

PHY-765 SS19 Gravitational Lensing Week 10

Searching for Exoplanets with Gravitational Lensing

Kasper B. Schmidt

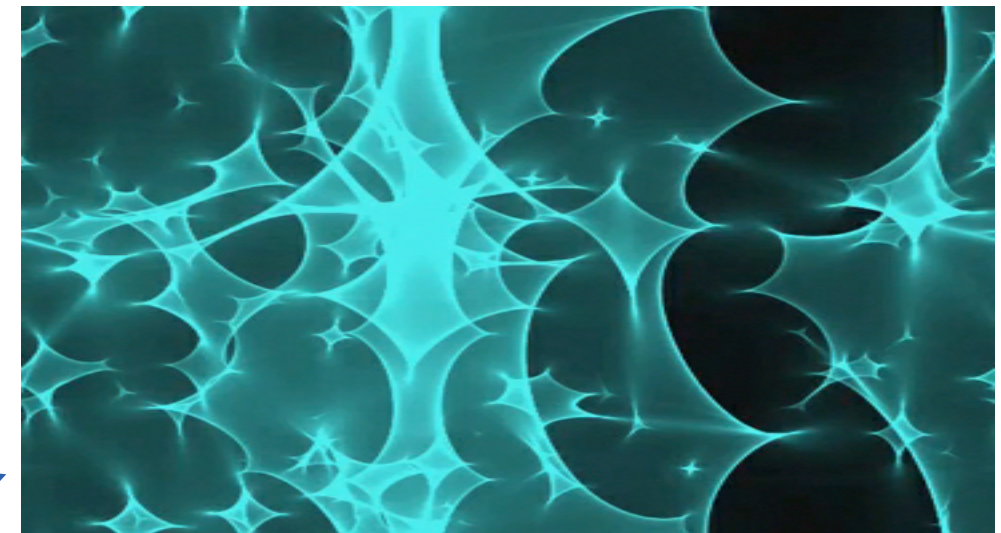
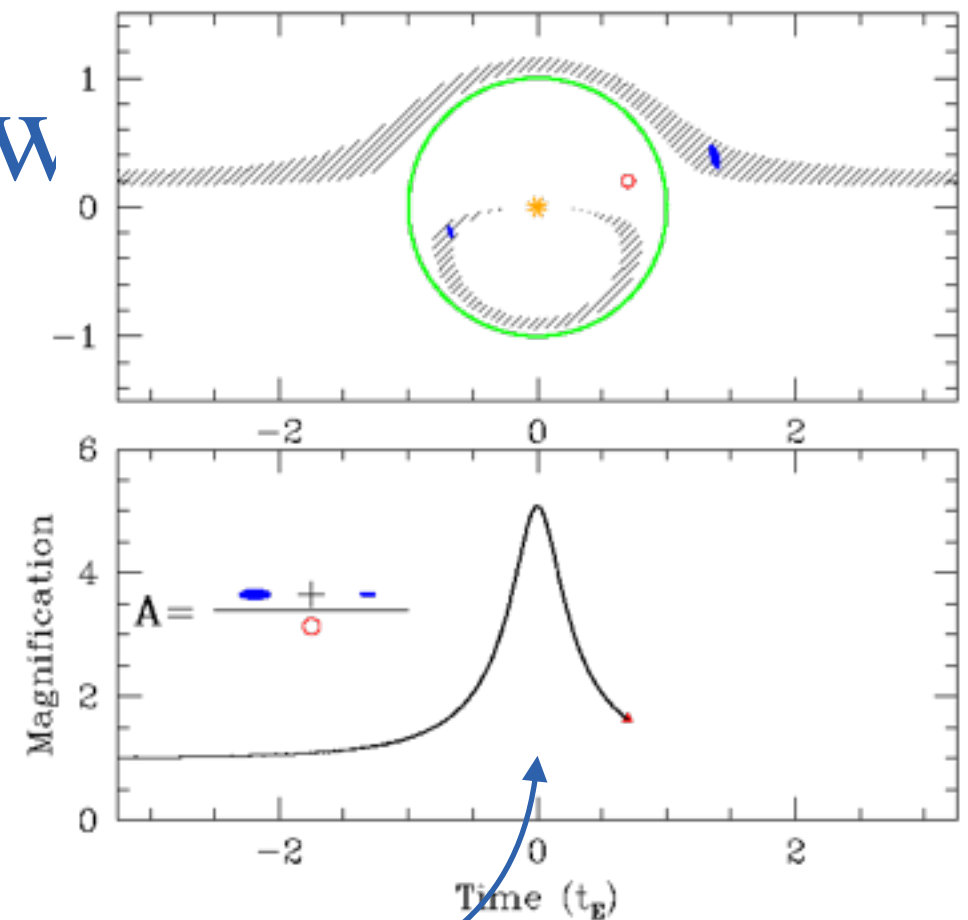
Leibniz-Institut für Astrophysik Potsdam (AIP)

Last week - what did w

- Microlensing
 - Time-variability often seen in lens-frame
- The point mass lens (un-resolved images):

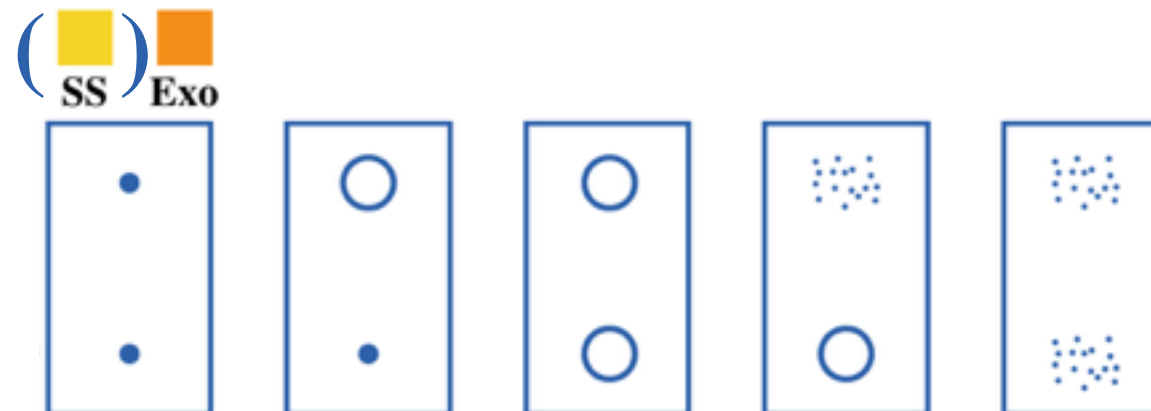
$$\mu = \frac{y^2 + 2}{y\sqrt{y^2 + 4}} \quad \text{where} \quad y = \frac{\beta}{\theta_E}$$

- The Paczynski curve describing source ‘light curve’
- Define the lensing optical depth: Probability of sight-line with microlensing
- Showed examples of detected events from MACHO, EROS and OGLE
- Using Microlensing to estimate DM MACHO-fraction in MW halo
 - ~20% from the MACHO survey
 - MW halo mostly non-MACHO
- Extra-galactic microlensing of multiple stars in lens-galaxies of QSO lenses. “μ-map”



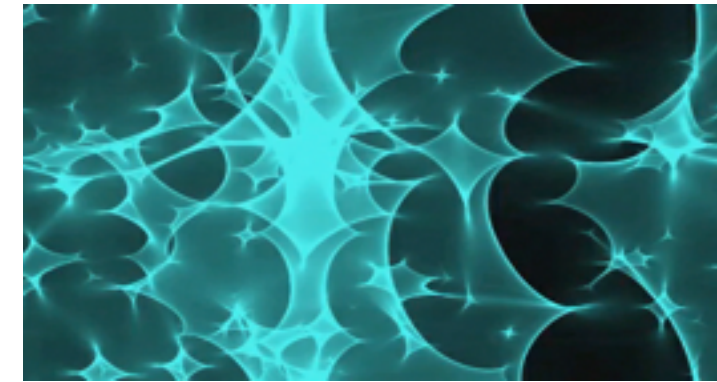
The aim of today

- The case of a multiple point mass lens system
- Specialize to the double point mass lens
- Image location and magnification for a star + planet lens
- Other methods for finding exoplanets
 - Radial velocities
 - Transits
 - Astrometry
 - Direct imaging
- Why do we need another method, i.e., microlensing?
- Examples of planets found with microlensing



The Multiple Point Mass Lens

- Want to generalize the concept of microlensing
 - which is also needed for extra-galactic microlensing



- Starting from the lens equation $\beta = \theta - \alpha(\theta)$
- The deflection angle for multiple point sources can be expressed as:

$$\alpha(\theta) = \sum_i \theta_{E,i}^2 \frac{\theta - \theta_i}{|\theta - \theta_i|^2}$$

Week 3/5 reminder:
Spherical Mass Distribution

$$\alpha(\theta) = \frac{1}{\pi} \int d^2\theta' \kappa(\theta') \frac{\theta - \theta'}{|\theta - \theta'|^2}$$

- Here θ_i are positions of the lenses in the lens plane.
 - But these are unlensed so the position in lens and source plane are identical
- So the only unknown in the lens equation is the source position θ
 - *relative* source position changes in time \rightarrow brightness (μ) of source changes
- Can define the combined Einstein radius of system as

$$\theta_E^2 \equiv \sum_i \theta_{E,i}^2 = \frac{MGD_{LS}}{D_L D_S c^2} \quad \text{where} \quad M \equiv \sum_i m_i$$

The Multiple Point Mass Lens

- Introduced normalized components of lens equation last week:

$$\mathbf{y} = \frac{\boldsymbol{\beta}}{\theta_E} \quad \mathbf{x} = \frac{\boldsymbol{\theta}}{\theta_E}$$

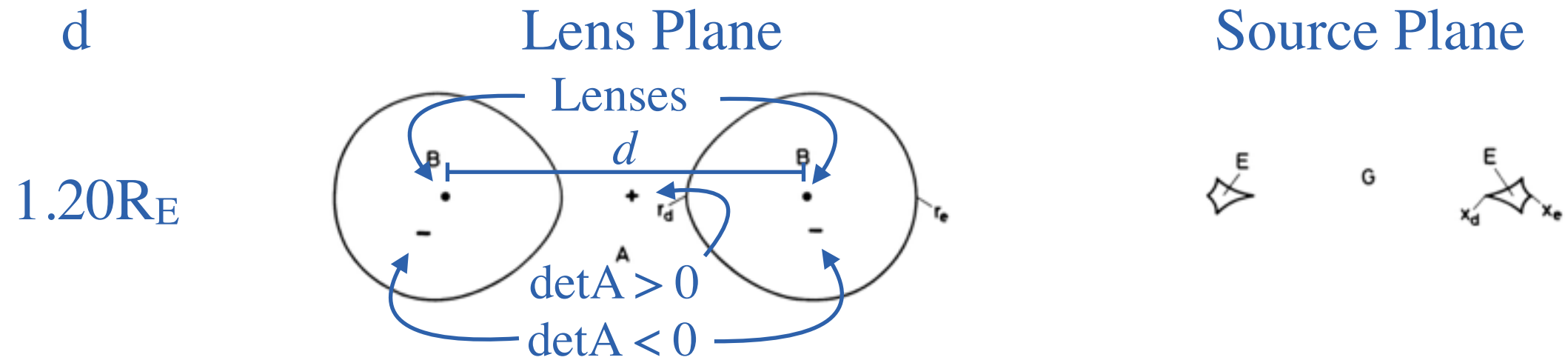
- Using definition of combined Einstein radius we get the lens equation

$$\mathbf{y} = \mathbf{x} - \sum_i \frac{m_i}{M} \frac{\mathbf{x} - \mathbf{x}_i}{|\mathbf{x} - \mathbf{x}_i|^2}$$

