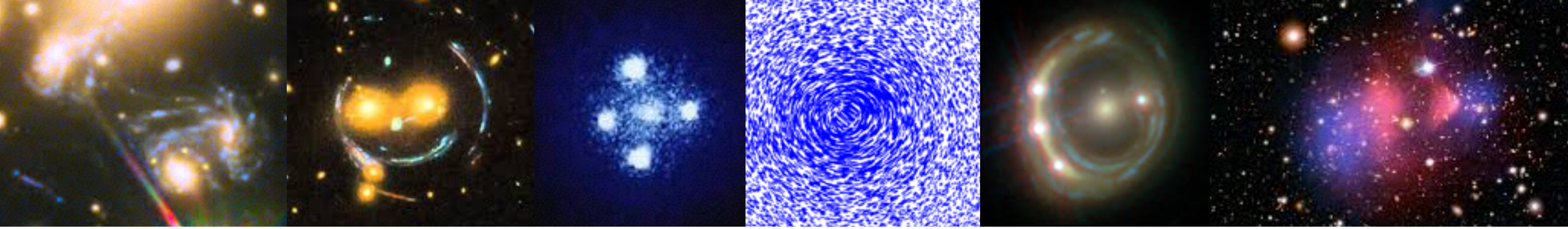

PHY-765 SS18 Gravitational Lensing Week 9

Searching for Exoplanets with Gravitational Lensing

Kasper B. Schmidt
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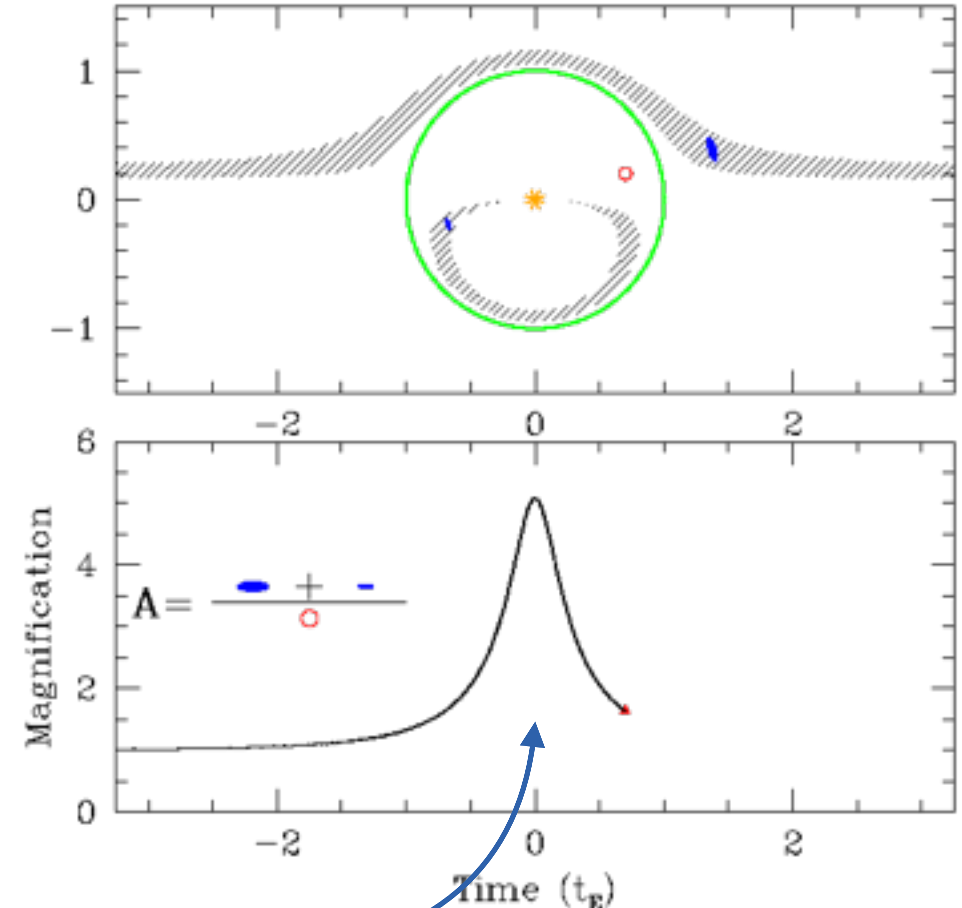
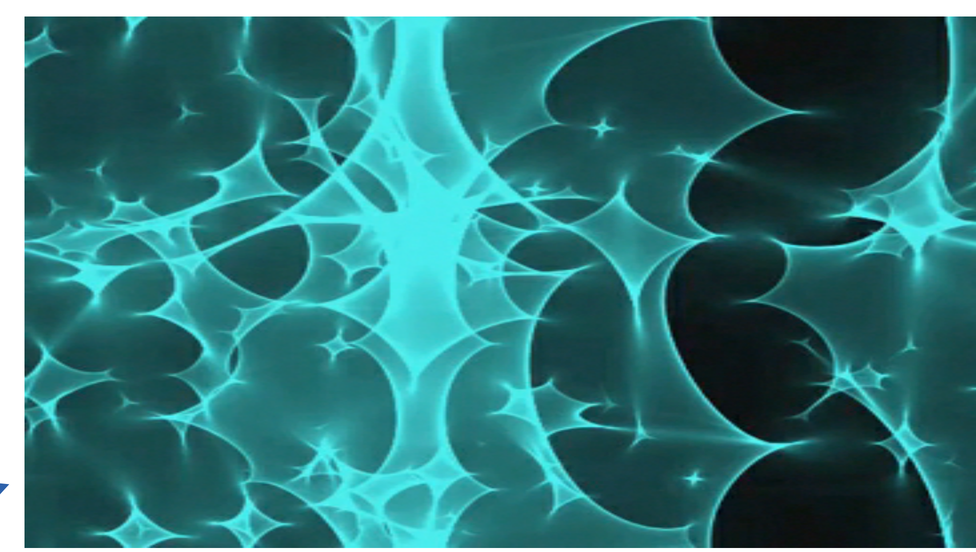
Last week



- Microlensing
  - Time-variability often seen in lens-frame
- The point mass lens (un-resolved images):

$$\mu = \frac{y^2+2}{y\sqrt{y^2+4}} \quad \text{where} \quad y = \frac{\beta}{\theta_E}$$

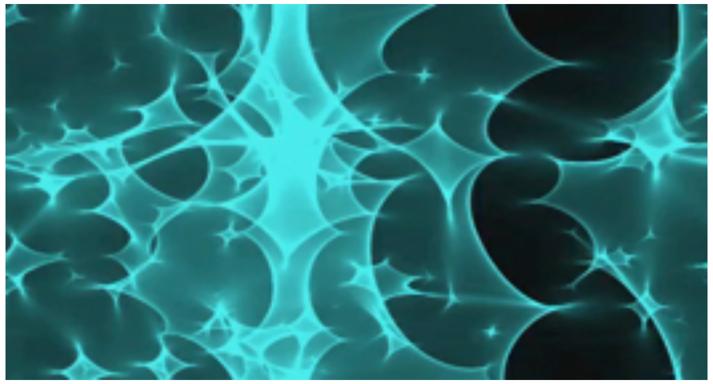
- The Paczynski curve describing source ‘light curve’
- Define the lensing optical depth: Probability of sight-line with microlensing
- Showed examples of detected events from MACHO, EROS and OGLE
- Using Microlensing to estimate DM MACHO-fraction in MW halo
  - ~20% from the MACHO survey
  - MW halo mostly non-MACHO
- Extra-galactic microlensing of multiple stars in lens-galaxies of QSO lenses. “μ-map”



The aim of today

- The case of a multiple point mass lens system
- Specialize to the double point mass lens
- Image location and magnification for a star + planet lens
- Other methods for finding exoplanets
  - Radial velocities
  - Transits
  - Astrometry
  - Direct imaging
- Why do we need another method, i.e., microlensing?
- Examples of planets found with microlensing

The Multiple Point Mass Lens



- Want to generalize the concept of microlensing
  - what is also needed for extra-galactic microlensing
- Starting from the lens equation $\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta})$
- The deflection angle for multiple point sources can be expressed as:

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) = \sum_i \theta_{\mathrm{E},i}^2 \frac{\boldsymbol{\theta} - \boldsymbol{\theta}_i}{|\boldsymbol{\theta} - \boldsymbol{\theta}_i|^2}$$

