model"
- Surface brightness is conserved (week 7) so $\mathcal{S}(\boldsymbol{\beta}) = \mathcal{S}(\boldsymbol{\theta})$

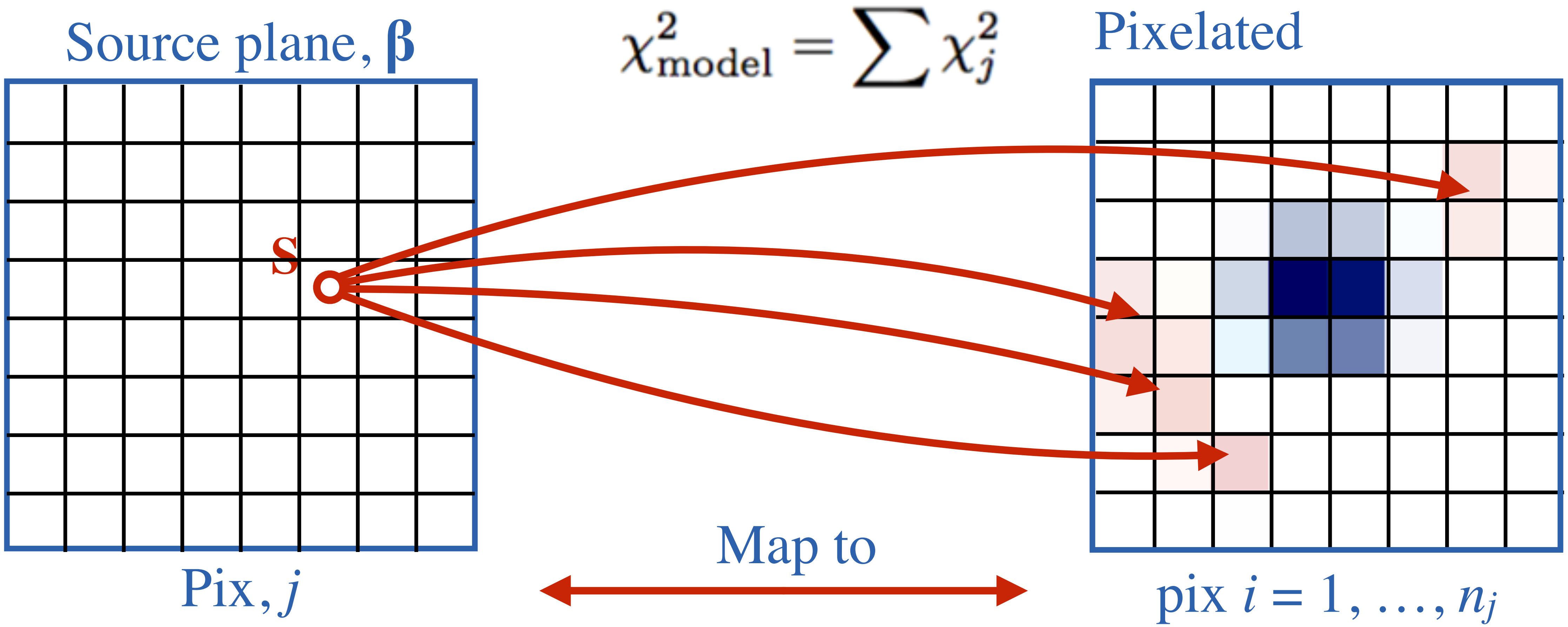


Parametric or **Non-Parametric** Modeling

- Basic idea: “There is an optimal estimate of source structure for any model”
- Surface brightness is conserved (week 7) so $\mathcal{S}(\boldsymbol{\beta}) = \mathcal{S}(\boldsymbol{\theta})$
- The lens equation describes the ‘source position—image position’ relation
- The goodness of fit can be estimated with

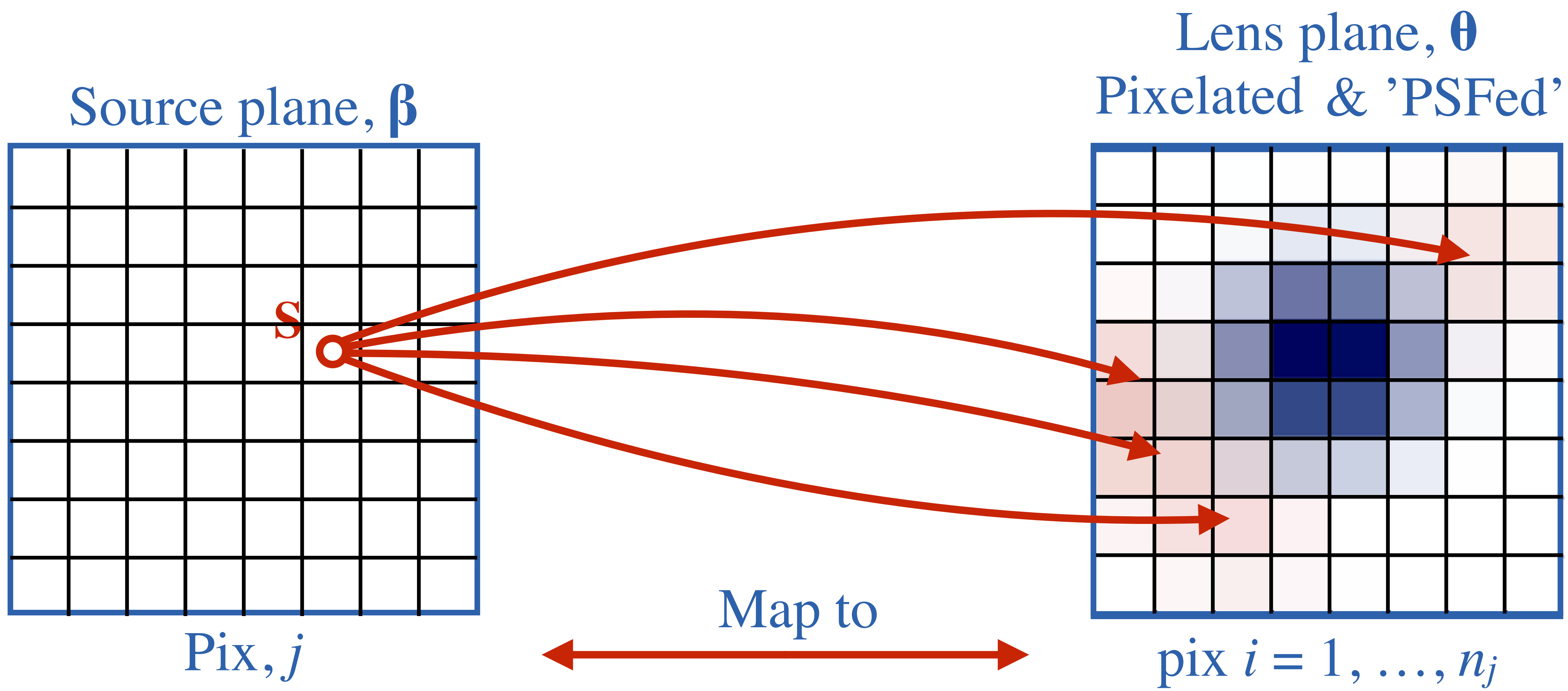
$$\chi_{\mathrm{j}}^2 = \sum_{i=1}^{n_j} \left(\frac{\mathcal{S}_i - \mathcal{S}_{\text{source plane}}}{\sigma_i} \right)^2$$

$$\chi_{\text{model}}^2 = \sum \chi_j^2$$



Parametric or **Non-Parametric** Modeling

- But… we never have a true surface brightness mapping
- The point spread function (PSF) of the telescope needs to be accounted for