- Lens Equation non-linear, so combined effect is *not* just sum of effects

The Double Point Mass Lens

- Schneider & Weiss (1986) explored the double point mass lens in detail
 - Assuming $m_1 = m_2$, so considering binary stars, not planets
- Described the critical curves and caustics as a function of star separation, d

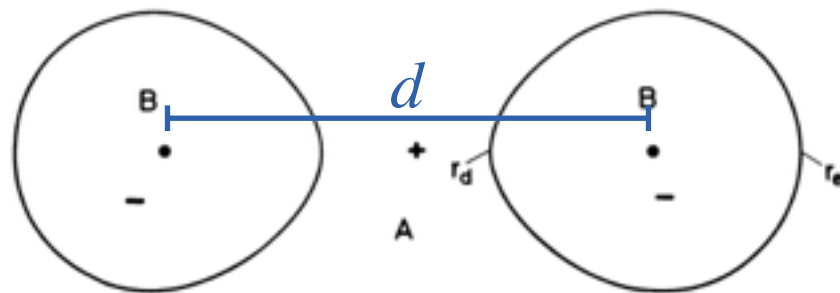


d

Lens Plane

Source Plane

$1.20R_E$

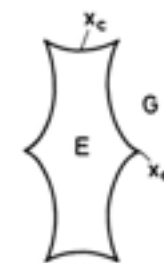
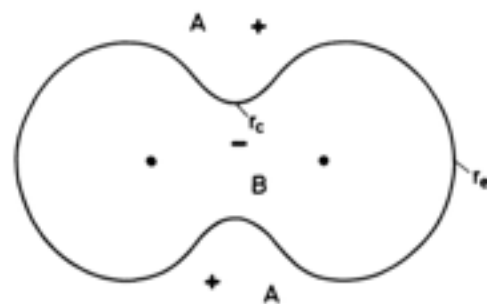


1st regime

$1.00R_E$

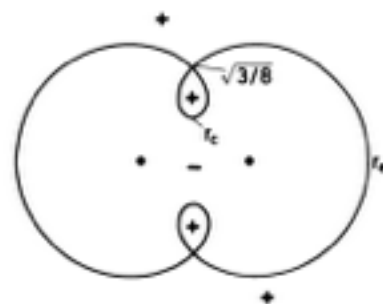


$0.50R_E$

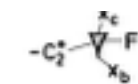
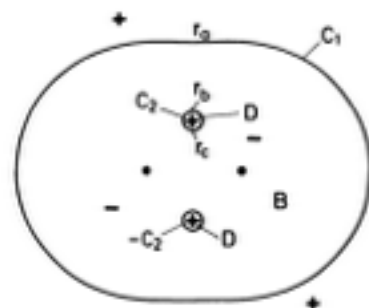


2nd regime

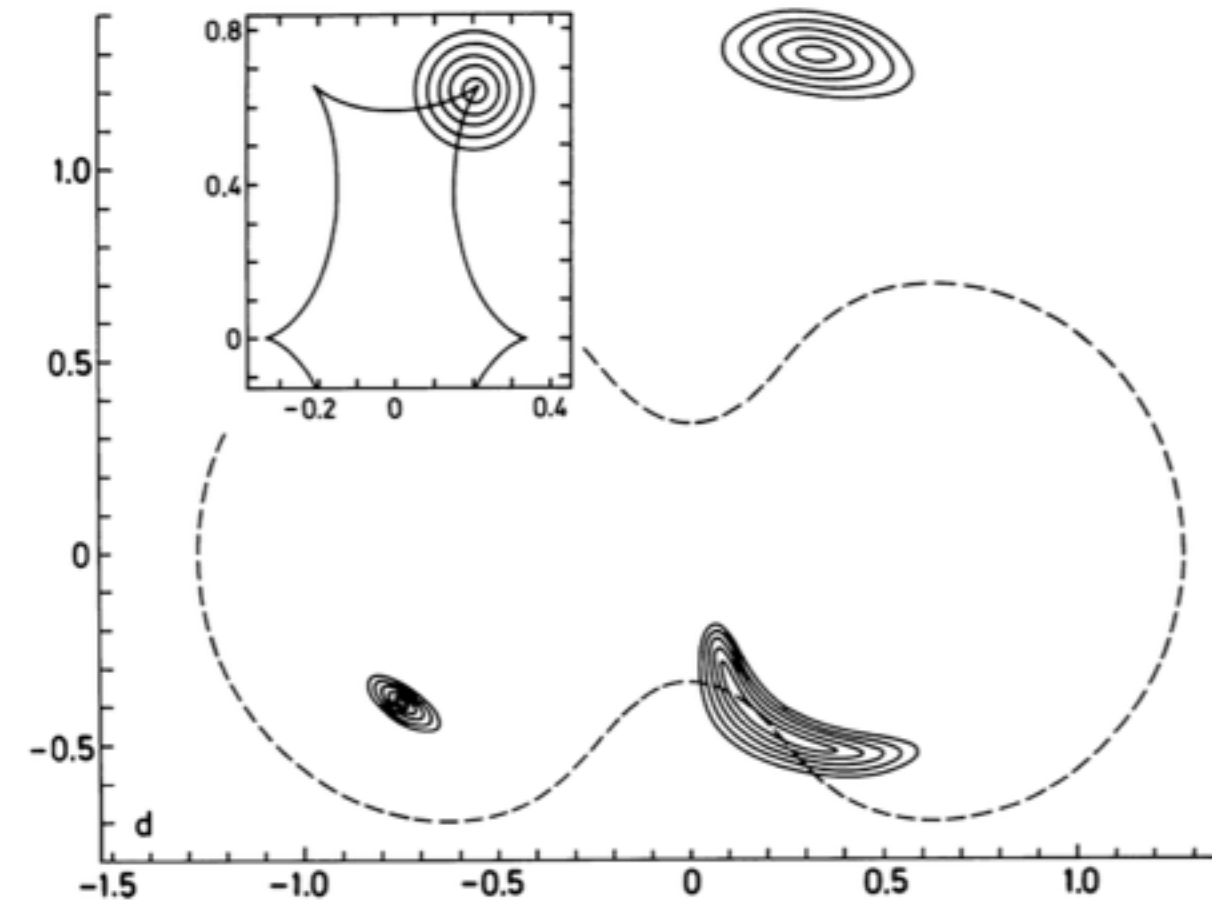
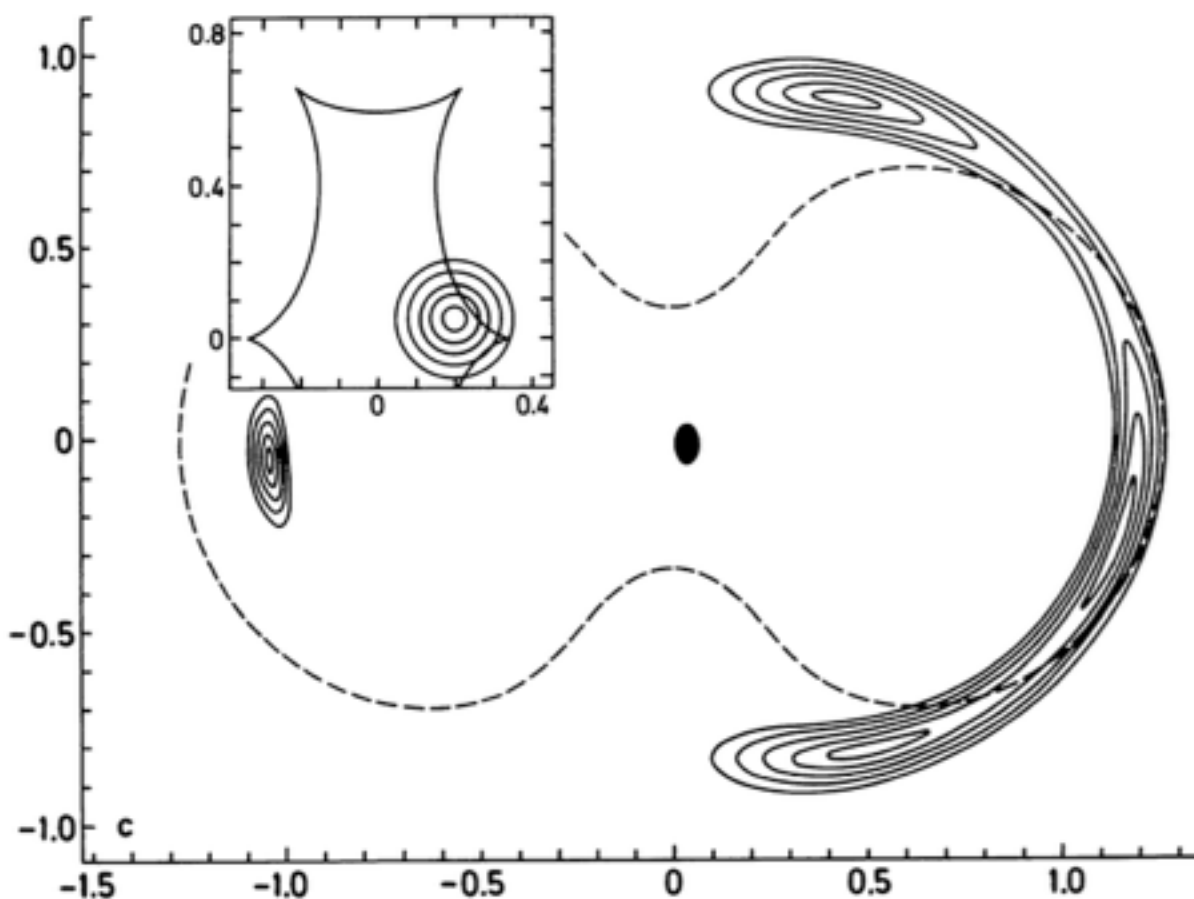
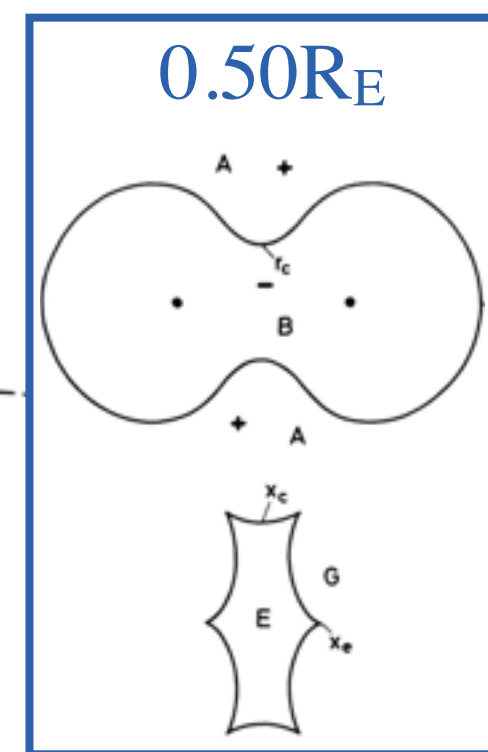
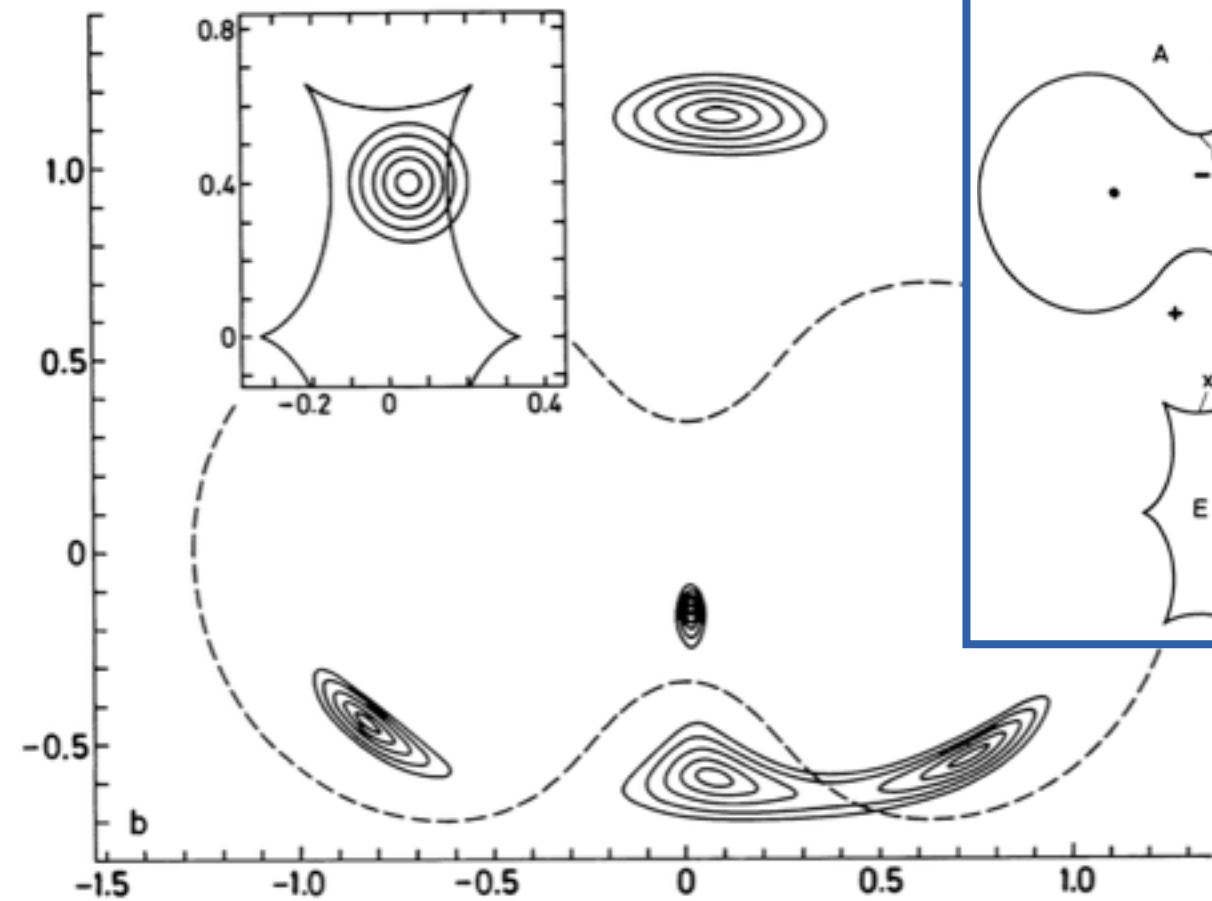
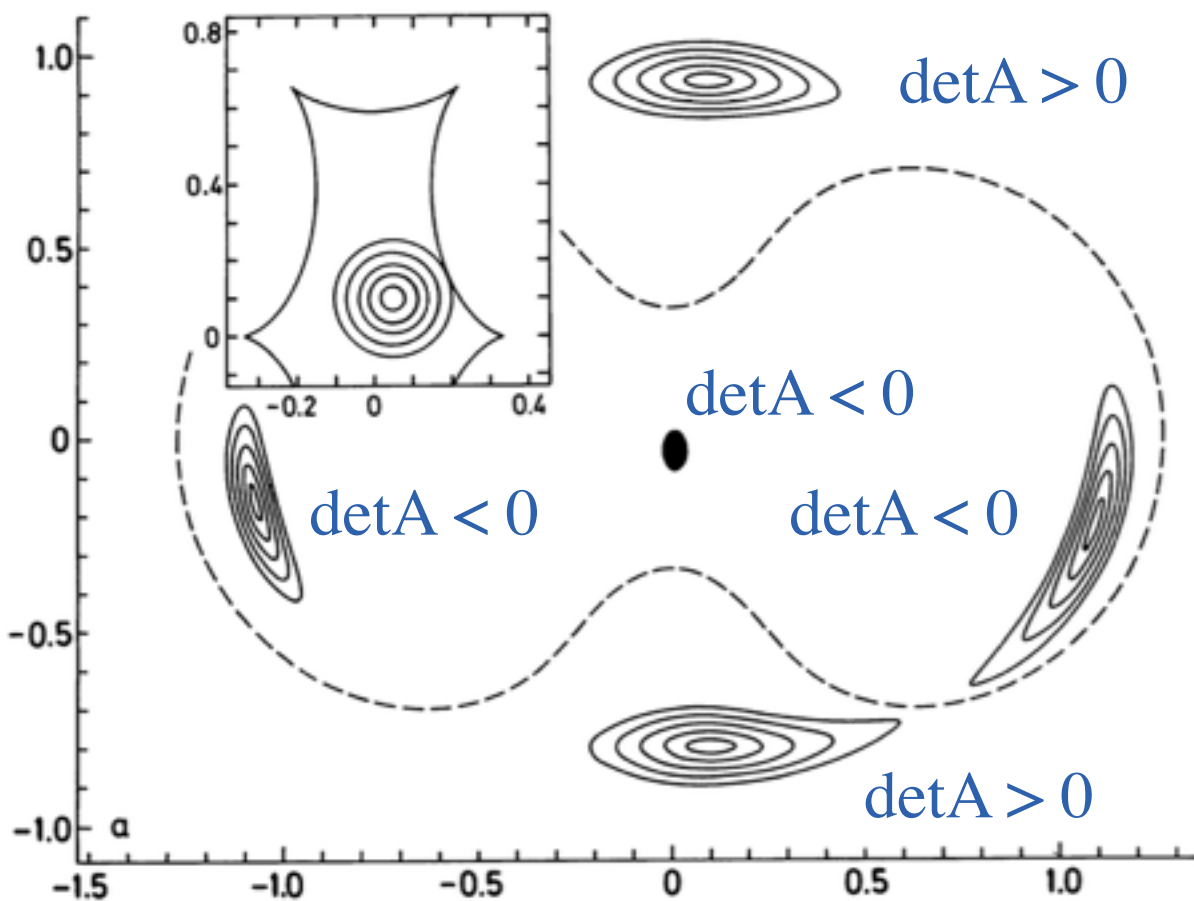
$0.35R_E$