Week 3/4 reminder:
Spherical Mass Distribution

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) = \frac{1}{\pi} \int d^2\theta' \kappa(\boldsymbol{\theta}') \frac{\boldsymbol{\theta} - \boldsymbol{\theta}'}{|\boldsymbol{\theta} - \boldsymbol{\theta}'|^2}$$

- Here $\boldsymbol{\theta}_i$ are positions of the lenses in the lens plane.
  - But these are unlensed so the position in lens and source plane are identical
- So the only unknown in the lens equation is the source position $\boldsymbol{\theta}$
  - *relative* source position change in time → μ (brightness) of source change
- Can define the combined Einstein radius of system as

$$\theta_{\mathrm{E}}^2 \equiv \sum_i \theta_{\mathrm{E}_i}^2 = \frac{MGD_{\mathrm{LS}}}{D_{\mathrm{L}} D_{\mathrm{S}} c^2} \quad \text{where} \quad M \equiv \sum_i m_i$$

The Multiple Point Mass Lens

- Introduced normalized components of lens equation last week:

$$\boldsymbol{y} = \frac{\boldsymbol{\beta}}{\theta_{\mathrm{E}}} \qquad \boldsymbol{x} = \frac{\boldsymbol{\theta}}{\theta_{\mathrm{E}}}$$

- Using definition of combined Einstein radius we get the lens equation

$$\boldsymbol{y} = \boldsymbol{x} - \sum_i \frac{m_i}{M} \frac{\boldsymbol{x} - \boldsymbol{x}_i}{|\boldsymbol{x} - \boldsymbol{x}_i|^2}$$

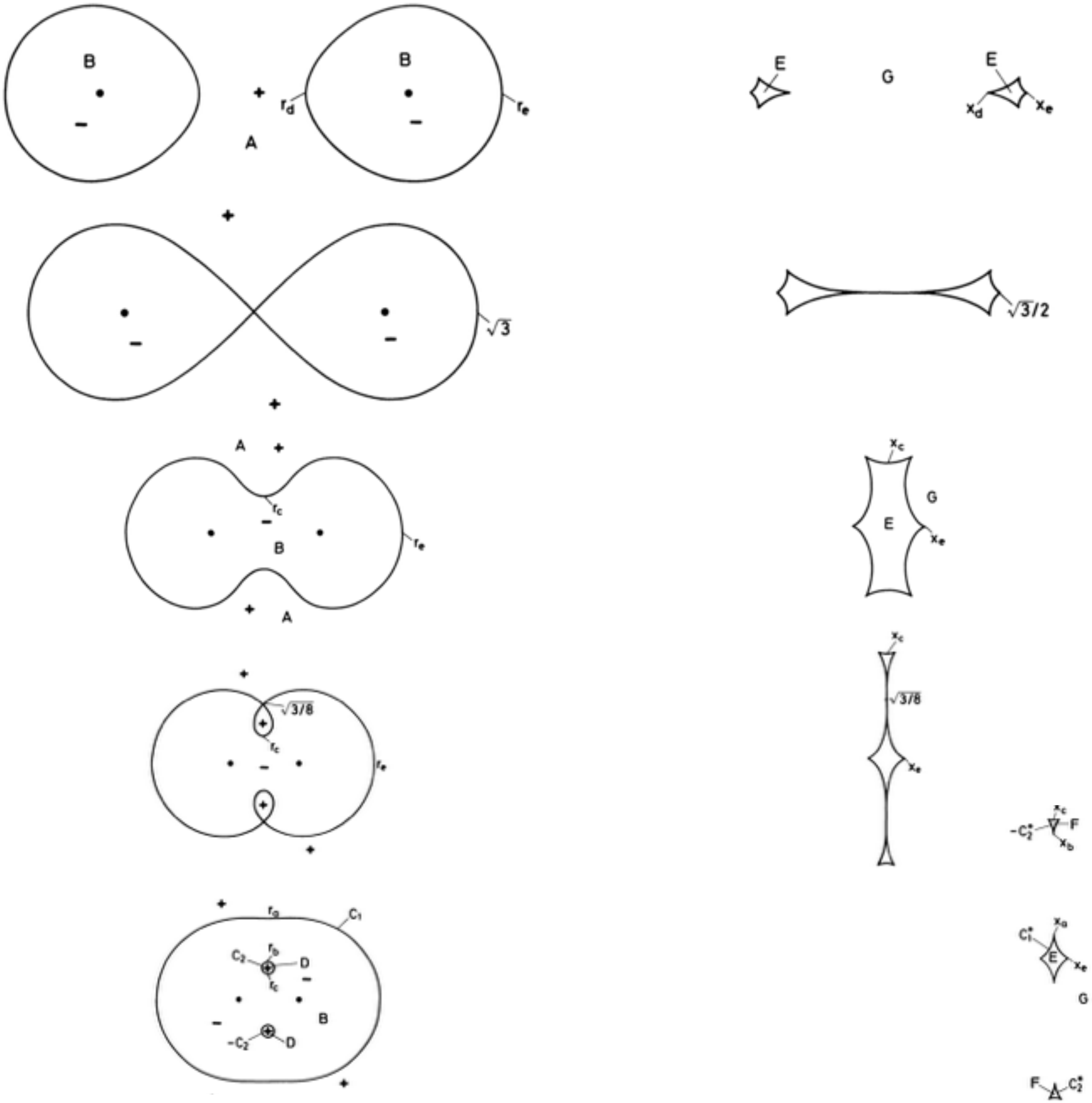
- Lens Equation non-linear, so combined effect is ***not*** just sum of effects

The Double Point Mass Lens

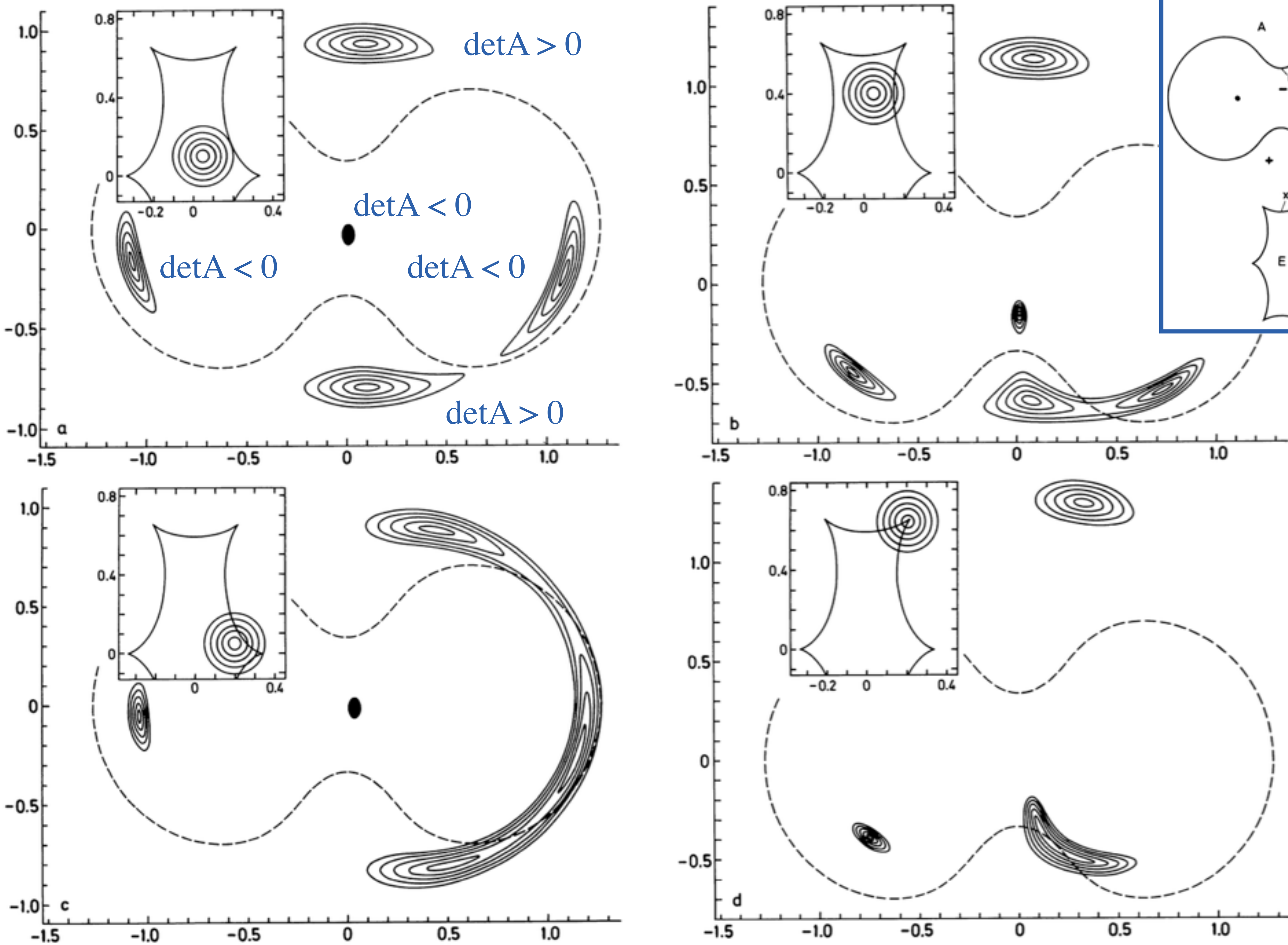
- Schneider & Weiss (1986) explored the double point mass lens in detail
  - Assuming $m_1 = m_2$, so considering binary stars, not planets
- Described the critical curves and caustics as a function of star separation, d