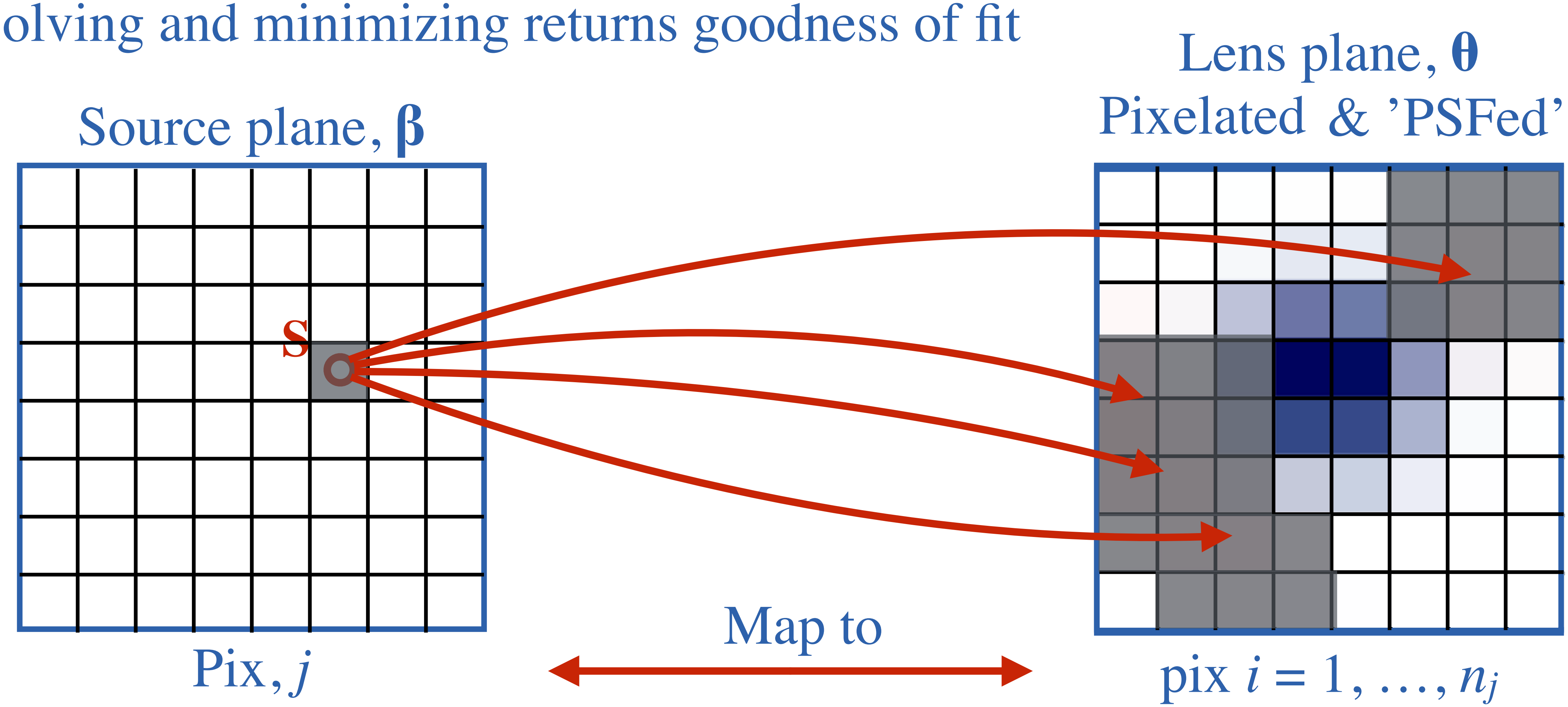
Parametric or **Non-Parametric** Modeling

- But… we never have a true surface brightness mapping
- The point spread function (PSF) of the telescope needs to be accounted for
- This can be described in terms of a set of linear equations (matrix eq.)

$$\chi^2 = \frac{\left|\boldsymbol{\mathcal{S}}_I - P(\mathrm{PSF},\ \text{lens model})\ \boldsymbol{\mathcal{S}}_{\text{source plane}}\right|^2}{\sigma^2}$$

  - Where P accounts for the PSF and lens model

- Solving and minimizing returns goodness of fit



The Mass Sheet Degeneracy

- But how unique can these (parametric or non-parametric) models become?
  - Even when assuming plenty of observational constraints
- Assume that your good model predicts some surface mass density, $\kappa(\boldsymbol{\theta})$
  - satisfying the Poisson equation (week 3) $\nabla^2\psi = 2\kappa$
- Then an equally good fit is obtained from the family of lens models with

$$\kappa_\lambda(\boldsymbol{\theta}) = (1-\lambda) + \lambda\kappa(\boldsymbol{\theta})$$

Adding homogeneous surface mass density, κ_c

Scaling of original κ

- To prove this statment, first consider the lens equation for κ_λ

$$\boldsymbol{\beta}_\lambda = \boldsymbol{\theta} - \boldsymbol{\alpha}_\lambda(\boldsymbol{\theta}) \quad \text{where} \quad \boldsymbol{\alpha}_\lambda(\boldsymbol{\theta}) = (1-\lambda)\boldsymbol{\theta} + \lambda\boldsymbol{\alpha}(\boldsymbol{\theta})$$

- Using (week 3) $\boldsymbol{\alpha} = \nabla\psi$ we also have for the scaled case that

$$\boldsymbol{\alpha}_\lambda(\boldsymbol{\theta}) = \nabla\psi_\lambda(\boldsymbol{\theta}) \quad \text{where} \quad \psi_\lambda(\boldsymbol{\theta}) = \frac{1-\lambda}{2}|\boldsymbol{\theta}|^2 + \lambda\psi(\boldsymbol{\theta})$$

The Mass Sheet Degeneracy

- This makes sure that the Poisson equation holds in the scaled case, i.e.

$$\nabla^2 \psi_\lambda = 2\kappa_\lambda \qquad \text{(Exercise 3.1)}$$

- Combining the two equations we get

$$\frac{\boldsymbol{\beta}_\lambda}{\lambda} = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta}) \qquad \text{(Exercise 3.2)}$$

- So the κ_λ lens equation deviates from the original lens equation through λ only
  - The source plane coordinates are scaled by the factor λ
  - You can't observe the source plane so effect is unobservable

Week 7

$$\mathcal{A}(\boldsymbol{\theta}) = \begin{pmatrix} \frac{\partial \beta_i}{\partial \theta_i} & \frac{\partial \beta_i}{\partial \theta_j} \\ \frac{\partial \beta_j}{\partial \theta_i} & \frac{\partial \beta_j}{\partial \theta_j} \end{pmatrix}$$

$$\mu \equiv \frac{1}{\det \mathcal{A}(\boldsymbol{\theta})}$$

- Hence, the Jacobian matrix and the magnification behave like

$$\mathcal{A}_\lambda = \lambda \mathcal{A} \qquad \mu_\lambda = \frac{\mu}{\lambda^2}$$

- So from the definitions of shear and convergence (week 7) we get

$$\gamma_\lambda(\boldsymbol{\theta}) = \lambda \gamma(\boldsymbol{\theta}) \qquad (1-\kappa_\lambda) = \lambda(1-\kappa) \qquad \text{(Exercise 3.3)}$$

- In agreement with our initial statement: $\kappa_\lambda(\boldsymbol{\theta}) = (1-\lambda) + \lambda\kappa(\boldsymbol{\theta})$