$0.30R_E$

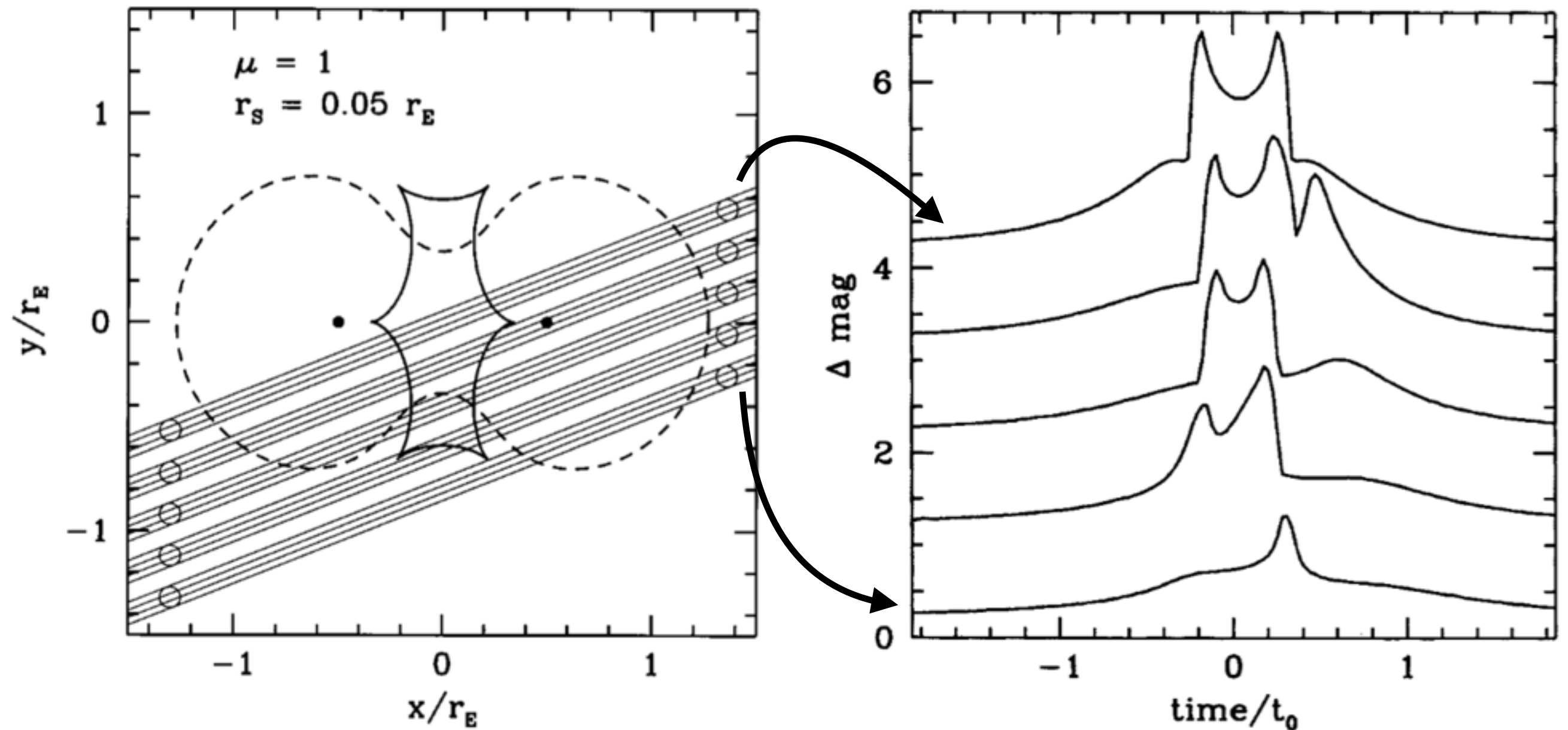


3rd regime



Light Curves of Double Point Mass Lens

Paczynski (1986)