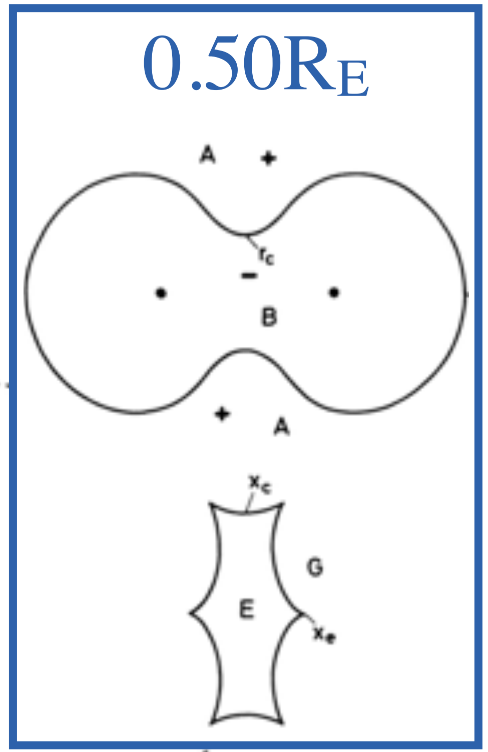
Schneider & Weiss (1986)

d	Lens Plane	Source Plane
$1.20R_E$		
$1.00R_E$		
$0.50R_E$		
$0.35R_E$		
$0.30R_E$		



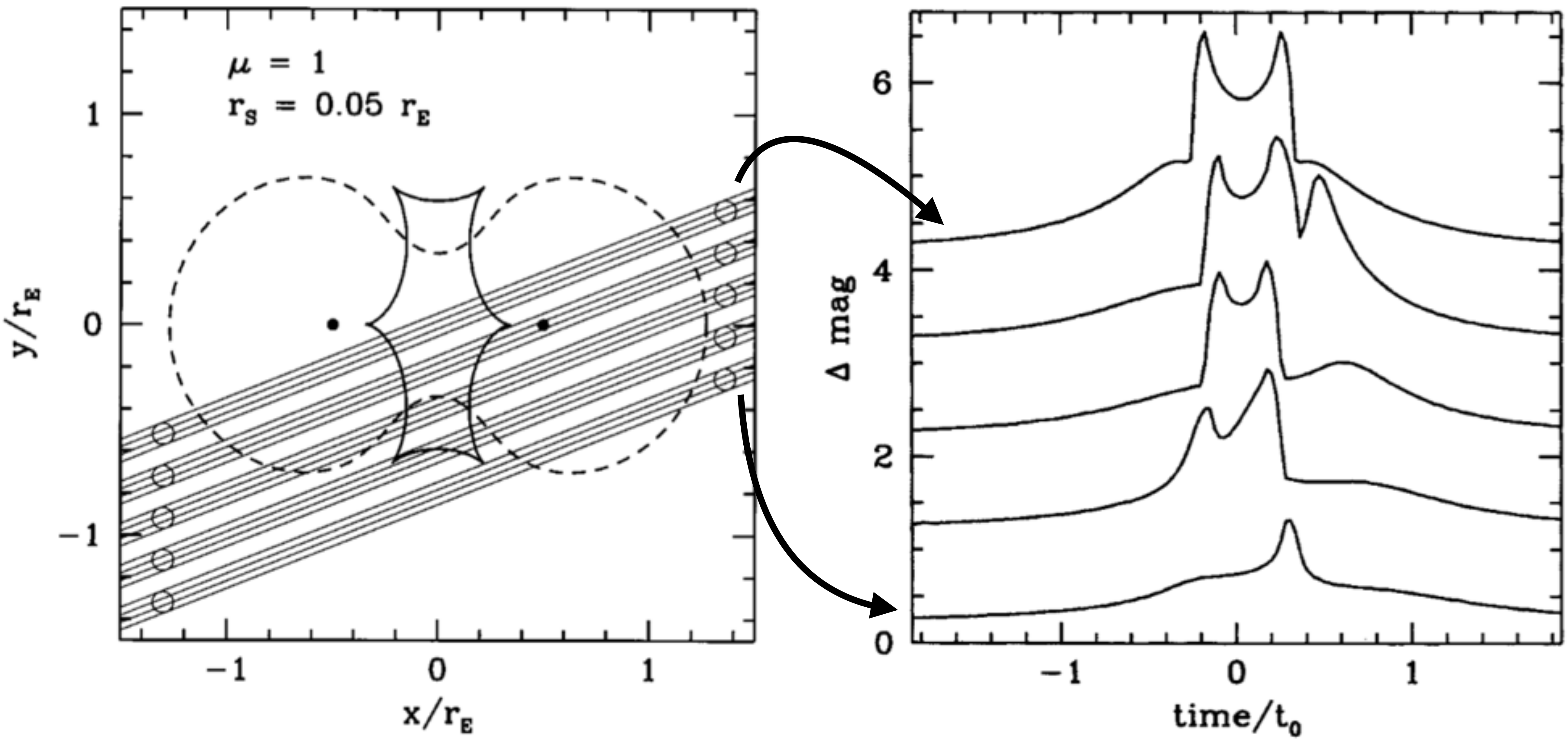
Schneider & Weiss (1986)





Light Curves of Double Point Mass Lens

Paczynski (1986)



The Double Point Mass Lens

- In the case of planet-star point mass lens the mass ratio is

$$q = \frac{m_{\rm p}}{M_\star} \ll 1$$

- In this case the lens equation then reduces to

$$\boldsymbol{y} \simeq \boldsymbol{x} - \frac{\boldsymbol{x}}{|\boldsymbol{x}|^2} - q\frac{\boldsymbol{x}-\boldsymbol{x}_{\rm p}}{|\boldsymbol{x}-\boldsymbol{x}_{\rm p}|^2} \qquad \text{as} \quad \frac{M_\star}{M}\frac{\boldsymbol{x}-\boldsymbol{x}_\star}{|\boldsymbol{x}-\boldsymbol{x}_\star|^2} \simeq \frac{\boldsymbol{x}}{|\boldsymbol{x}|^2}$$

  - choosing the origin to be the heavy foreground star, i.e. $\boldsymbol{x}_\star = 0$

- Multiplying through with denominators and rearranging gives

$$q|\boldsymbol{x}|^2(\boldsymbol{x}-\boldsymbol{x}_{\rm p}) \simeq |\boldsymbol{x}|^2\,|\boldsymbol{x}-\boldsymbol{x}_{\rm p}|^2\,\boldsymbol{x} - \boldsymbol{y}\,|\boldsymbol{x}|^2\,|\boldsymbol{x}-\boldsymbol{x}_{\rm p}|^2 - |\boldsymbol{x}-\boldsymbol{x}_{\rm p}|^2\,\boldsymbol{x}$$

  - 5th order polynomial in $\boldsymbol{x}$
  - Potentially has 5 solutions in agreement with figures from Schneider & Weiss

The Double Point Mass Lens

- Assume that star, planet and source all lie on one line (call it x-axis)
- Then all components off this axis (call them y-components) vanish
- Hence, the fifth order polynomial in $\boldsymbol{x}$ reduces to