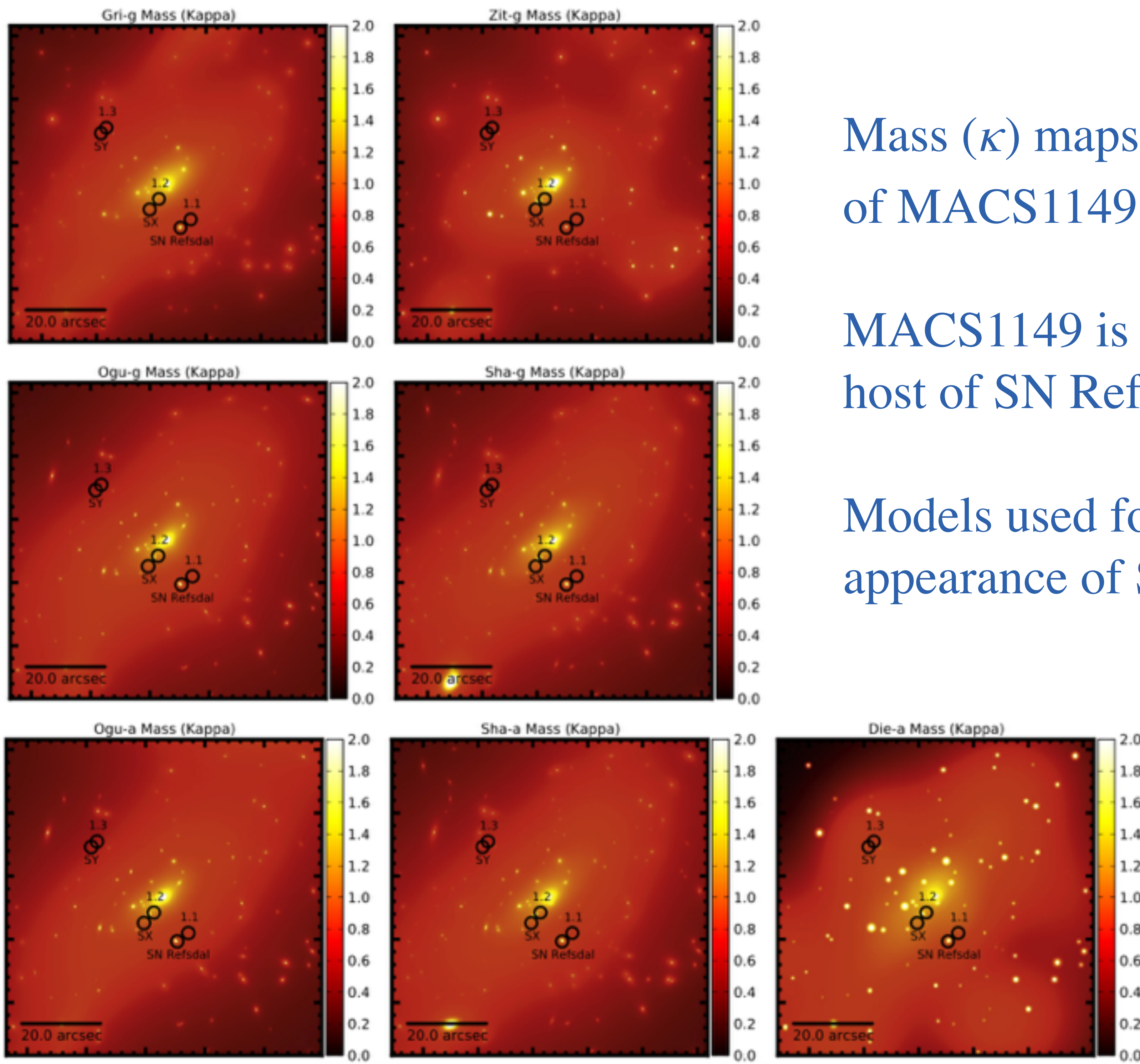
The Mass Sheet Degeneracy

- So this illustrates that:

  For any good lens model, an equally good lens model can be obtained by adding a ‘sheet’ of mass to the surface mass density of the model and scaling it by a corresponding factor, call it λ

- To break this degeneracy, modelers need prior information on either
- The absolute scale of the source
  - By knowing size or luminosity (scale) of the object
- An absolute mass scale for lens
  - obtained from stellar kinematics or cluster velocity dispersions
- Source positions as a function of redshift
  - multiple lensed systems at different redshifts (distances, D_S)
  - κ differs with source redshift as it depends on Σ_{cr} which depends on D_S

Treu et al. 2016 Mass Models



Mass (κ) maps for different lens models of MACS1149 shown in week 7

MACS1149 is the cluster lensing the host of SN Refsdal

Models used for predicting re-appearance of SN Refsdal

Treu et al. 2016 Modelers

Diego et al.: `WSLAP+`

- Galaxies and cluster 'diffuse' mass components
- Galaxies assumed fixed M/L (except BCG) with NFW profile
- Diffuse mass determined by adaptive grid pixillation

Grillo et al.: `GLEE`

- 300 cluster galaxies modeled as "pseudoisothermal elliptical" (dPIE)
- Scaling M/L of individual galaxies to match empirical $M/L \propto L^{0.2}$ relation
- 3 extra "dark matter" halos are added

Zitrin et al.:

- Light traces mass
- Scaling and smoothing of power-law distributions
- Best-fit obtained via MCMC chain conversion

Short name	Team	Type	rms	Images
Die-a	Diego et al.	Free-form	0.78	gold+sil
Gri-g	Grillo et al.	Simply param	0.26	gold
Ogu-g	Oguri et al.	Simply param	0.43	gold
Ogu-a	Oguri et al.	Simply param	0.31	all
Sha-g	Sharon et al.	Simply param	0.16	gold
Sha-a	Sharon et al.	Simply param	0.19	gold+sil
Zit-g	Zitrin et al.	Light-tr-mass	1.3	gold