The Double Point Mass Lens

- In the case of planet-star point mass lens the mass ratio is

$$q = \frac{m_p}{M_\star} \ll 1$$

$$\mathbf{y} = \mathbf{x} - \sum_i \frac{m_i}{M} \frac{\mathbf{x} - \mathbf{x}_i}{|\mathbf{x} - \mathbf{x}_i|^2}$$

- In this case the lens equation reduces to

$$\mathbf{y} \simeq \mathbf{x} - \frac{\mathbf{x}}{|\mathbf{x}|^2} - q \frac{\mathbf{x} - \mathbf{x}_p}{|\mathbf{x} - \mathbf{x}_p|^2} \quad \text{as} \quad \frac{M_\star}{M} \frac{\mathbf{x} - \mathbf{x}_\star}{|\mathbf{x} - \mathbf{x}_\star|^2} \simeq \frac{\mathbf{x}}{|\mathbf{x}|^2}$$

- choosing the origin to be the heavy foreground star, i.e. $\mathbf{x}_\star = 0$
- Multiplying through with denominators and rearranging gives

$$q|\mathbf{x}|^2(\mathbf{x} - \mathbf{x}_p) \simeq |\mathbf{x}|^2 |\mathbf{x} - \mathbf{x}_p|^2 \mathbf{x} - \mathbf{y} |\mathbf{x}|^2 |\mathbf{x} - \mathbf{x}_p|^2 - |\mathbf{x} - \mathbf{x}_p|^2 \mathbf{x}$$
 - 5th order polynomial in \mathbf{x}
 - Potentially has 5 solutions in agreement with figures from Schneider & Weiss

The Double Point Mass Lens

- Assume that star, planet and source all lie on one line (call it x-axis)
- Then all components off this axis (call them y-components) vanish
- Hence, the fifth order polynomial in x reduces to

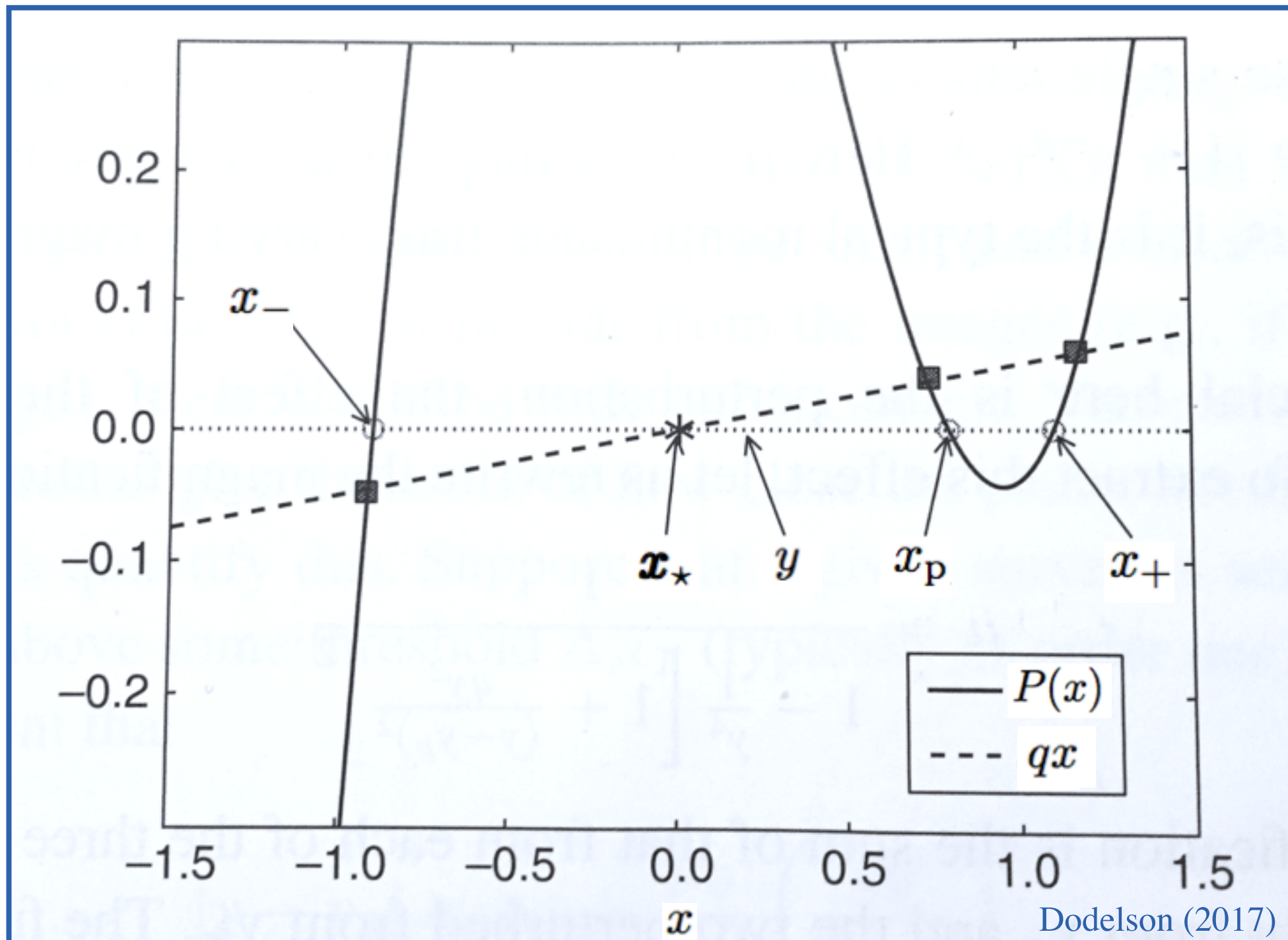
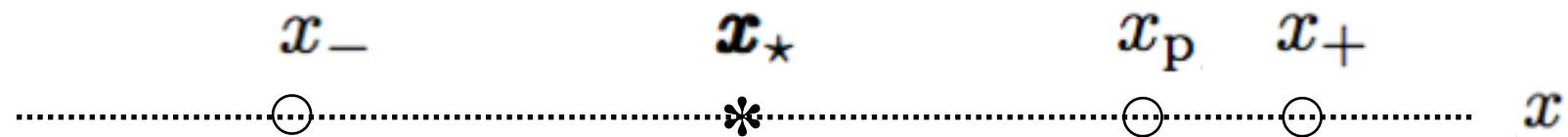
$$qx \simeq x^2(x - x_p) - yx(x - x_p) - (x - x_p)$$

- For star-planet lens q is small, so solution to polynomial is \sim full solution
- The solutions (zeros) are given by (Exercise 4)

$$(x - x_p) = 0, \quad (x - x_+) = 0 \quad \text{and} \quad (x - x_-) = 0$$

- Where x_+ and x_- are the image position for the point mass lens (week 5)
- So for negligible mass there are three images of the background source
 - The nonzero mass of planet changes image positions slightly

The Double Point Mass Lens



Total Magnification from Double Point Mass Lens

- Multiple images occur; but in microlensing they are unresolved!
- To estimate total magnification of the system we can use $\mu \equiv \frac{1}{\det \mathcal{A}(\boldsymbol{\theta})}$ (Week 7)
- Hence, using the Lens equation for the double point mass lens:

$$\boldsymbol{y} \simeq \boldsymbol{x} - \frac{\boldsymbol{x}}{|\boldsymbol{x}|^2} - q \frac{\boldsymbol{x} - \boldsymbol{x}_p}{|\boldsymbol{x} - \boldsymbol{x}_p|^2}$$

- Calculating derivatives for Jacobian matrix: $\frac{\partial \beta_i}{\partial \theta_j} = \frac{\partial y_i}{\partial x_j}$
- Calculating determinant of Jacobian matrix: $\det \mathcal{A}$
- For the ‘x-axis aligned’ case it can be shown that the magnification is:

$$\mu = \left[1 - \left(\frac{1}{x_x^2} + \frac{q}{(x - x_p)_x^2} \right)^2 \right]^{-1} \quad (\text{Exercise 5})$$

- where the x -subscript refers to the x -component of the image positions
- where the y -components in the terms of the Jacobian matrix were set to 0

Total Magnification from Double Point Mass Lens

- For the single-lens we know that $\mu_0 = \frac{1}{1 - \frac{\theta_E^4}{\theta^4}}$ from week 7
- ‘Isolating’ this part gives

$$\mu = \frac{1}{1 - \frac{1}{x^4} \left(1 + \frac{qx^2}{(x-x_p)^2} \right)^2}$$

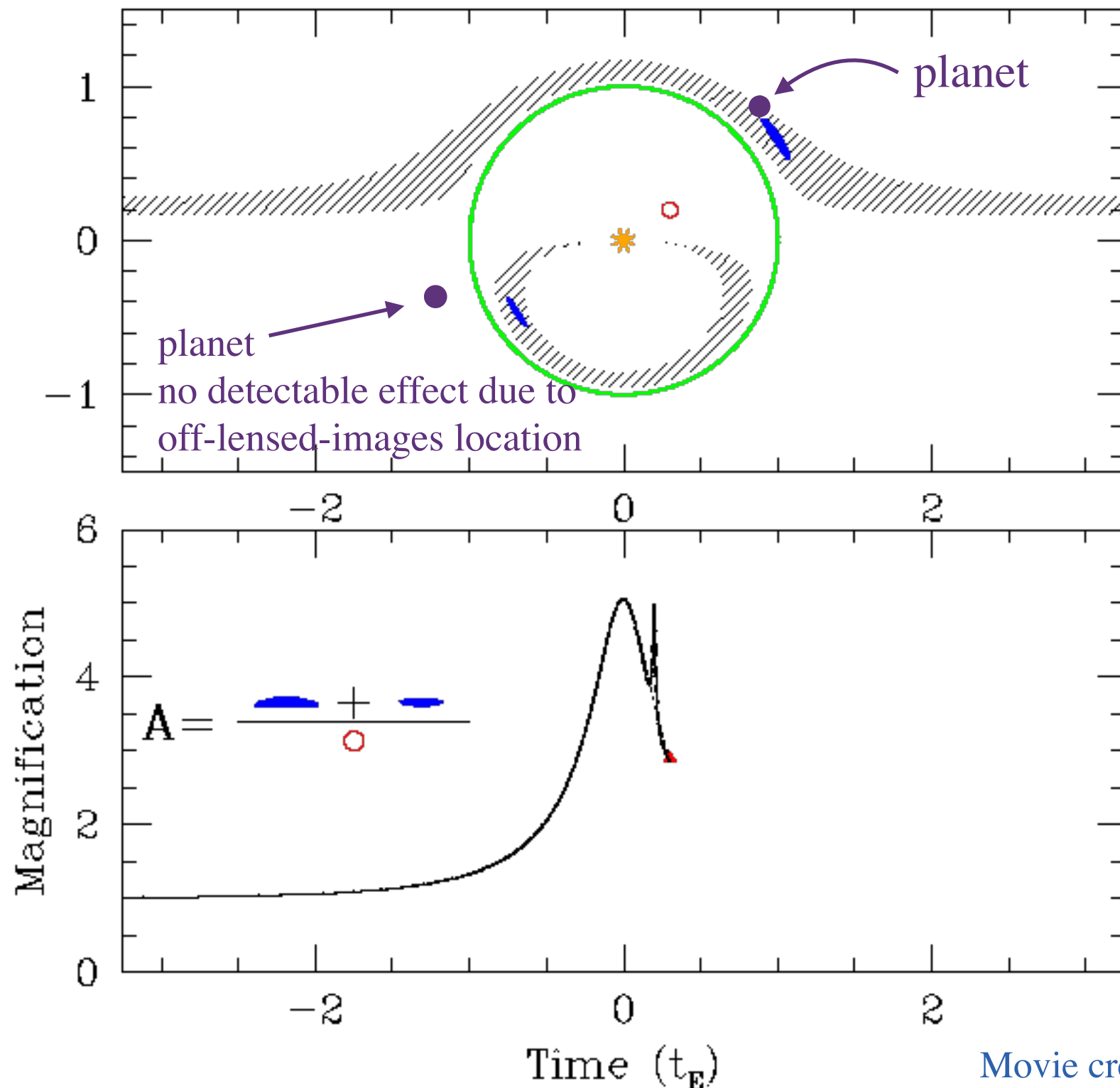
- Which can be used to show that the excess μ introduced by the planet is