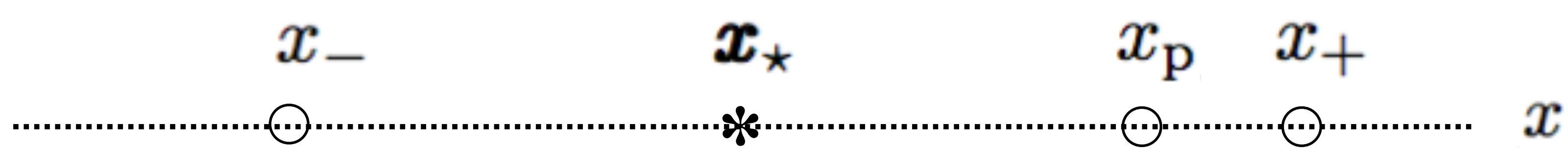
$$qx \simeq x^2(x - x_{\mathrm{p}}) - yx(x - x_{\mathrm{p}}) - (x - x_{\mathrm{p}})$$

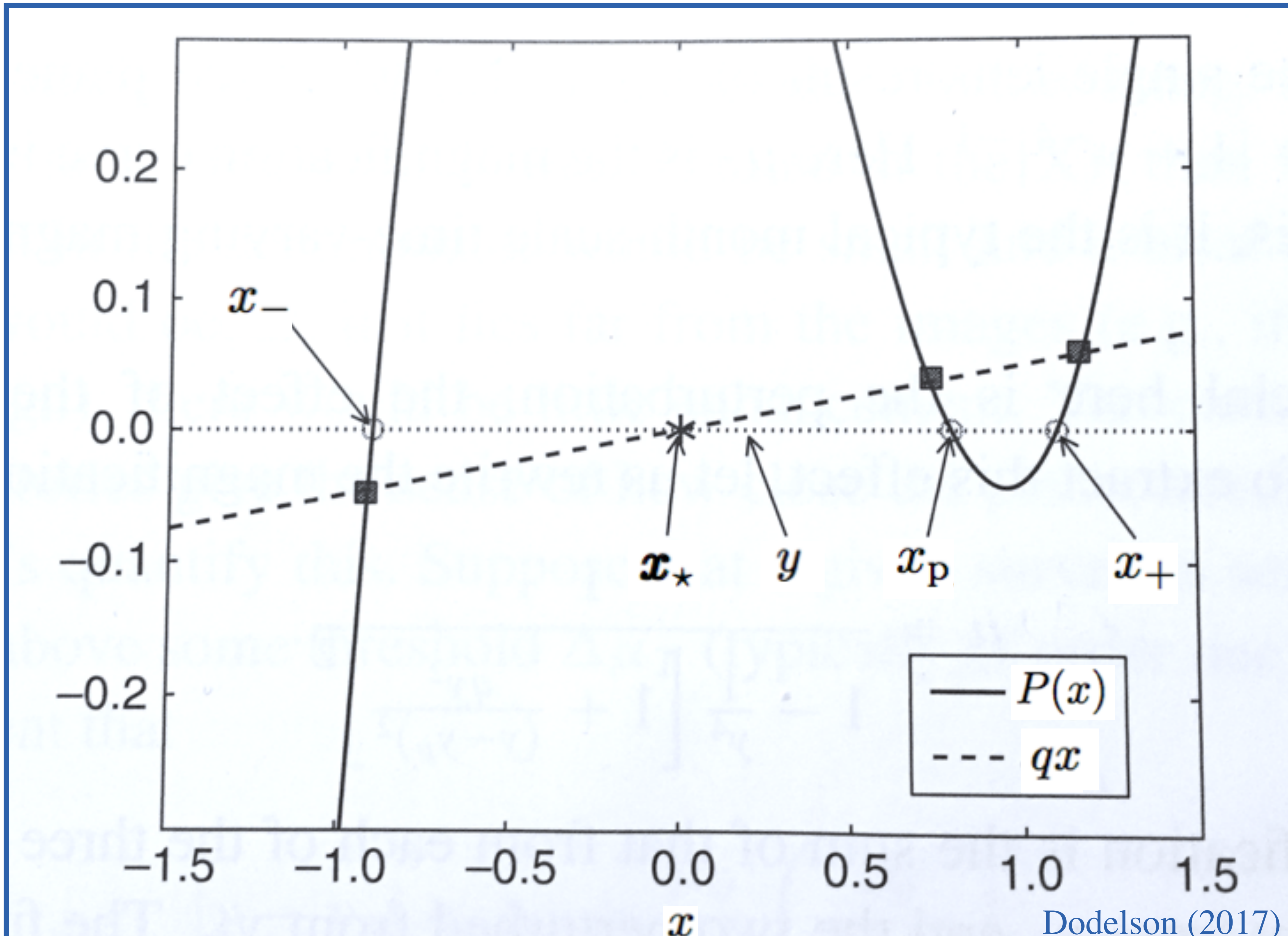
- For star-planet lens q is small, so solution to polynomial is ~full solution
- The solutions (zeros) are given by (Exercise 3)

$$(x - x_{\mathrm{p}}) = 0, \quad (x - x_+) = 0 \quad \text{and} \quad (x - x_-) = 0$$

- Where x_+ and x_- are the image position for the point mass lens (week 4)
- So for negligible mass there are three images of the background source
  - The nonzero mass of planet changes image positions slightly

The Double Point Mass Lens





Dodelson (2017)

Total Magnification from Double Point Mass Lens

- Multiple images occur; but in microlensing they are unresolved!
- To estimate total magnification of the system we can use $\mu \equiv \frac{1}{\det \mathcal{A}(\boldsymbol{\theta})}$ (Week 6)
- Hence, using the Lens equation for the double point mass lens:

$$\boldsymbol{y} \simeq \boldsymbol{x} - \frac{\boldsymbol{x}}{|\boldsymbol{x}|^2} - q \frac{\boldsymbol{x} - \boldsymbol{x}_\mathrm{p}}{|\boldsymbol{x} - \boldsymbol{x}_\mathrm{p}|^2}$$

- Calculating derivatives for Jacobian matrix: $\frac{\partial \beta_i}{\partial \theta_j} = \frac{\partial y_i}{\partial x_j}$
- Calculating determinant of Jacobian matrix: $\det \mathcal{A}$
- For the 'x-axis aligned' case it can be shown that the magnification is:

$$\mu = \left[1 - \left(\frac{1}{x_x^2} + \frac{q}{(x - x_\mathrm{p})_x^2}\right)^2\right]^{-1}$$ (Exercise 4)

  - where the *x*-subscript refers to the *x*-component of the image positions
  - where the y-components in the terms of the Jacobian matrix were set to 0

Total Magnification from Double Point Mass Lens

- So for the single-lens we find $\mu = \frac{1}{1 - \frac{\theta_E^4}{\theta^4}}$ from week 6
- 'Isolating' this part gives