rms: root mean square of obs. vs. model img positions in arcsec

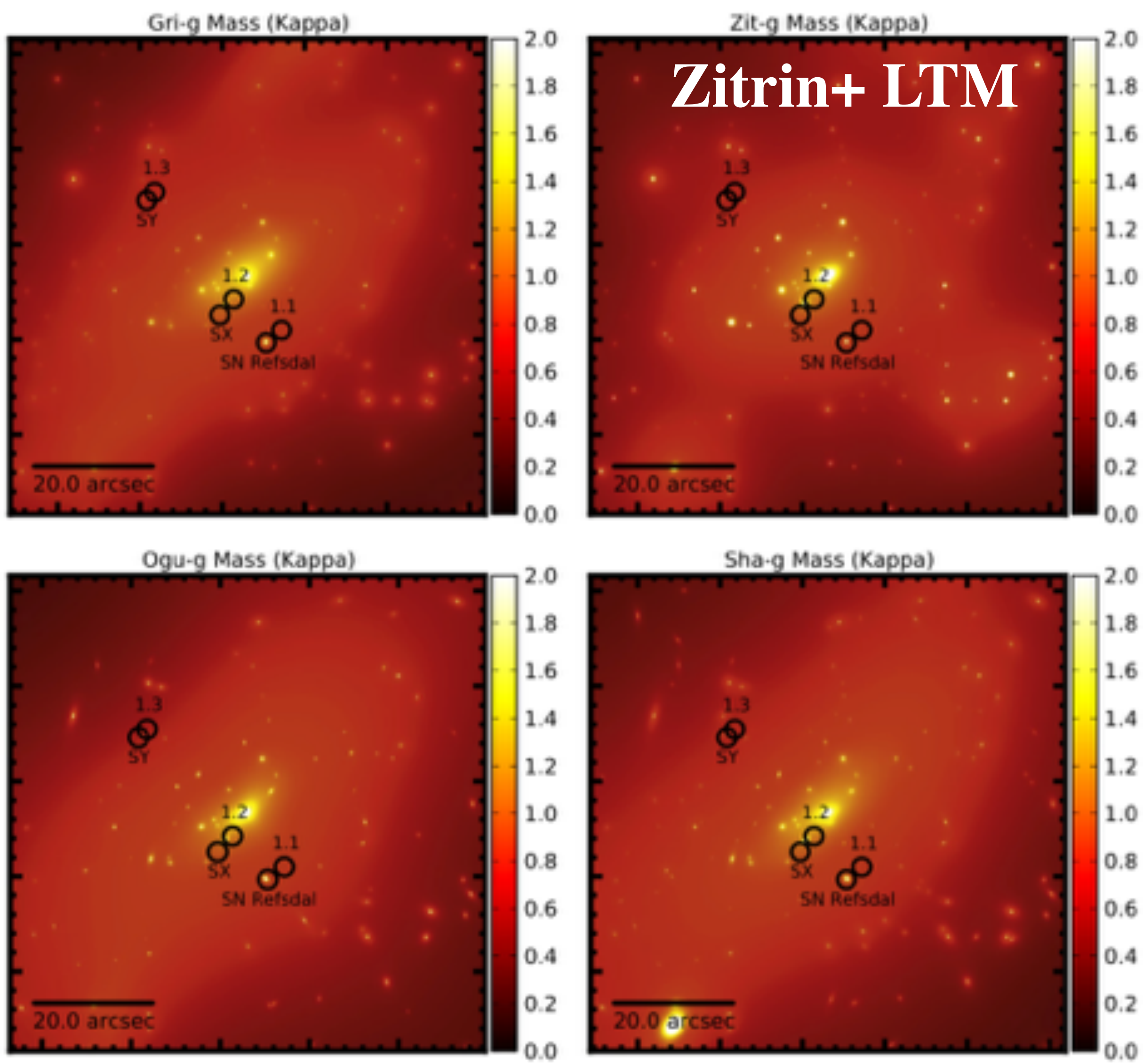
Oguri et al.: `GLAFIC`

- Assumes small number of matter components: some follow galaxies (Jaffe profiles), some 'free' (NFW)
- Best model obtained from direct χ^2 minimization

Sharon et al.: `Lenstool`

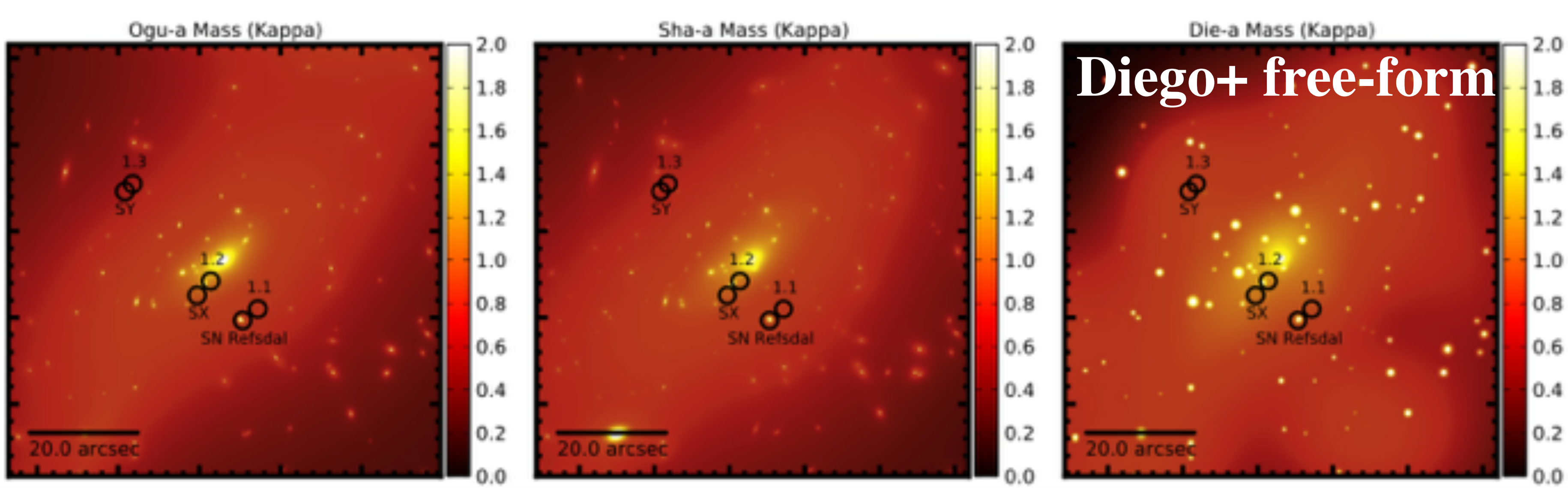
- Assumes elliptical mass distributions (functional for of mass components
- Cluster and galaxy scale halos
- Cluster scale halo positions free to vary

Treu et al. 2016 Mass Models

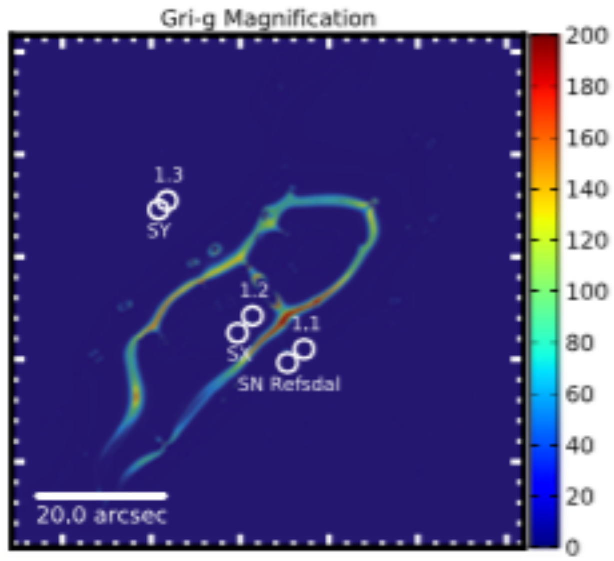


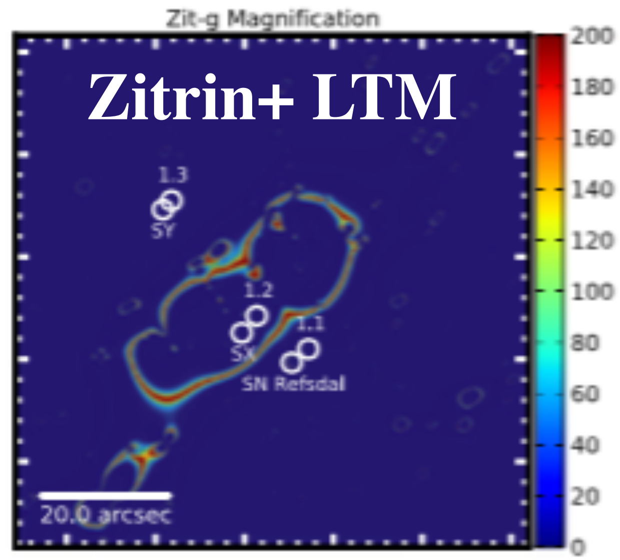
Mass (κ) maps for different lens models

Team	Type	rms	Images
Diego et al.	Free-form	0.78	gold+sil
Grillo et al.	Simply param	0.26	gold
Oguri et al.	Simply param	0.43	gold
Oguri et al.	Simply param	0.31	all
Sharon et al.	Simply param	0.16	gold
Sharon et al.	Simply param	0.19	gold+sil
Zitrin et al.	Light-tr-mass	1.3	gold

