PHY-765 SS18: Gravitational Lensing. Worksheet Week 3

1 Finish the Scientific Poster for Presentation Next Week

Finish your poster presentation for next week. If you didn't already send a link to for instance the ADS page of the paper you plan to prepare a poster on to kbschmidt@aip.de, please do so.

2 Determine Characteristic Sizes of the Schwarzschild Lens

In the following you will estimate the characteristic sizes for a point source lens, giving you an idea of the quantities involved.

2.1

Using the geometric relation between ξ and θ, express the lens equation in terms of β, θ, $R_{\rm S}$ and length scales for the point source deflector.

2.2

Show that the characteristic angle in the lens plane is given by the Einstein radius, α_0 (expressed in terms of $R_{\rm S}$ and the involved distances).

2.3

In a similar way, determine the characteristic length scale in the lens plane, ξ_0.

2.4

And finally, also in terms of $R_{\rm S}$ and the involved distances, determine the characteristic length scale in the source plane, η_0.

2.5

Assuming that you have a lens of $M = 5 \times 10^{11} M_\odot$ at a redshift $z_{\rm L} = 0.54$ and a source at $z_{\rm S} = 1.49$, what is the characteristic deflection angle α_0. What if the mass of the lens is instead $M = 10^{14} M_\odot$. (Hint: use Ned Wright's calculator to get the distances involved).

2.6

For a lens much closer to the observer than the source, i.e., $D_{\rm L} \ll D_{\rm LS} \sim D_{\rm S}$, determine the constants K_α and K_ξ in the expressions

$$\alpha_0 = K_\alpha \left(\frac{D_{\rm L}}{1{\rm kpc}}\right)^{-1/2} \left(\frac{M}{M_\odot}\right)^{1/2} {\rm arcsec}$$

and

$$\xi_0 = K_\xi \left(\frac{D_{\rm L}}{1{\rm kpc}}\right)^{1/2} \left(\frac{M}{M_\odot}\right)^{1/2} {\rm cm}$$

2.7

Is the approximation $D_{\rm L} \ll D_{\rm LS} \sim D_{\rm S}$ used in 2.6 a fair description for the setup described in 2.5?

3 Some algebra

The following let you derive some of the relations quoted in this week's slides.

3.1

From the geometry of a gravitational lens, show that

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \frac{D_{\mathrm{LS}}}{D_{\mathrm{S}}}\hat{\boldsymbol{\alpha}}(D_{\mathrm{L}}\boldsymbol{\theta})$$

3.2

Using the realtions provided in this week's slides, express the scaled deflection angle in terms of $\boldsymbol{\theta}$:

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) = \frac{1}{\pi}\int d^2\theta' \kappa(\boldsymbol{\theta'})\frac{\boldsymbol{\theta} - \boldsymbol{\theta'}}{|\boldsymbol{\theta} - \boldsymbol{\theta'}|^2}$$

3.3

For any vector, show that

$$\nabla \ln |\boldsymbol{r}| = \boldsymbol{r}/|\boldsymbol{r}|^2$$