$$\mu = \frac{1}{1 - \frac{1}{x^4}\left(1 + \frac{qx^2}{(x - x_p)^2}\right)^2}$$

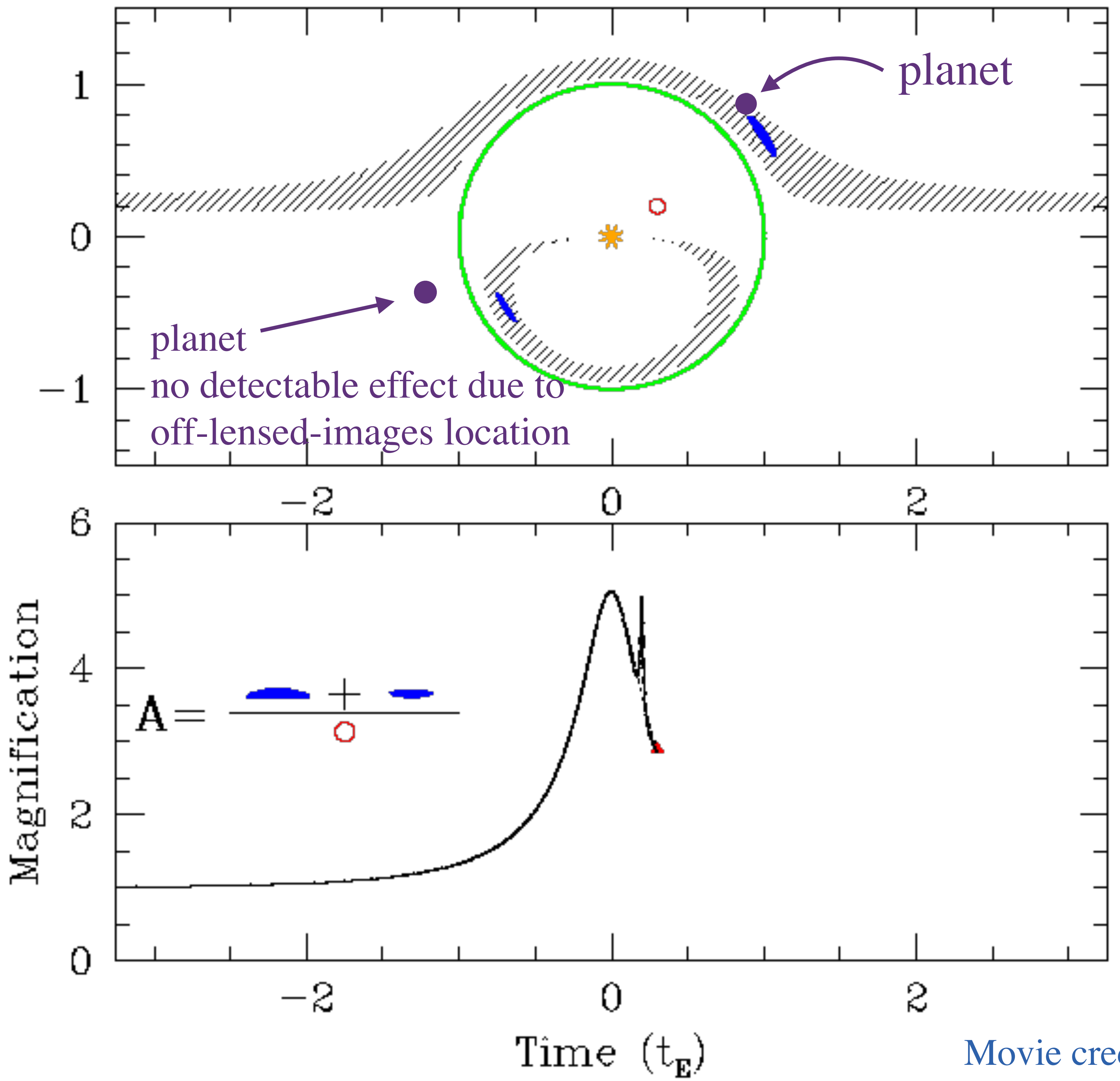
- Which can be used to show that the excess μ introduced by the planet is

$$\Delta\mu_p \simeq \frac{2\mu_0^2 q}{x^2(x - x_p)^2}$$

- So if the planet is close to the lensed image position $\Delta\mu_p$ becomes large
- If $(x\text{-}x_p)^2$ is large, i.e., the planet is away from images $\Delta\mu_p$ becomes small

The Double Point Mass Lens

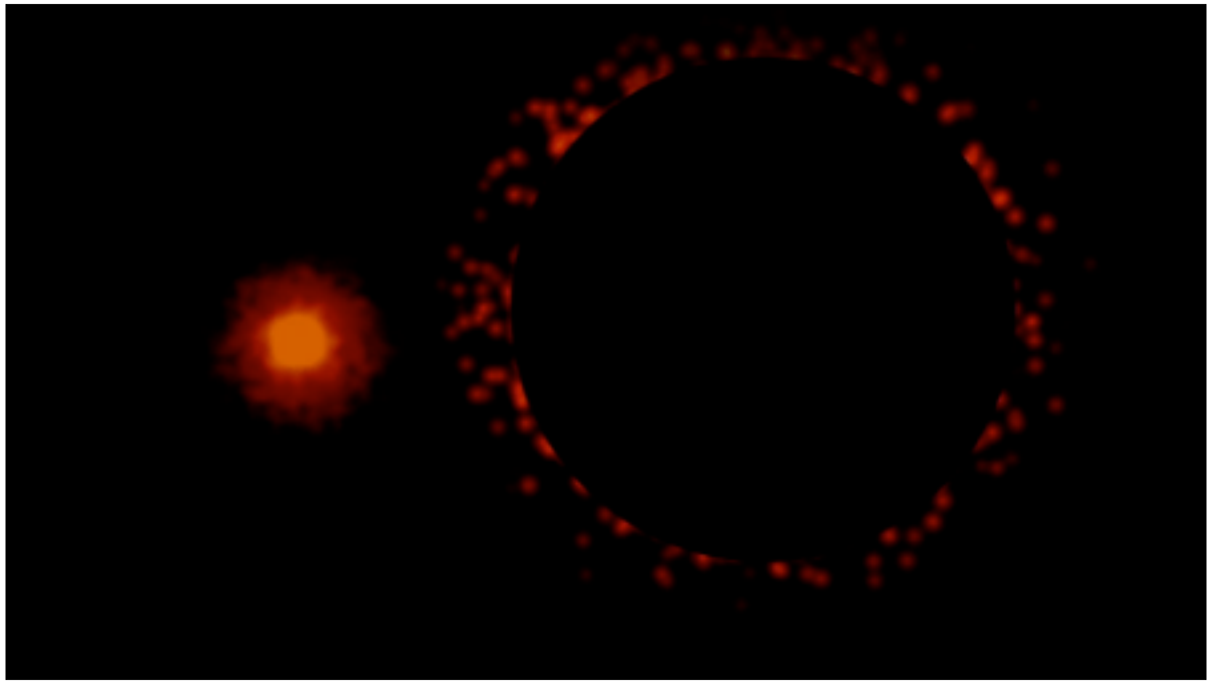
- Add planet around lens star … assume stationary during microlensing event