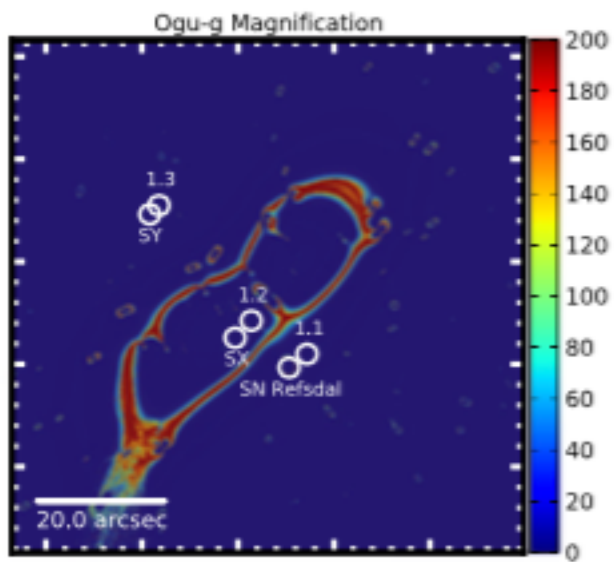


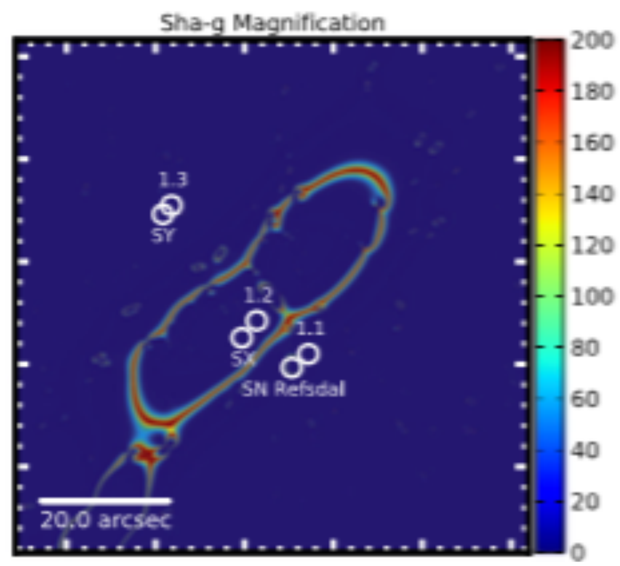
Treu et al. 2016 Magnification Models



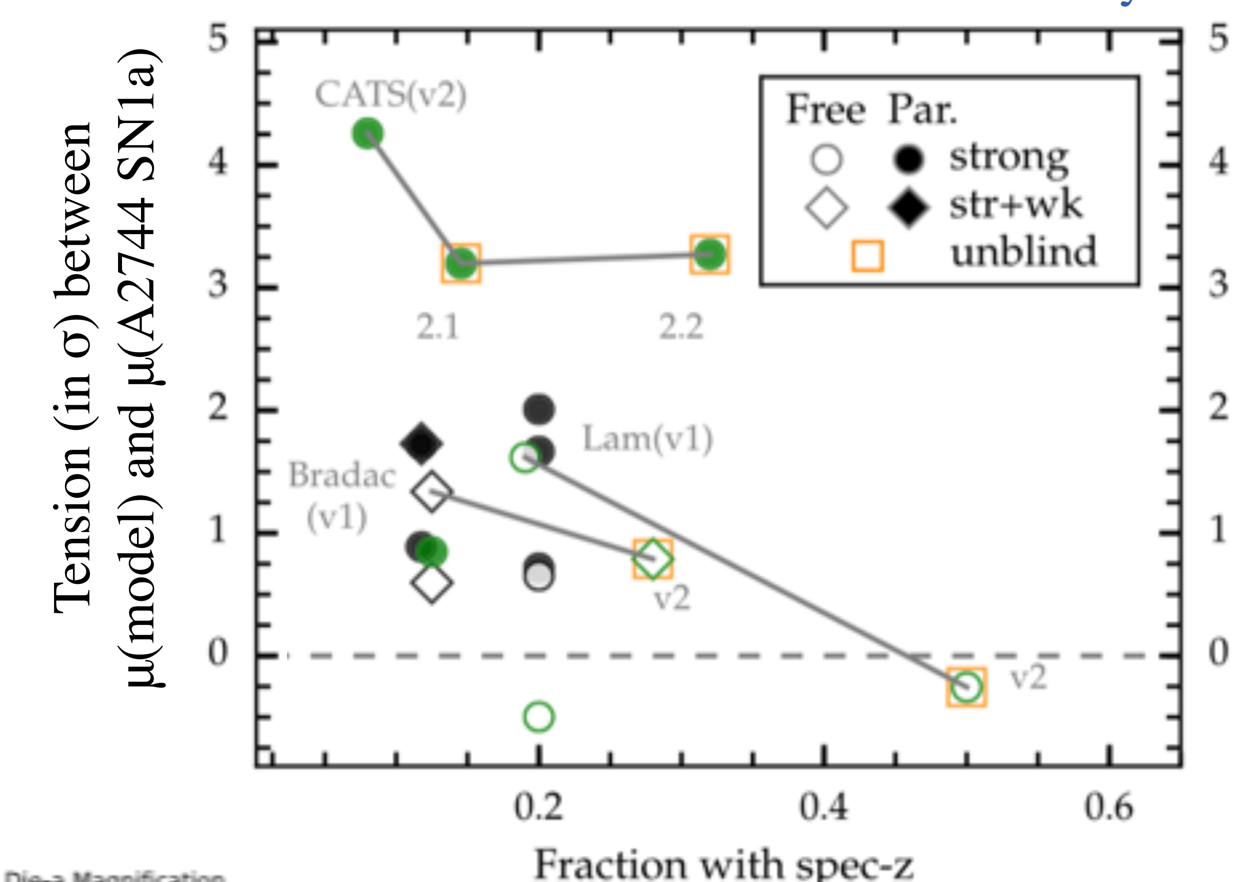


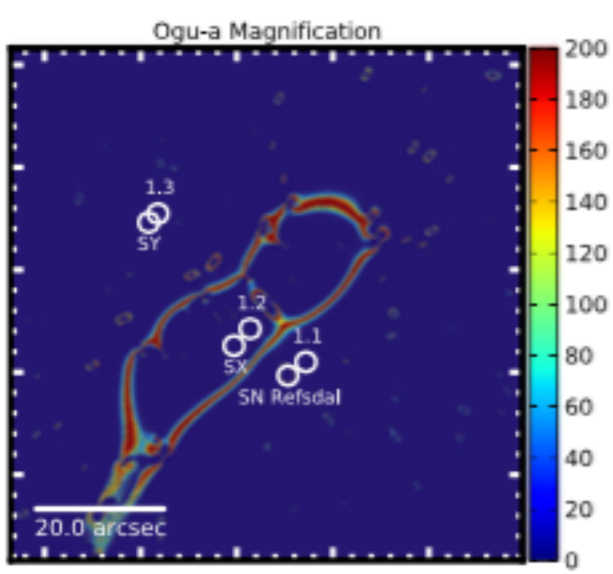
Magnification (μ=0-200) maps for different lens models