$$\Delta\mu_p \simeq \frac{2\mu_0^2 q}{x^2(x-x_p)^2}$$

- So if the planet is close to the lensed image position $\Delta\mu_p$ becomes large
- If $(x-x_p)^2$ is large, i.e., the planet is away from images $\Delta\mu_p$ becomes small

The Double Point Mass Lens

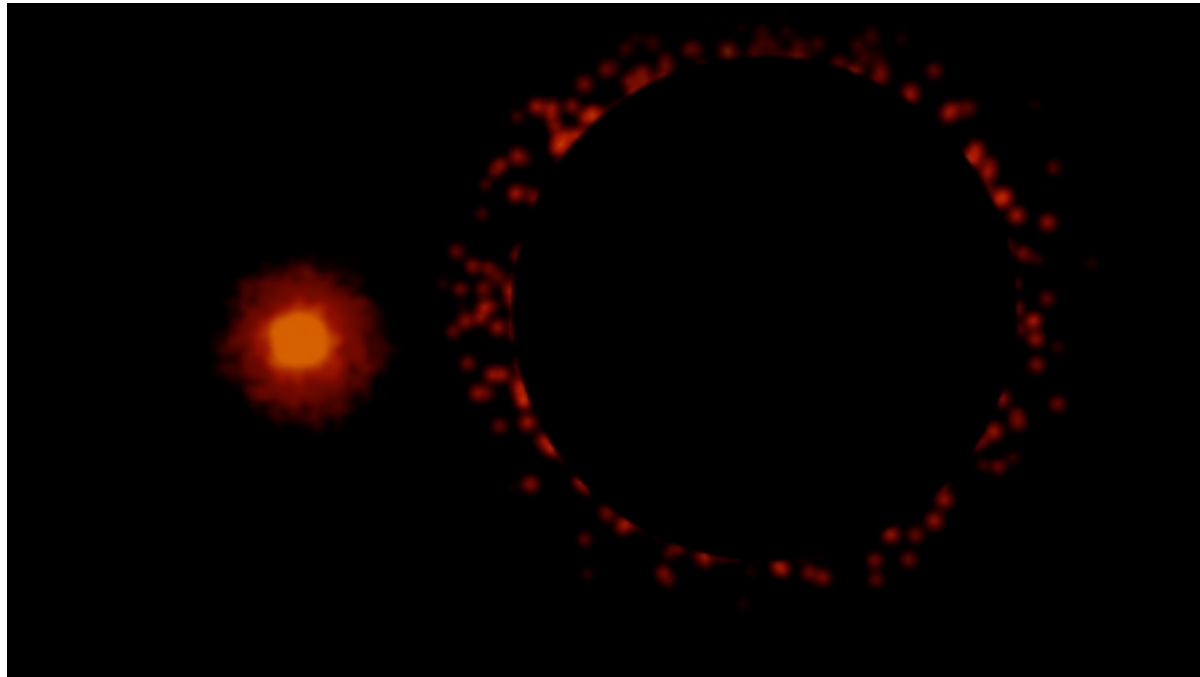
- Add planet around lens star ... assume stationary during microlensing event