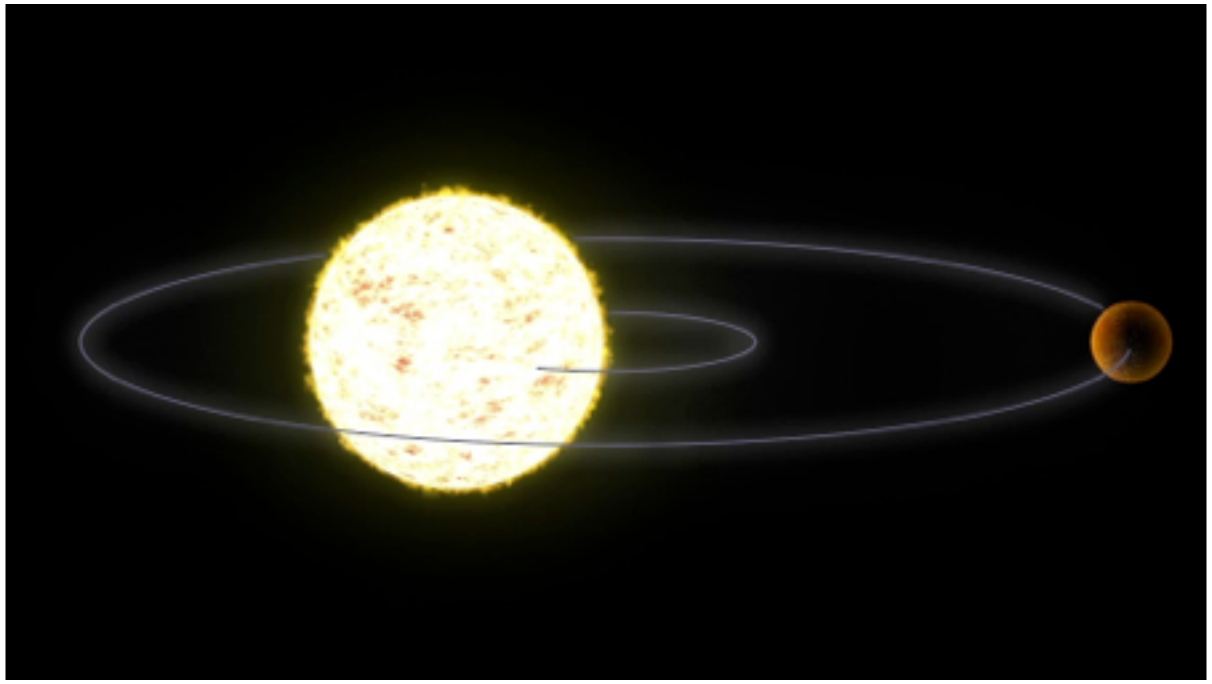
Movie credit: B. Scott Gaudi, OSU

Finding Planets

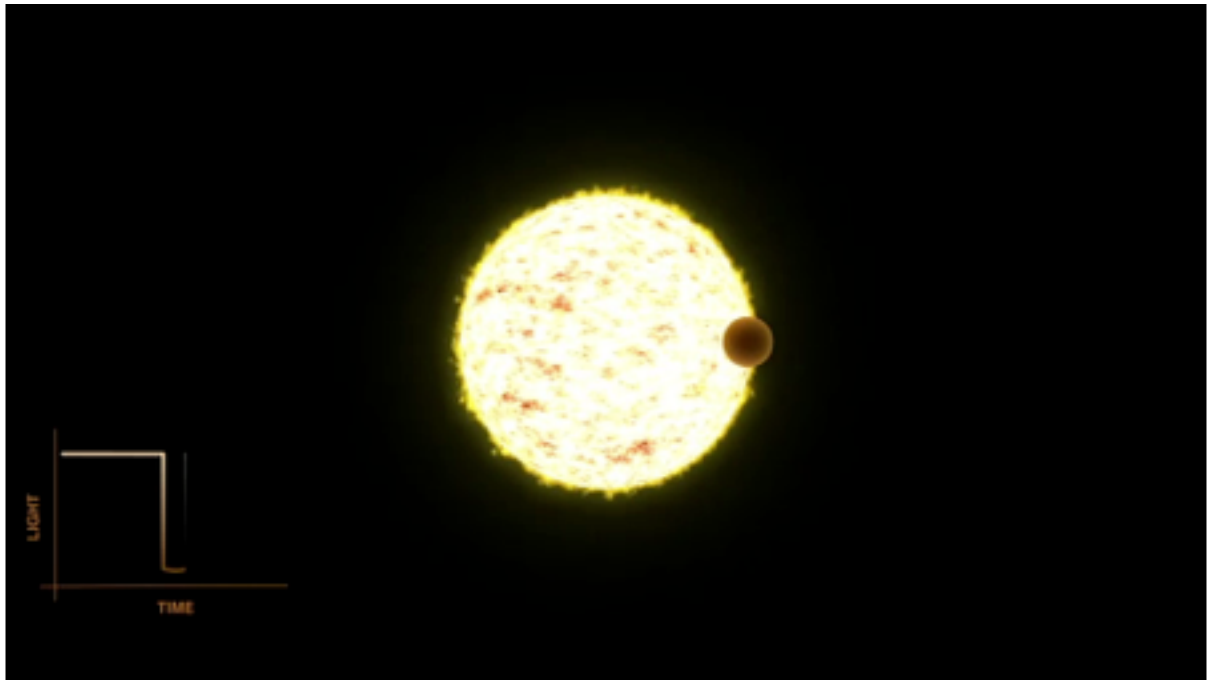
Direct detection; O(r_p, f); 1.2%



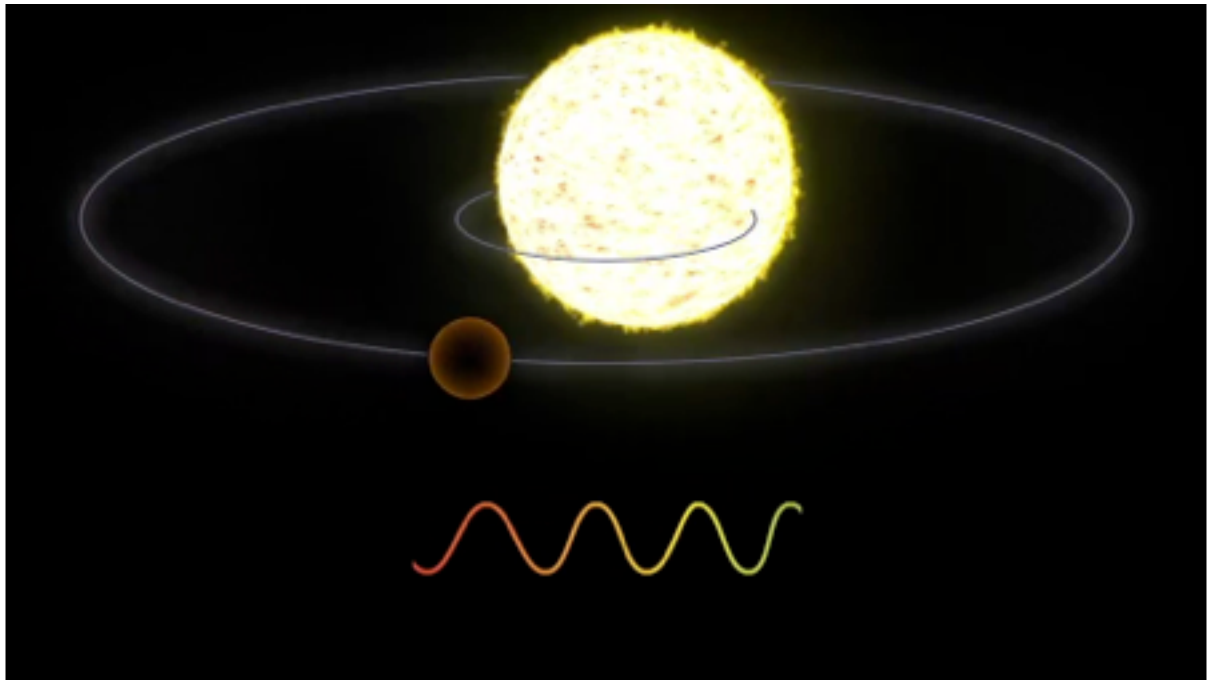
Astrometric Wobble; O(m/M); 0.03%



Planet Transit; O(r_p); 78.2%



Radial Velocity; O(m/M); 18.0%



Movies: https://exoplanets.nasa.gov

Why add Another Method - Microlensing?

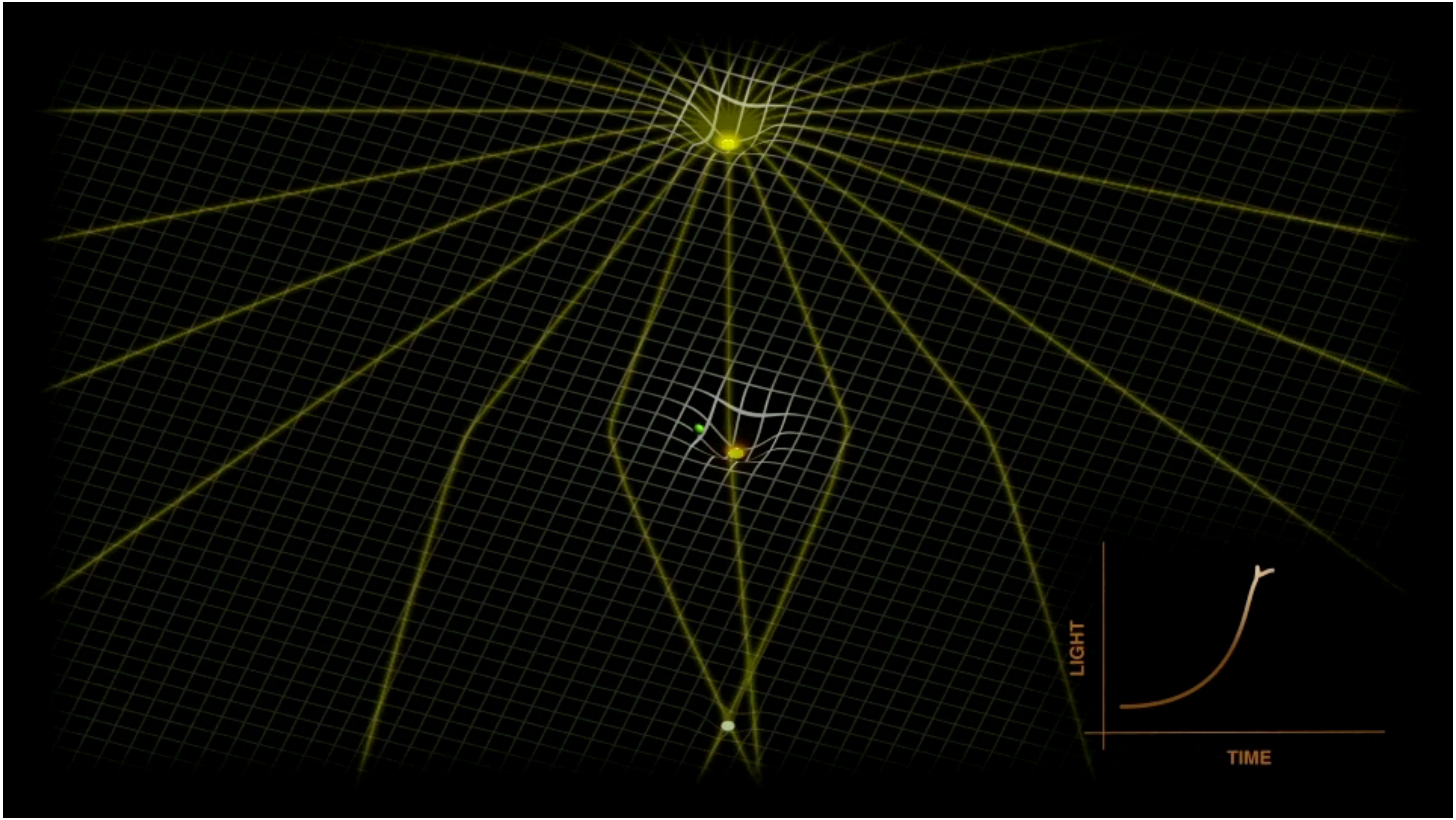
- Small probability of events
- Yes, of the order 10^6-10^7 but current surveys easily survey this many stars and 1000s of events and triggers have been found
- Duration very short (days - months)
- Coarse survey sampling with dense follow-up accommodate this
- Found planets are distant
- True, but a benefit of lensing - mapping planets further than other methods
- Impossible to do post-event follow-up
- But follow-up of host star might be possible
- Parameter degeneracy in modeling
- True for specific geometries, but good data allow braking these degeneracies
- Not m_p but q that is determined
- True, but knowing the host star mass (potentially statistically) you will know the planet masses to the same accuracy
- No independent confirmation
- True, but good enough data doesn't need independent confirmation