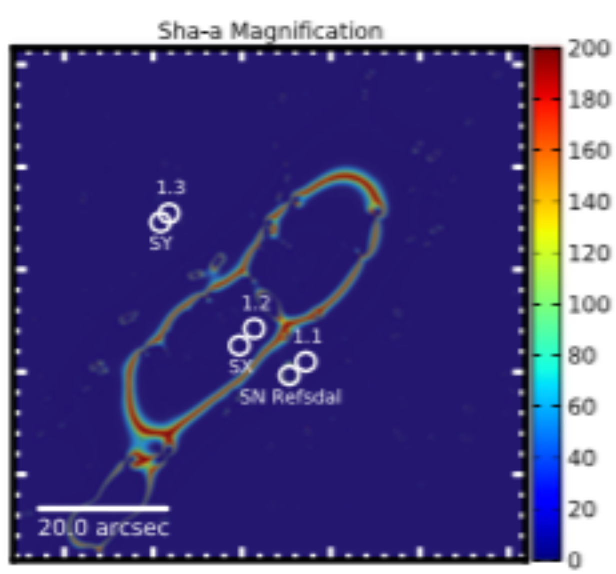


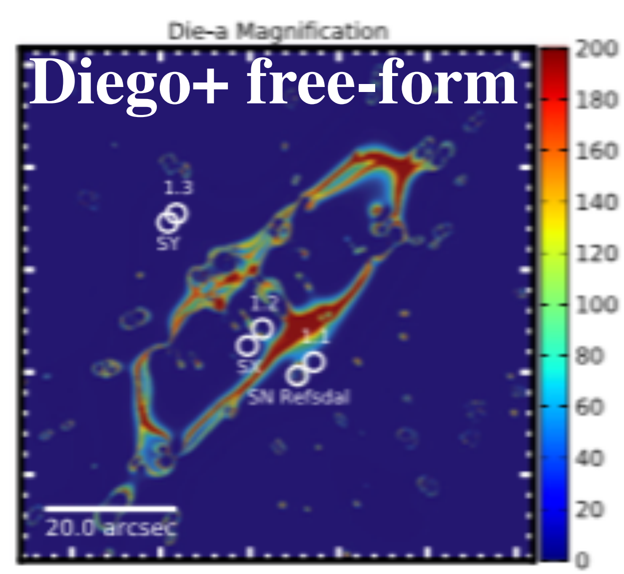


Rodney+15





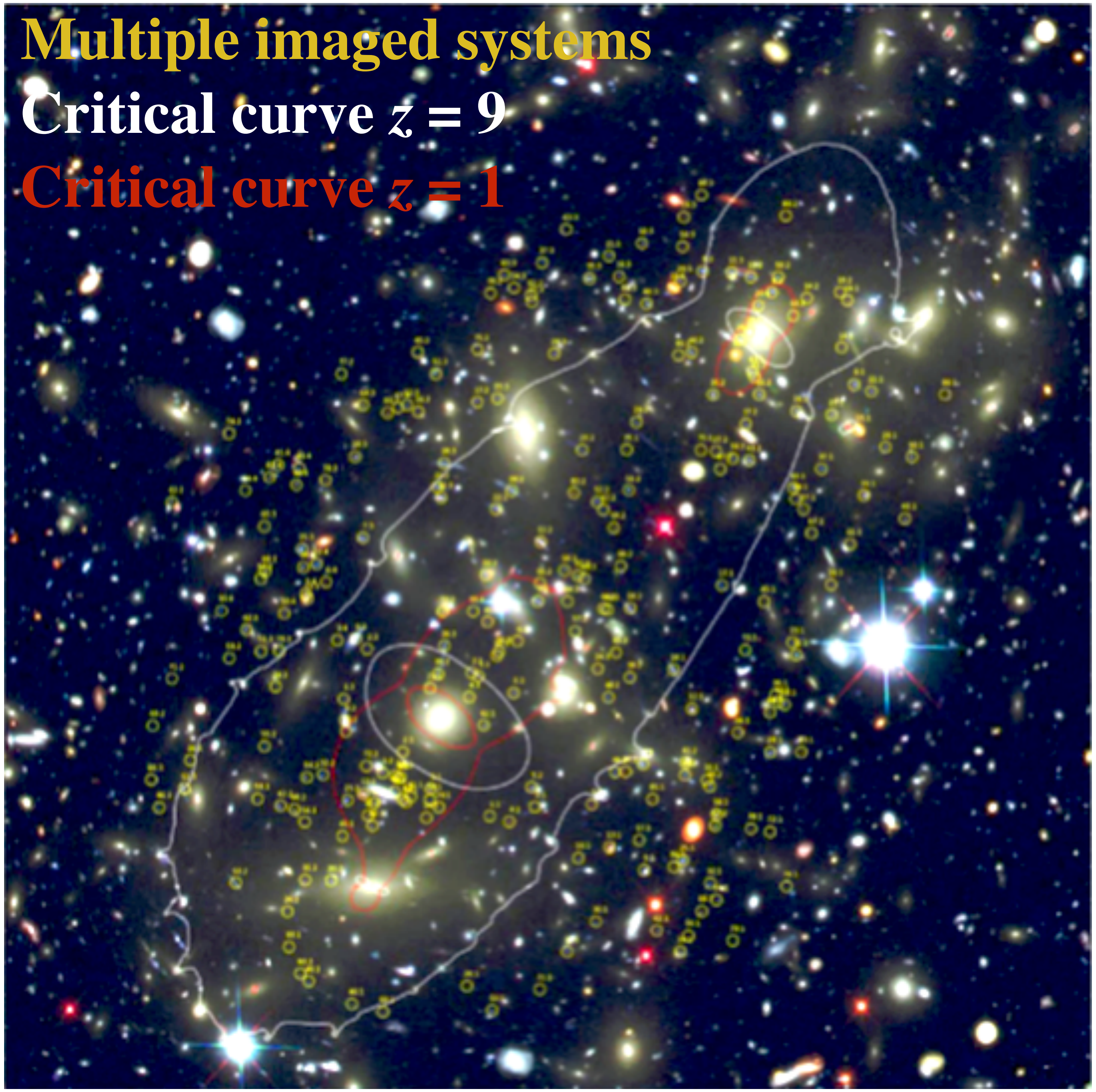




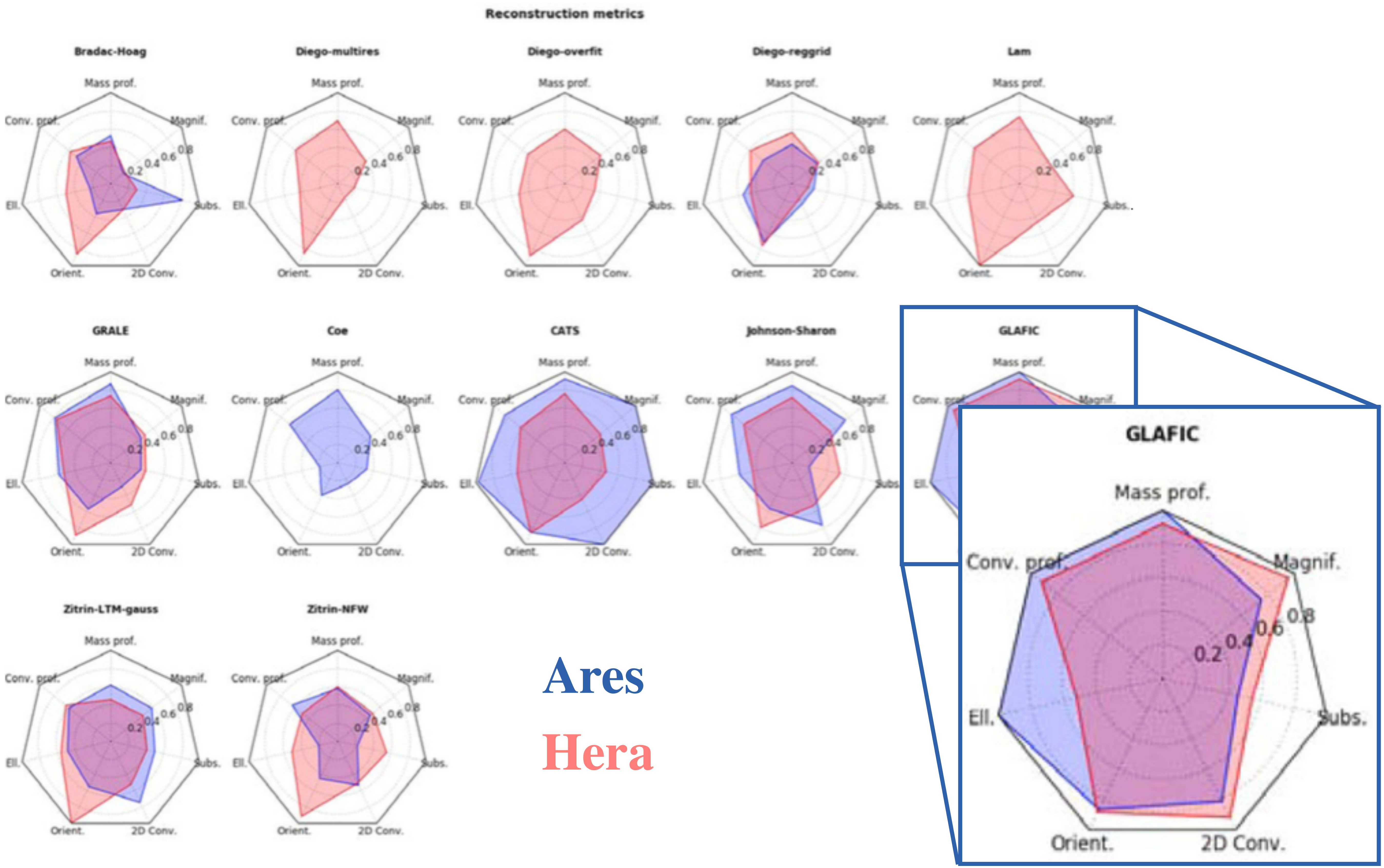
Meneghetti et al. (2017) Model Comparison