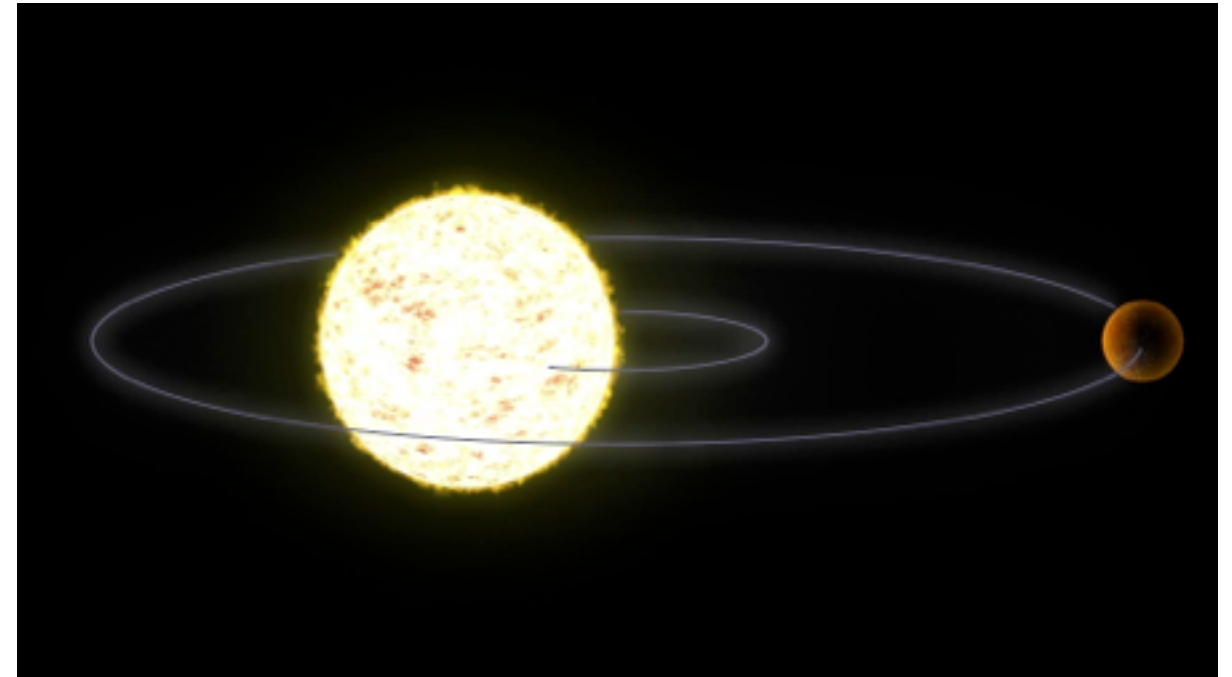
Movie credit: B. Scott Gaudi, OSU

Finding Planets

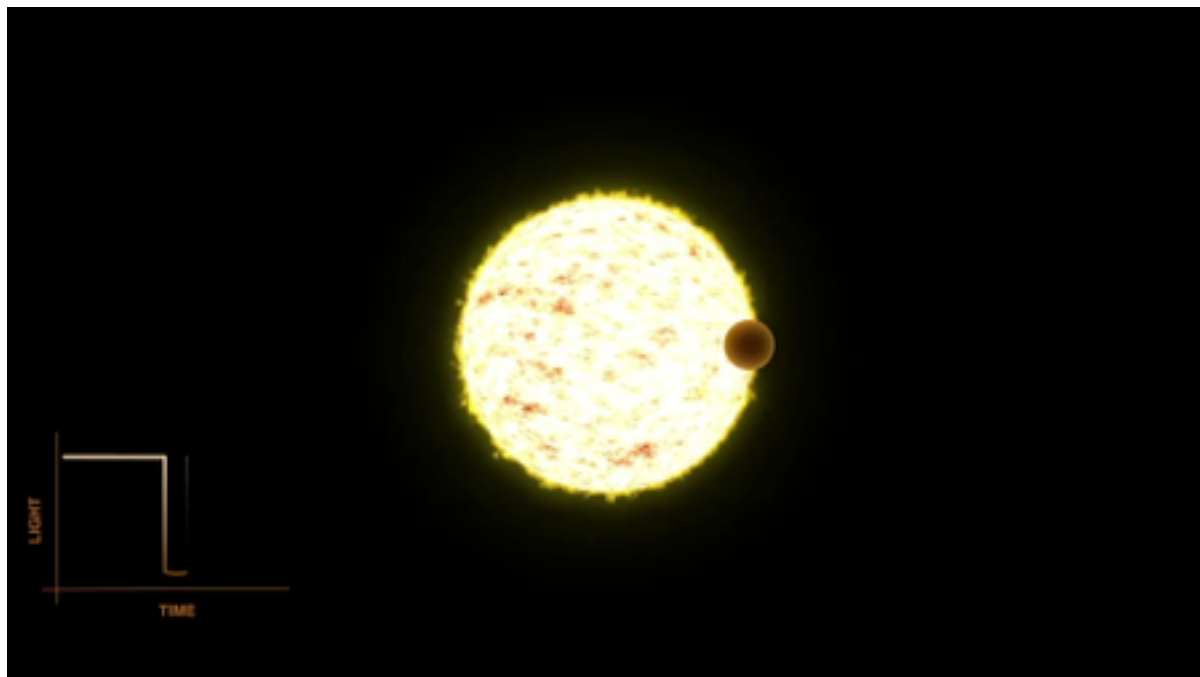
Direct detection; $O(r_p, f)$; 1.1%



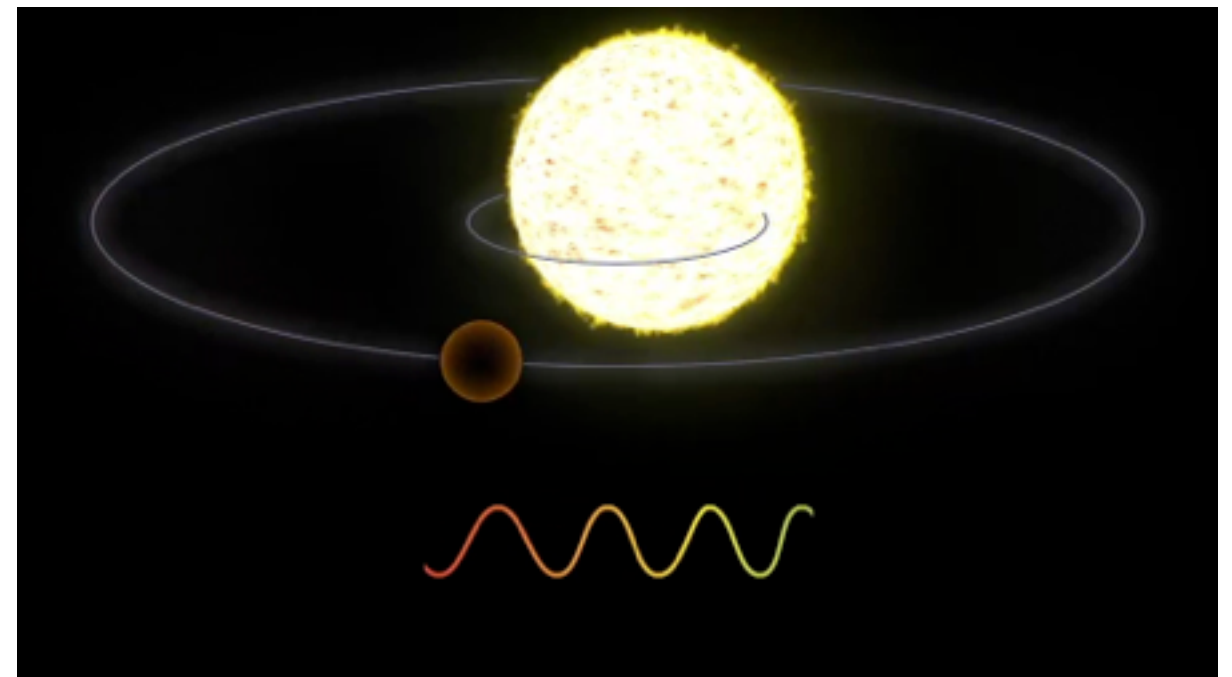
Astrometric Wobble; $O(m/M)$; 0.03%



Planet Transit; $O(r_p)$; 77.1%



Radial Velocity; $O(m/M)$; 18.9%



Movies: <https://exoplanets.nasa.gov>

Why add Another Method - Microlensing?

- Small probability of events
- Duration very short (days - months)
- Found planets are distant
- Impossible to do post-event follow-up
- Parameter degeneracy in modeling
- Not m_p but q that is determined
- No independent confirmation
- Yes, of the order 10^6 - 10^7 but current surveys easily survey this many stars and 1000s of events and triggers have been found
- Coarse survey sampling with dense follow-up accommodate this
- True, but a benefit of lensing - mapping planets further than other methods
- But follow-up of host star might be possible
- True for specific geometries, but good data allow breaking these degeneracies
- True, but knowing the host star mass (potentially statistically) you will know the planet masses to the same accuracy
- True, but good enough data doesn't need independent confirmation

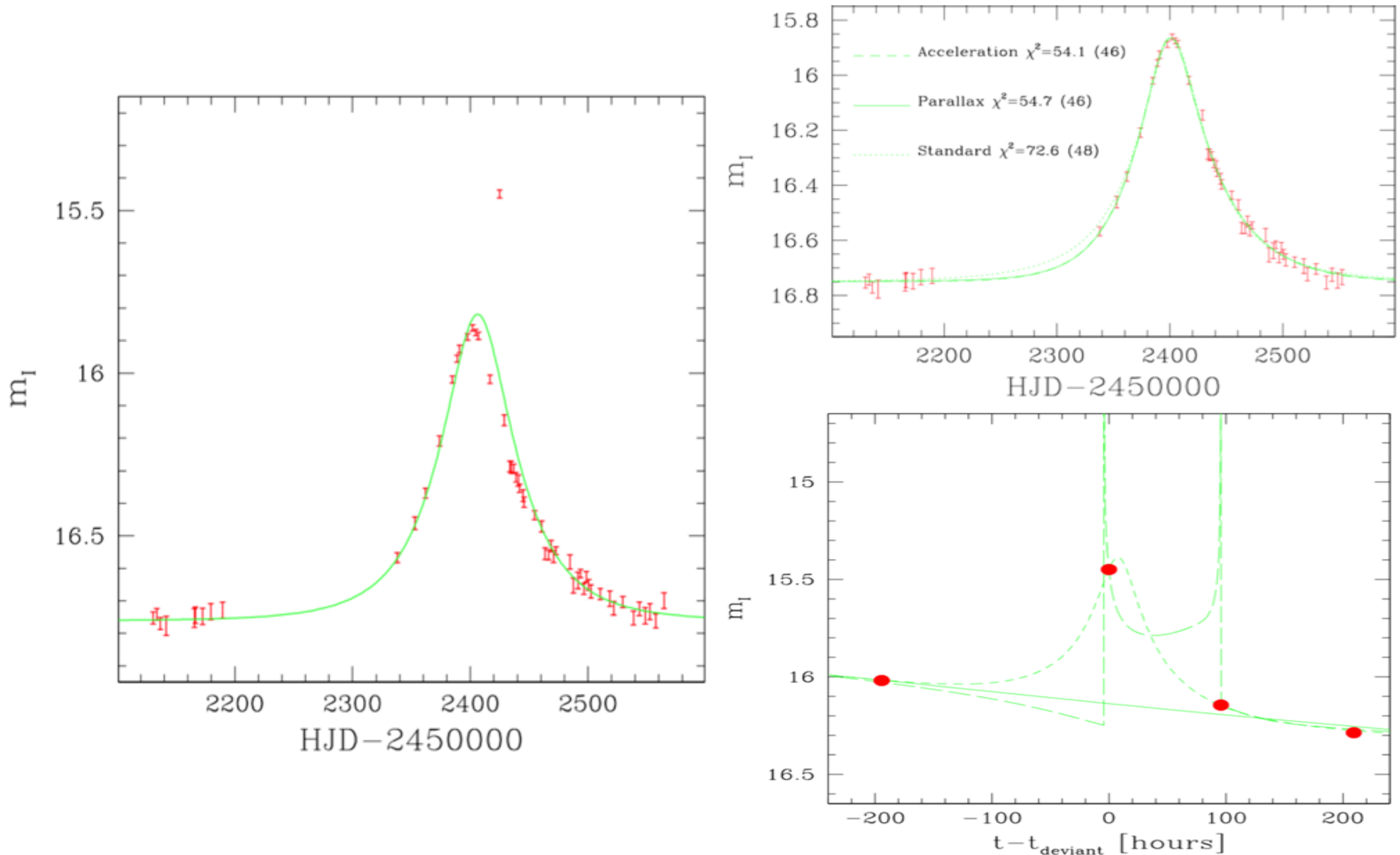
Schneider, Kochanek, Wambsganss (2006)