Schneider, Kochanek, Wambsganss (2006)

Microlensing Upsides for Planet Detections

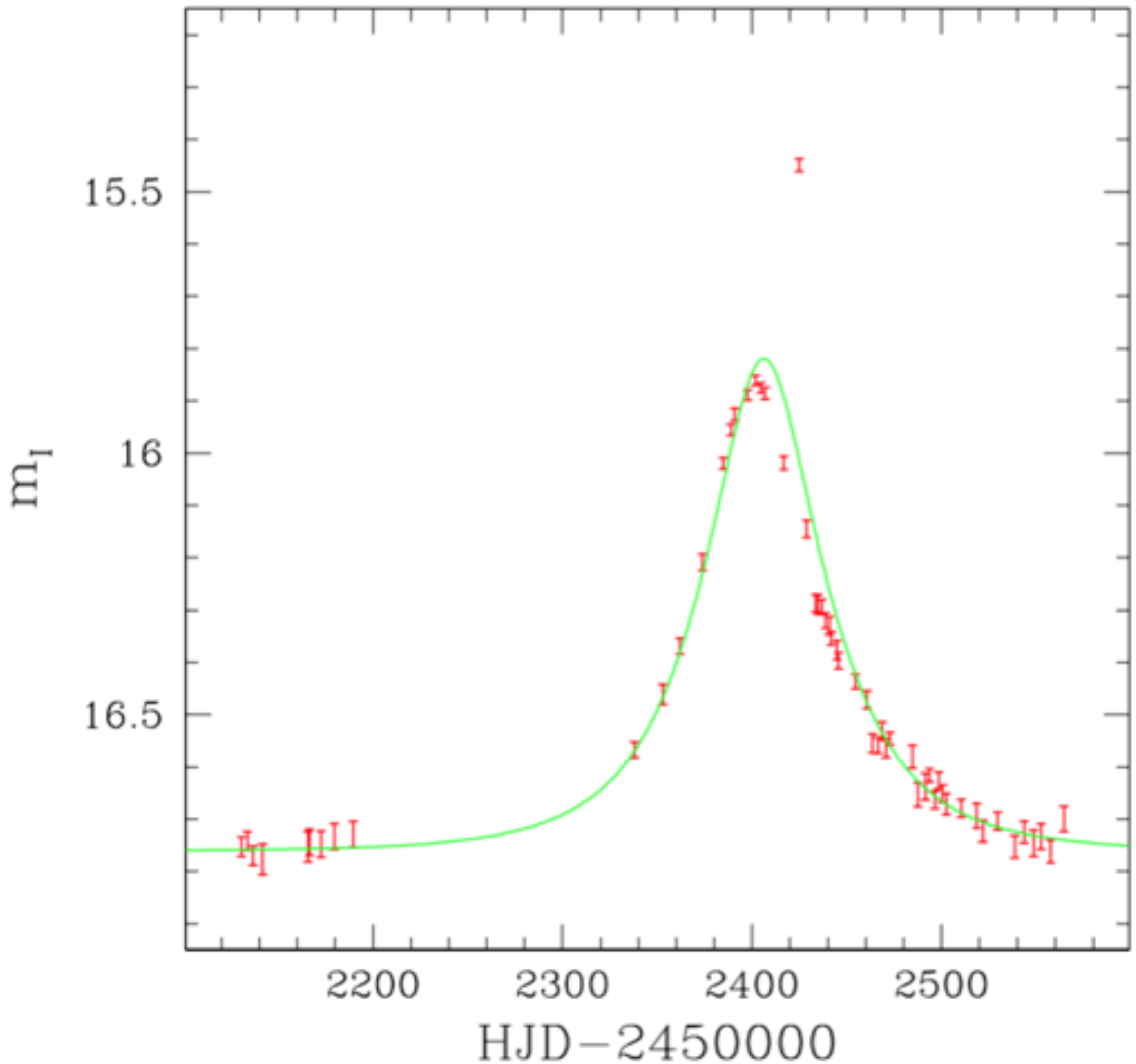
- No bias for nearby stars; i.e., a benefit that lensing needs a certain D_L
- No bias for planets around a certain type of star (solar or main sequence)
  - Other methods select and target hosts after a certain set of criteria
- No strong bias towards planets of large mass
- Microlensing sensitive down to masses of the order Earth mass
- Most effective for planets orbiting at R_E, overlaps with ‘habitable zone’
- Multiple planets detectable with same method
- Detection of free-floating planets (or other dark objects)
- Best statistical test (un-biased) of galactic sample of planets.
- (Caustic crossings of planet or planet satellites/moons can in principle be detected due to extreme magnification of reflected light)