- Model Cluster (Ares & Hera)
  - $z = 0.5$
  - $M_{tot} \sim 2 \times 10^{15}\, M_\odot$
- Produced by ray-tracing with
  - `MOKA` (Giocoly+12)
- HST images generated with
  - `SKYLENS` (Meneghetti+08,10)
- Asked cluster modelers to predict κ and μ (among other things)
- Provided:
  - Multiple images (with redshifts)
  - Cluster members
  - Large FoV image of background obj for shape measurements

Synthetic Galaxy Cluster 'Ares'



Meneghetti et al. (2017) Comparison Metrics

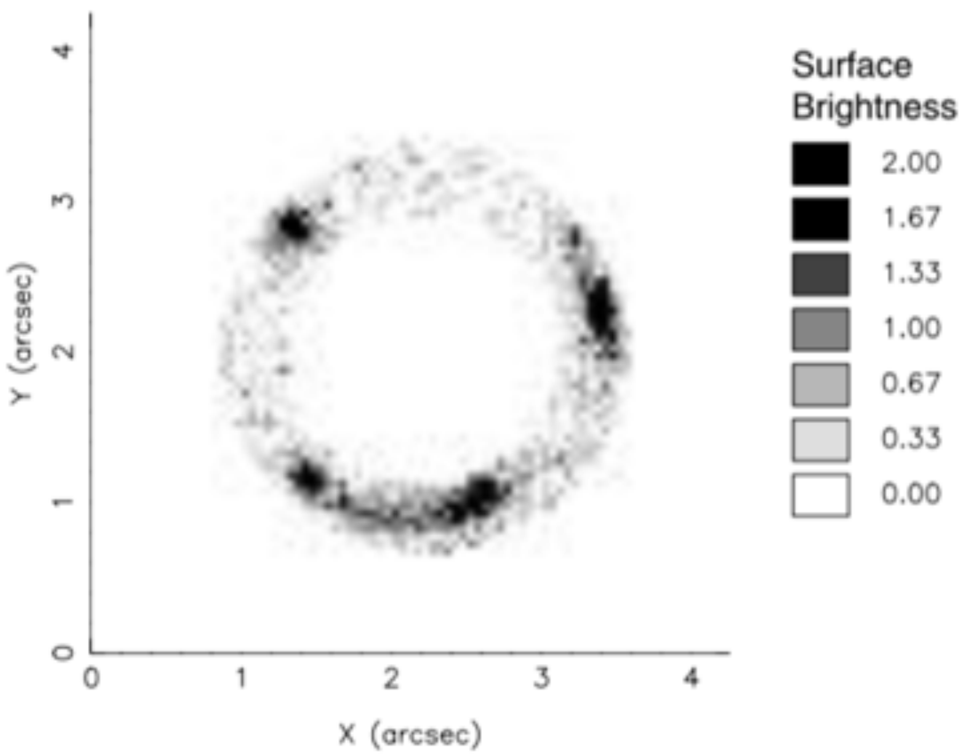


Meneghetti et al. (2017) Findings

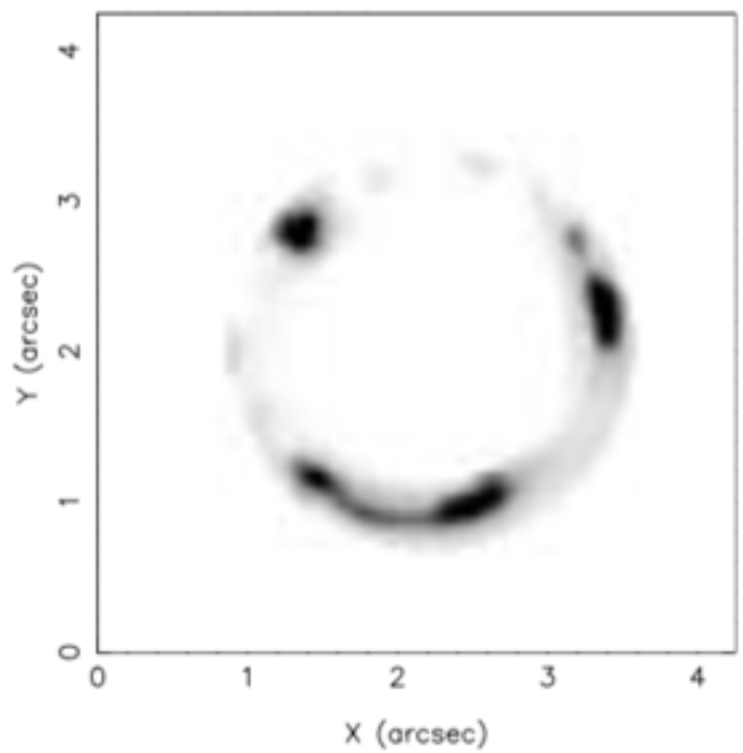
- First time such an extensive lens-comparison study was made
  - A good step on the way away from the “black art” of lens modeling
- Parametric models better at capturing 2D structure
- Non-parametric models competitive when determining 1D κ profiles
- Mass($<\theta_E$), i.e. where strong lensing happens, is of the order a few %(!)
  - Substructures (cluster members) around critical lines increase this to ~10%
- Strongest limitation of parametric models: determining asymmetries