Gravitational Microlensing Advantages for Planet Detections

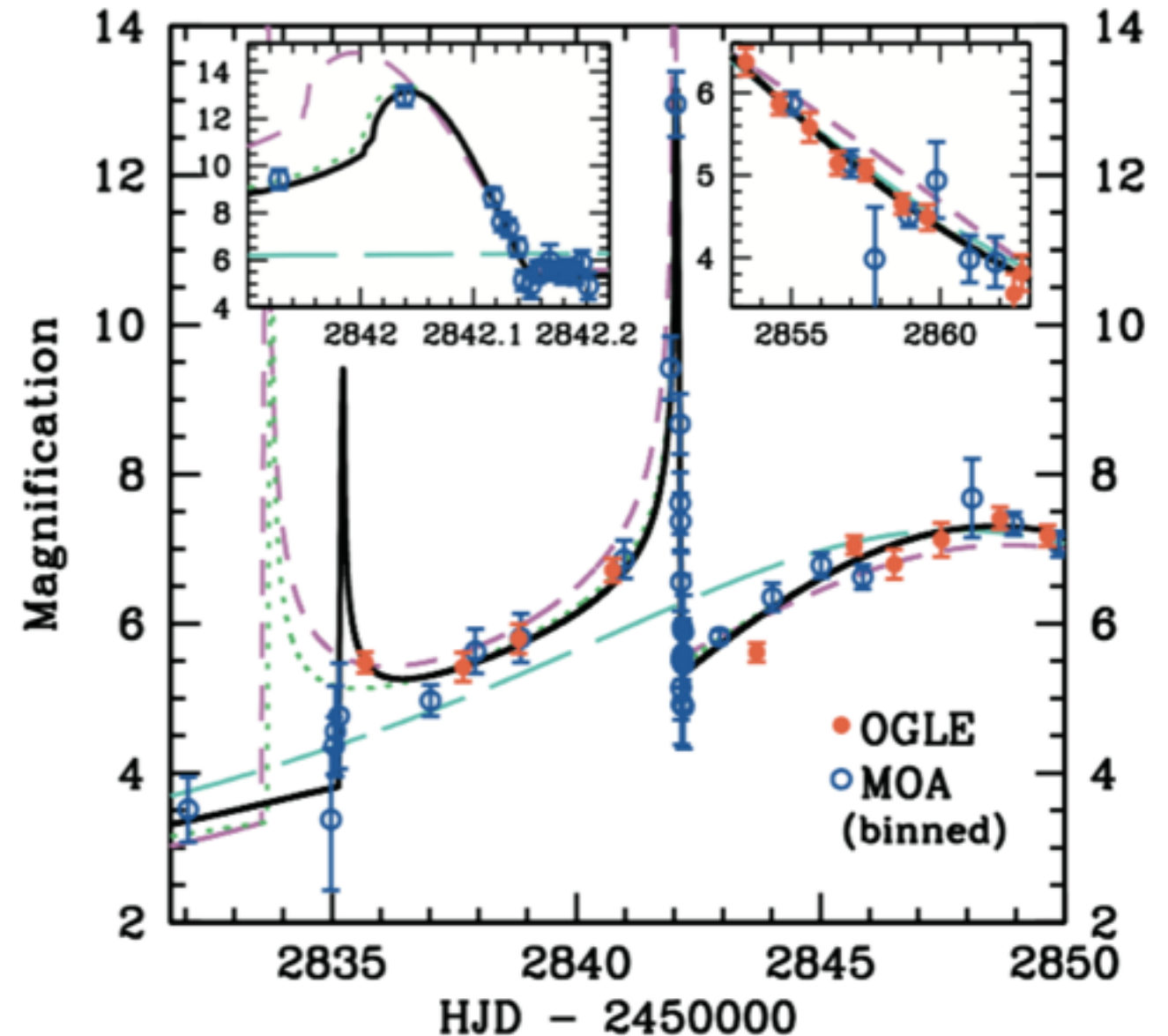
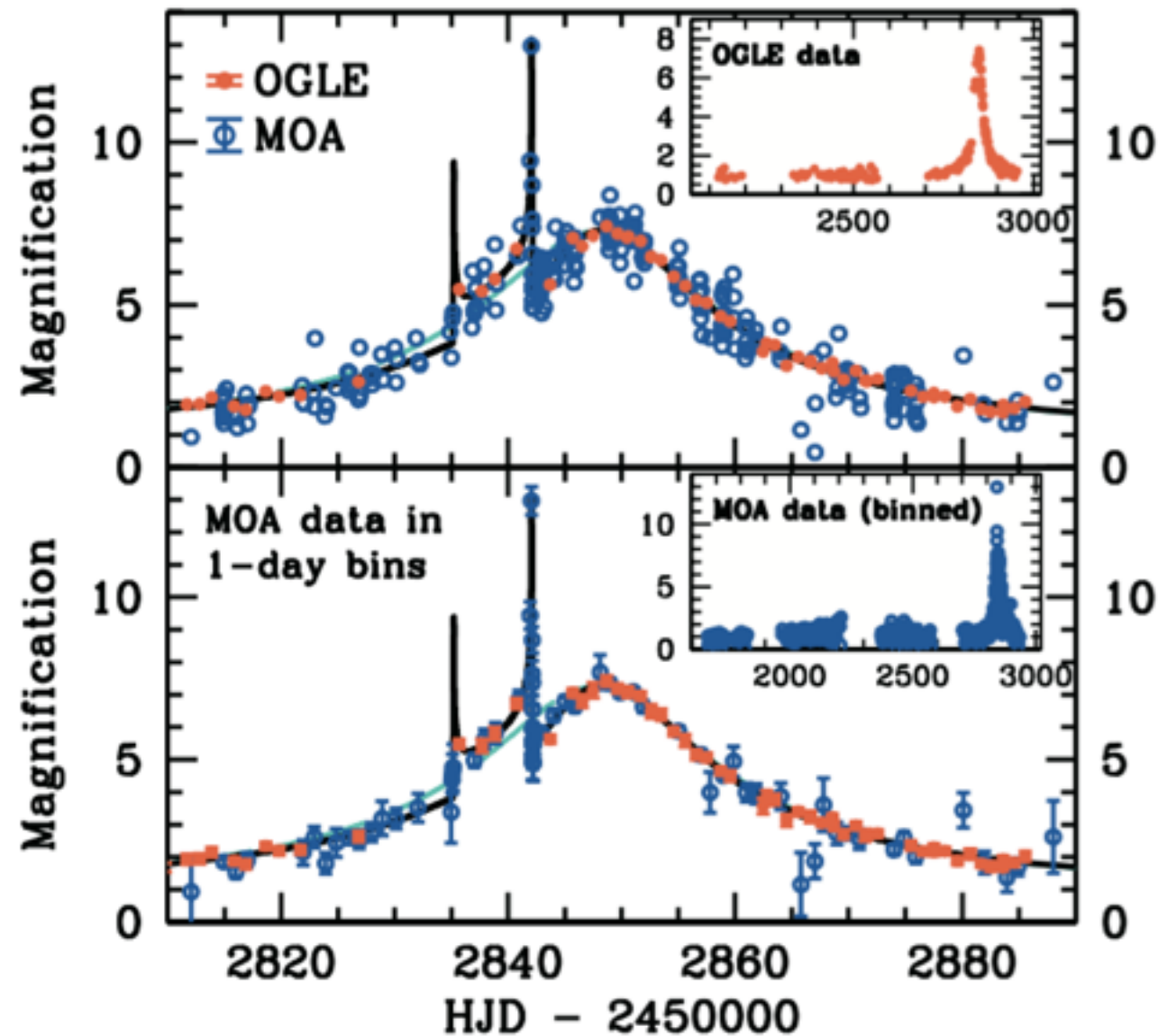
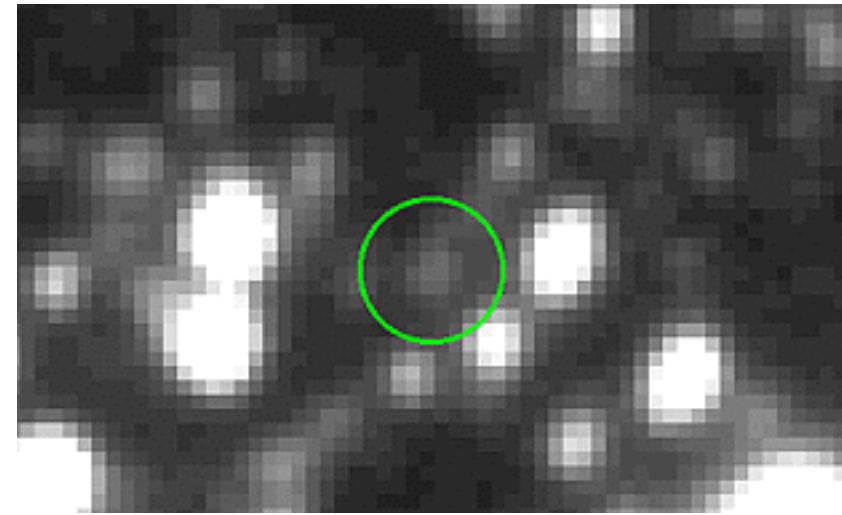
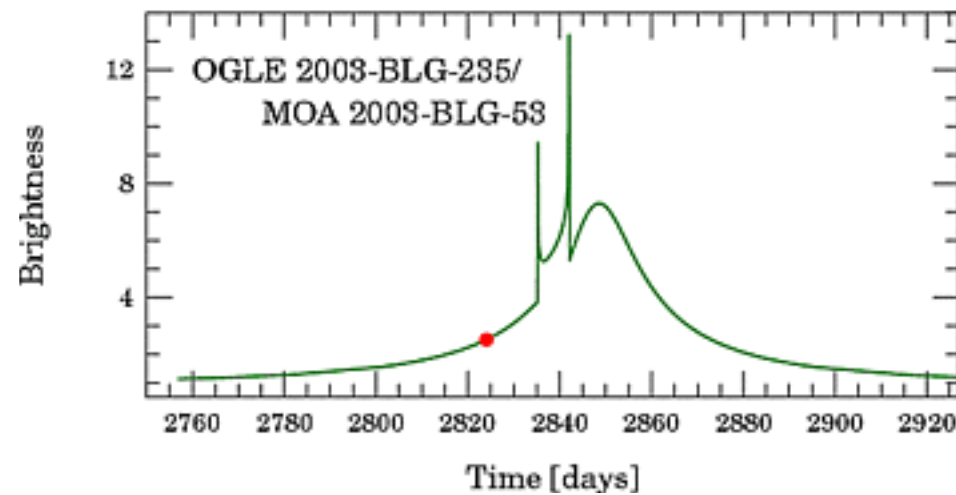
- No bias for nearby stars; i.e., a benefit that lensing needs a certain D_L
- No bias for planets around a certain type of star (solar or main sequence)
 - Other methods select and target hosts after a certain set of criteria
- No strong bias towards planets of large mass
- Microlensing sensitive down to masses of the order Earth mass
- Most effective for planets orbiting at R_E , overlaps with ‘habitable zone’
- Multiple planets detectable with same method
- Quick detection of large-separation systems (much quicker than seeing full period)
- Detection of free-floating planets (or other dark objects)
- Best statistical test (un-biased) of galactic sample of planets.
- (Caustic crossings of planet or planet satellites/moons can in principle be detected due to extreme magnification of reflected light)

Schneider, Kochanek, Wambsganss (2006)

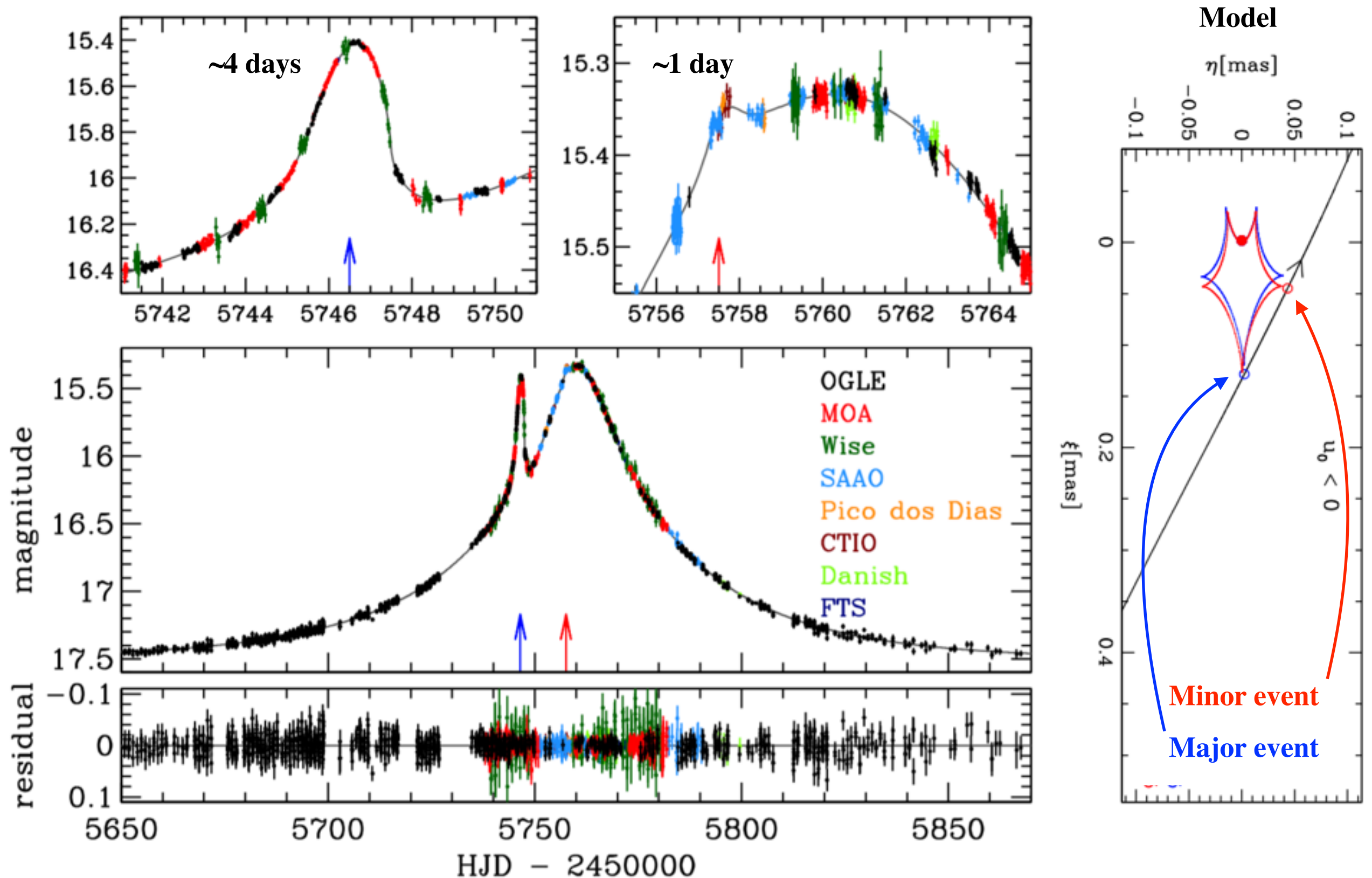
The First Planet Candidate - Jaroszynski et al. 2002



The First Clear Case - Bond et al. (2004)