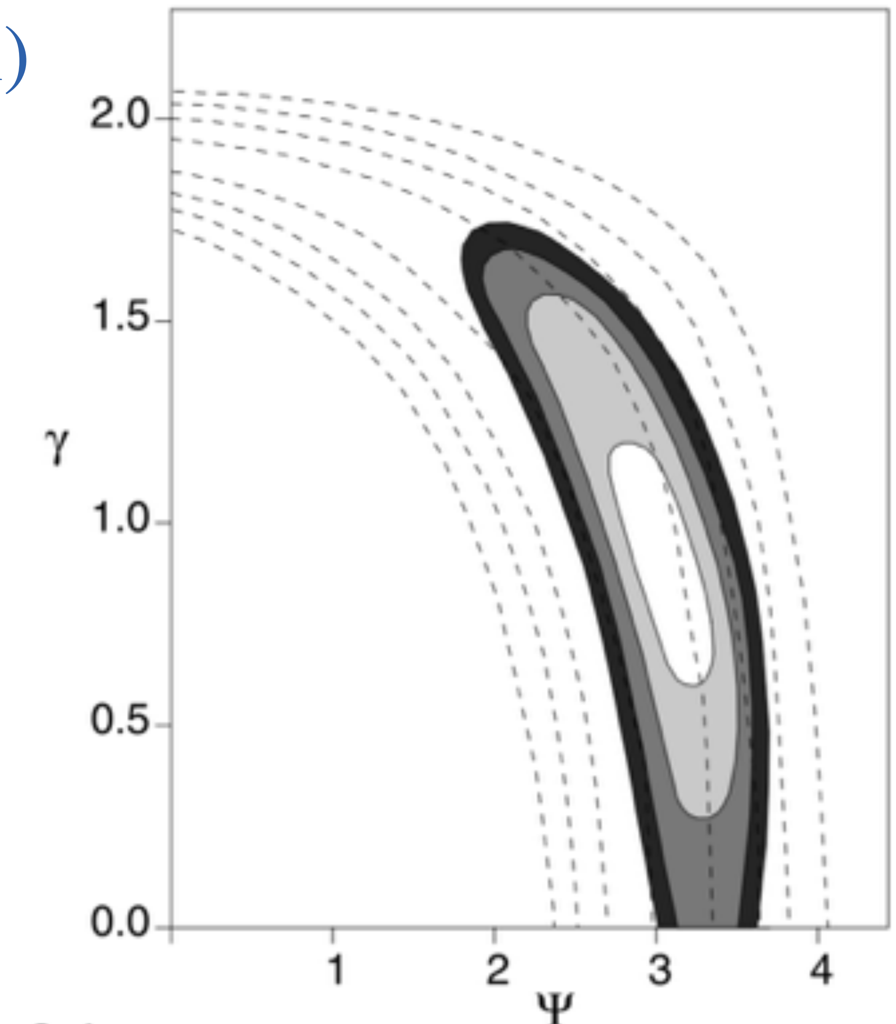
Model Example: Einstein Ring 0047-2808

Lens plane, $\boldsymbol{\theta}$ (obs.)

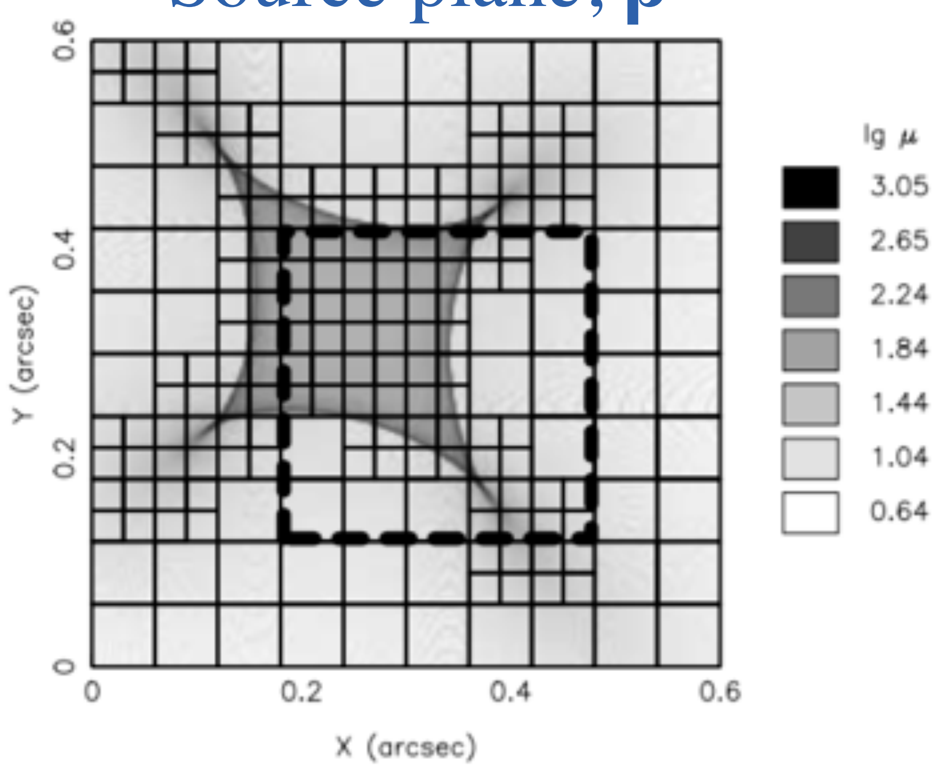


Lens plane, $\boldsymbol{\theta}$ (model)

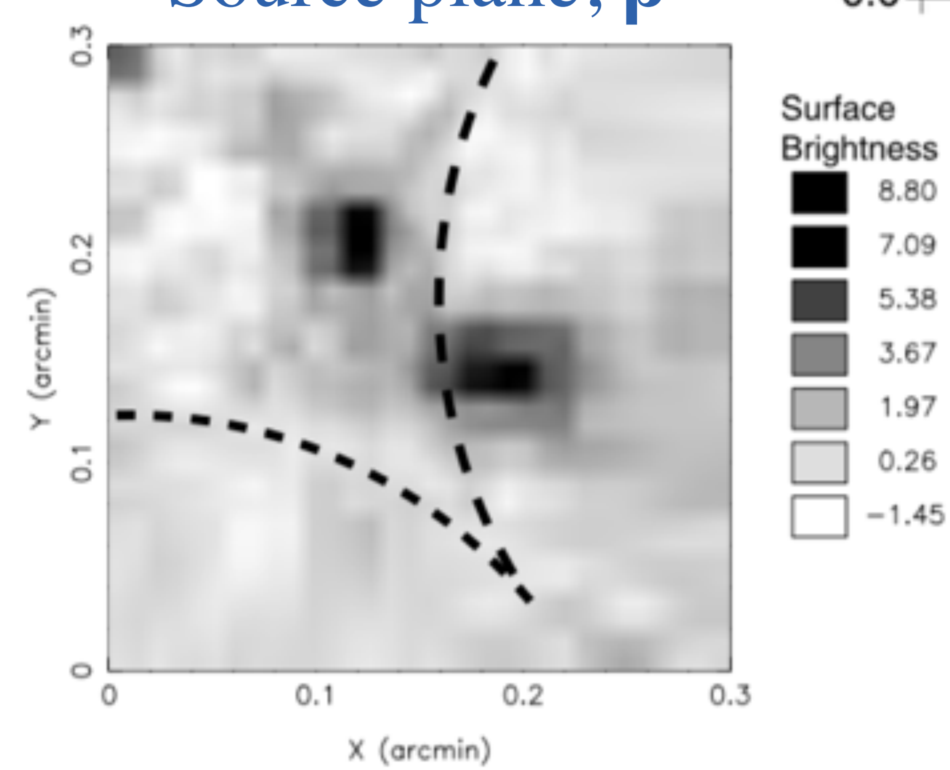




Source plane, $\boldsymbol{\beta}$



Source plane, $\boldsymbol{\beta}$



DM (γ) & M/L (Ψ) constraints from model

$$\rho(r) = \frac{\rho_s}{(r/r_s)^{\gamma}(1 + r/r_s)^{3-\gamma}}$$

Dye & Warren (2005)

So in summary…

- Lens models are split into parametric and non-parametric models
- The goal of models is to minimize disagreement with observations, e.g.,
  - in terms of image positions $$\chi^2_{\mathrm{img}} = \sum_i \left(\frac{\boldsymbol{\theta}_i(\beta) - \boldsymbol{\theta}_i}{\sigma_i} \right)^2$$
  - surface brightness measurements $$\chi^2 = \frac{\left| \boldsymbol{\mathcal{S}}_I - P(\mathrm{PSF},\ \mathrm{lens\ model})\, \boldsymbol{\mathcal{S}}_{\mathrm{source\ plane}} \right|^2}{\sigma^2}$$
- The Mass Sheet Degeneracy states that:

For any good lens model with $\kappa(\boldsymbol{\theta})$, an equally good lens model can be obtained by a model with $\kappa(\boldsymbol{\theta}) = (1 - \lambda) + \lambda\kappa(\boldsymbol{\theta})$

- MSD can be broken with multiple lensed systems or kinematic masses
- Improved efforts for comparison of (cluster) lens models are underway
  - Treu+16: Comparison of models to predict SN Refsdal re-appearance
  - Rodney+15: Comparison of models predicting SN 1a magnification
  - Meneghetti+17: Comparison of model predictions for (two) simulated cluster