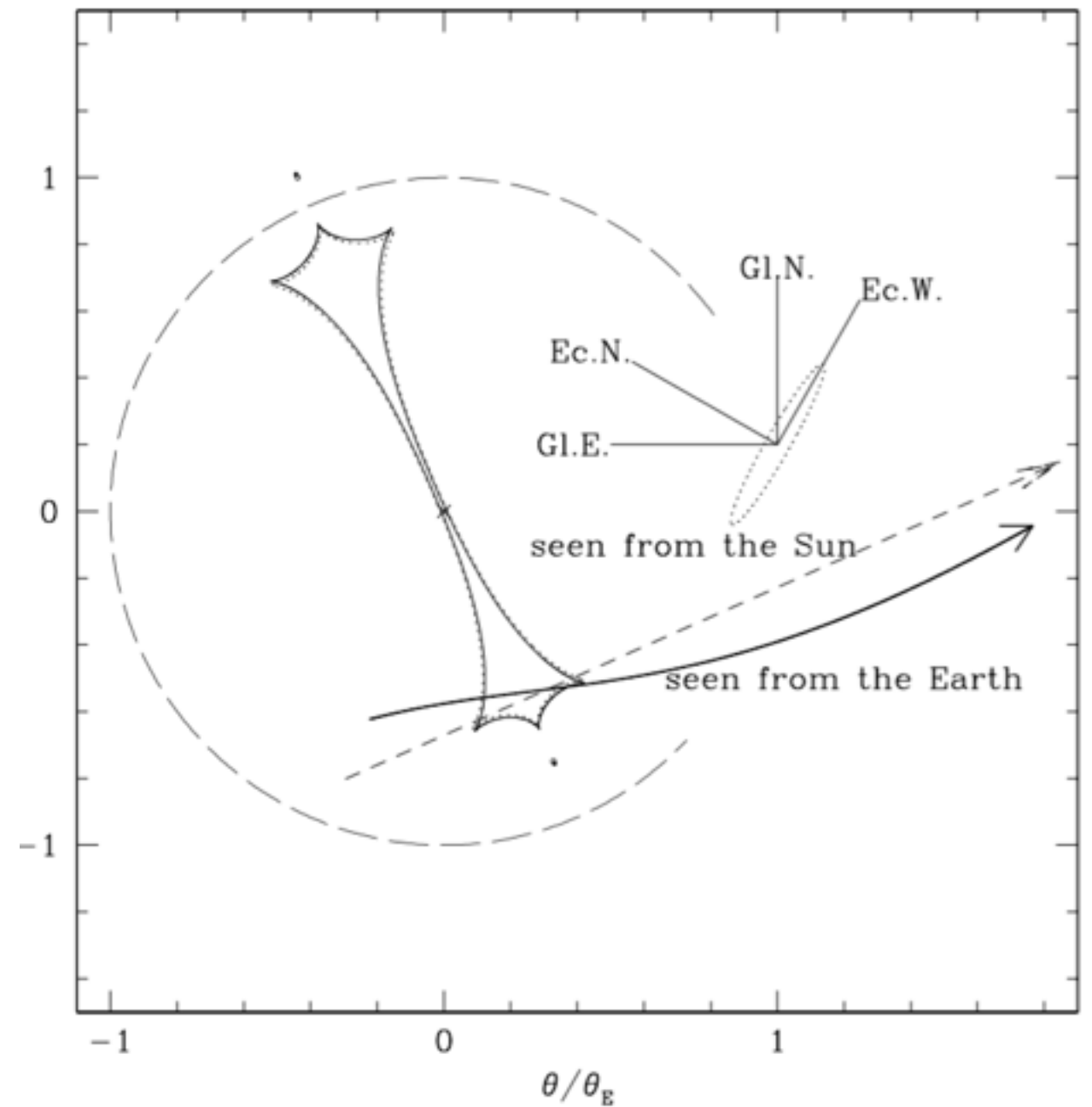
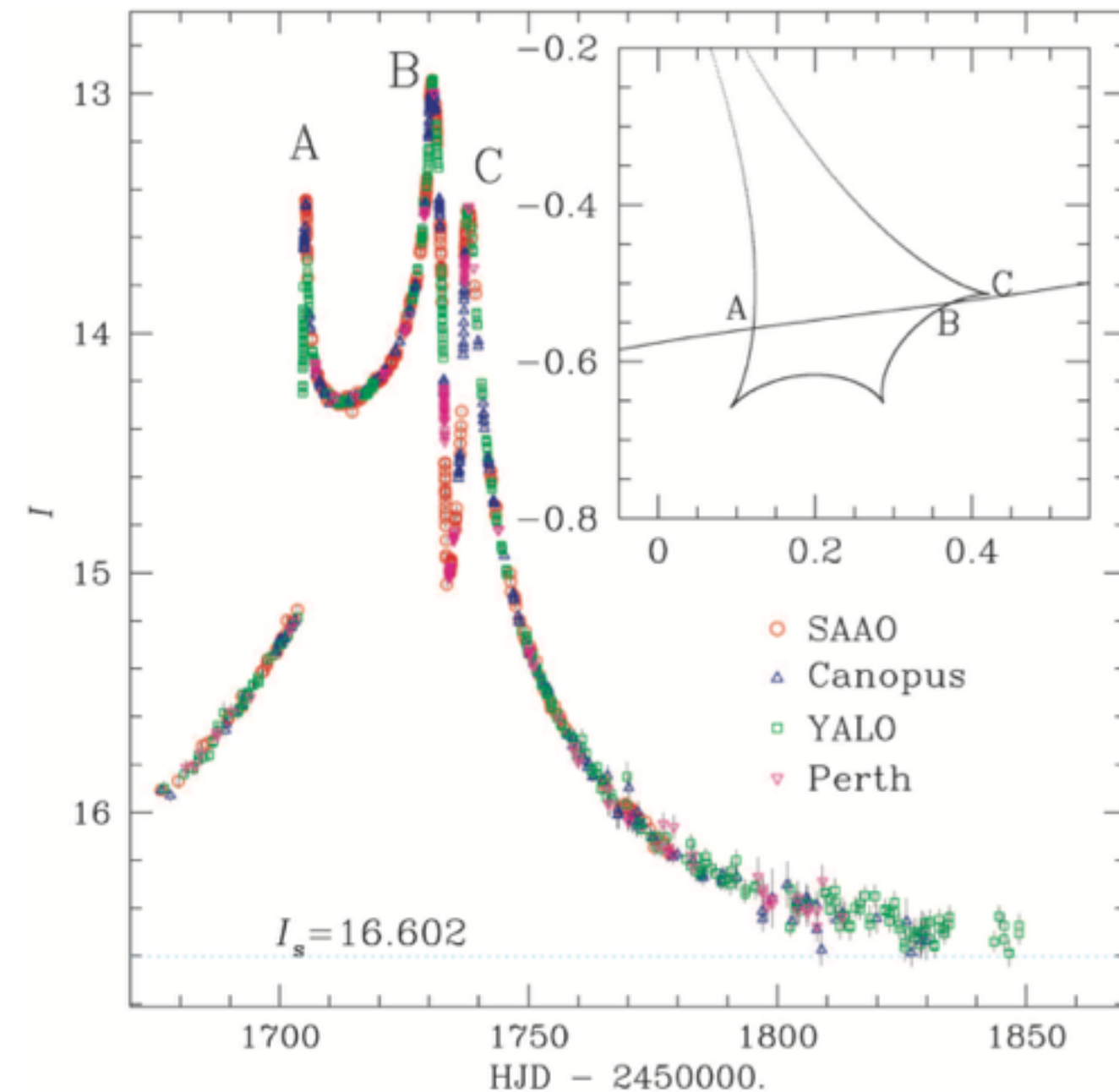
Today's data quality - Showron et al. 2015



Modeling Sensitive to the Parallax of the Earth

An+00

An+00

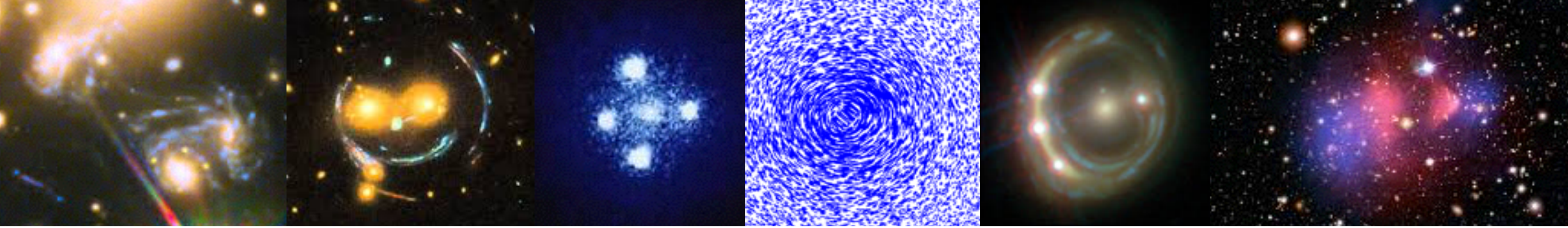


(Binary star event)

So in summary...

- The deflection from multiple point masses $\alpha(\boldsymbol{\theta}) = \sum_i \theta_{\text{E},i}^2 \frac{\boldsymbol{\theta} - \boldsymbol{\theta}_i}{|\boldsymbol{\theta} - \boldsymbol{\theta}_i|^2}$
- With the correspond lens equation $\boldsymbol{y} = \boldsymbol{x} - \sum_i \frac{m_i}{M} \frac{\boldsymbol{x} - \boldsymbol{x}_i}{|\boldsymbol{x} - \boldsymbol{x}_i|^2}$
- Using the definitions:

$$\boldsymbol{y} = \frac{\boldsymbol{\beta}}{\theta_{\text{E}}} \quad \boldsymbol{x} = \frac{\boldsymbol{\theta}}{\theta_{\text{E}}} \quad \theta_{\text{E}}^2 \equiv \sum_i \theta_{\text{E},i}^2 = \frac{MGD_{\text{LS}}}{D_{\text{L}}D_{\text{S}}c^2} \quad M \equiv \sum_i m_i$$
- For a lens consisting of a star and a planet with a mass ratio $q = \frac{m_{\text{p}}}{M_{\star}}$
 - the lens equation becomes $\boldsymbol{y} \simeq \boldsymbol{x} - \frac{\boldsymbol{x}}{|\boldsymbol{x}|^2} - q \frac{\boldsymbol{x} - \boldsymbol{x}_{\text{p}}}{|\boldsymbol{x} - \boldsymbol{x}_{\text{p}}|^2}$
- Microlensing is complimentary to other methods to find planets:
 - Can probe larger distances
 - Not biased towards high mass planets or large-orbit planets
 - Works best for planets close to the ‘habitable zone’
- The microlensing surveying teams are well-coordinated and produce well-sampled data presenting planet microlensing events of high quality



PHY-765 SS19 Gravitational Lensing Week 10

This Week's Worksheet

Giving “a good talk”

- Assume spending 2 minutes per slide (at least) when estimating total time
- Tell a story, i.e., answer the “whys”, “hows” and “whats” of the work
- Limit the use of colors on slides → No “rainbow” slides
 - Avoid poor color combinations like “blue on black” and “green on red”
- Don’t write text paragraphs unless you are going to read them out loud
 - Otherwise the audience will be reading instead of listening to you
- Use builds to avoid displaying the whole slide at once
 - Otherwise the audience will be reading instead of listening to you
- Use cartoons/sketches to explain concepts instead of just waving you hands
- Don’t have slides where you say “just ignore this and that”.
 - If things should be ignored, just remove them from the slides.
- **Rehearse the talk before giving it!**

Disclaimer: “A good talk” according to K. B. Schmidt... others might (surely will) disagree