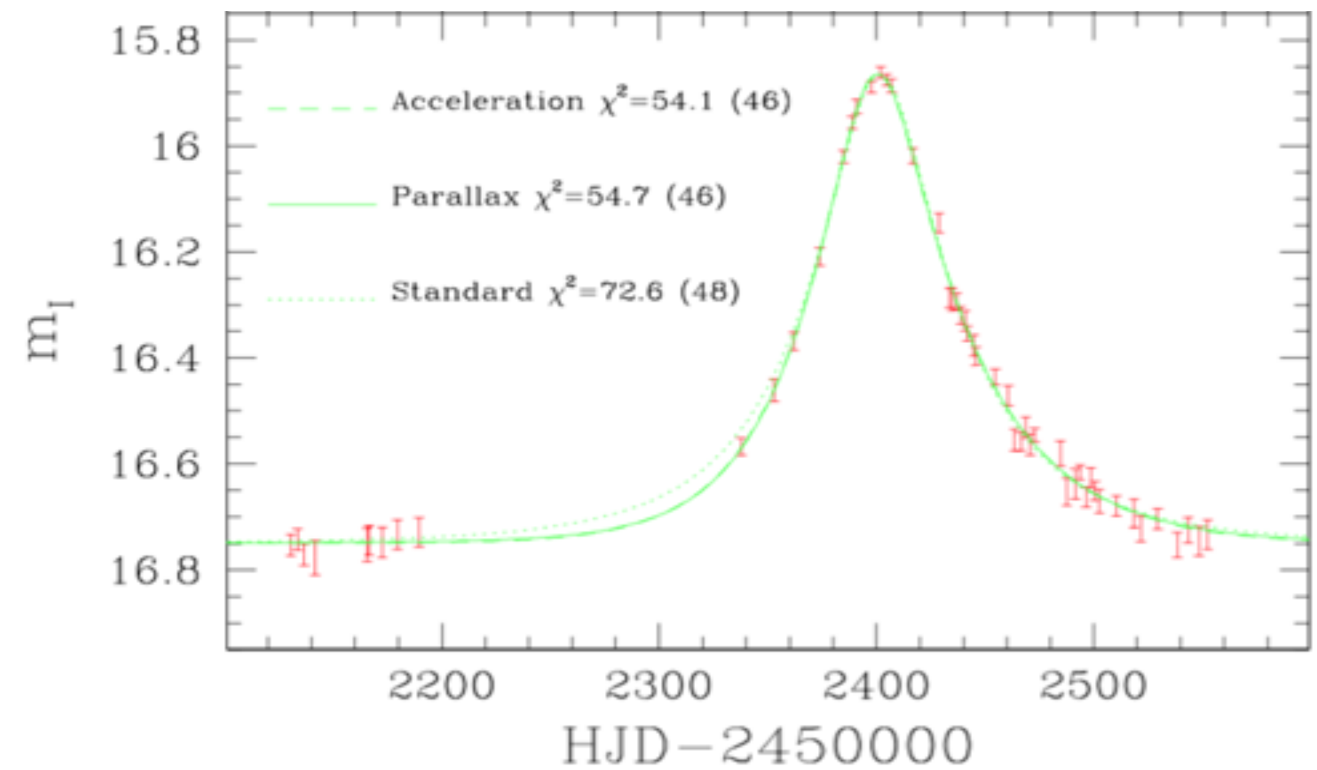
Microlensing with Double Point Mass

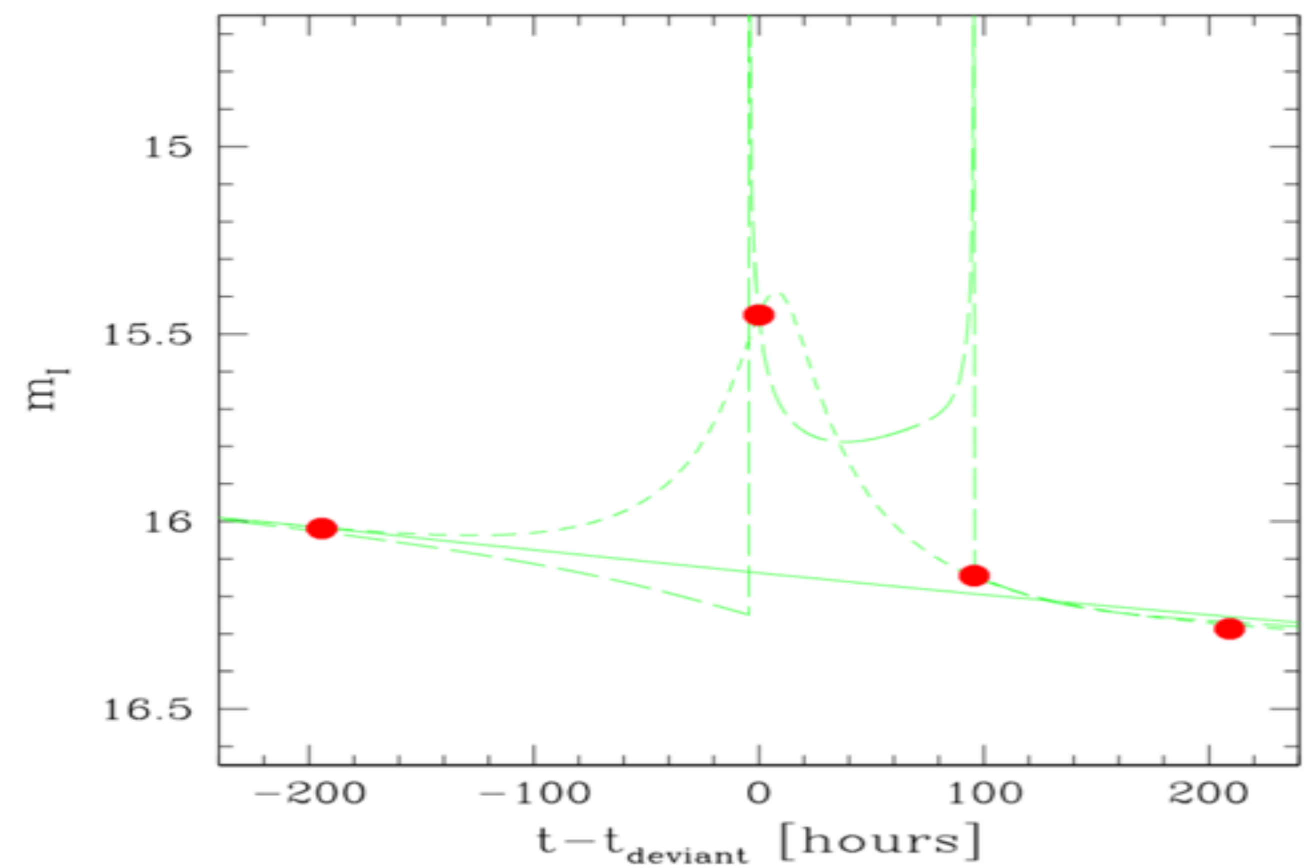


Movie: https://exoplanets.nasa.gov

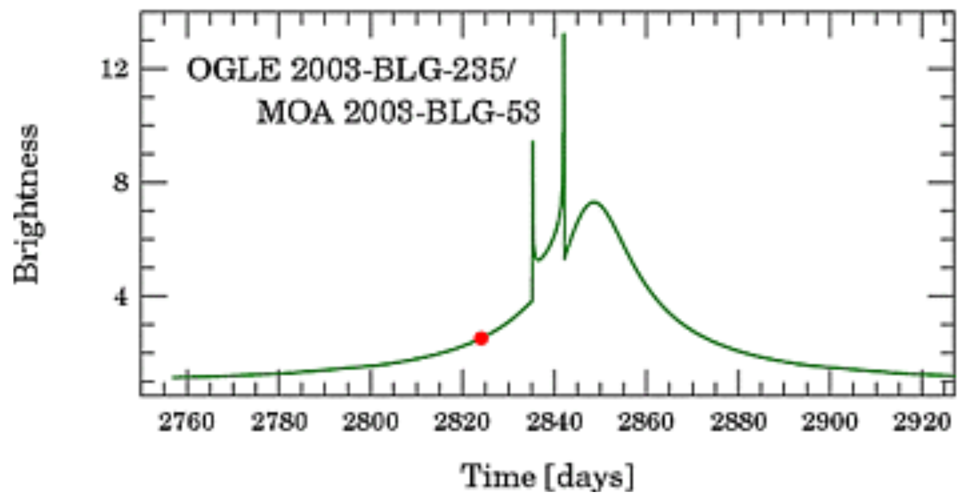
The First Planet Candidate - Jaroszynski et al. 2002

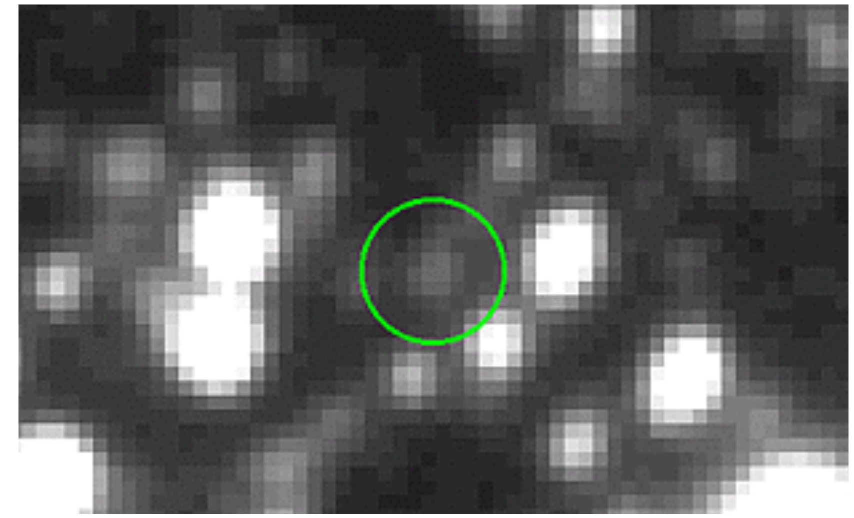


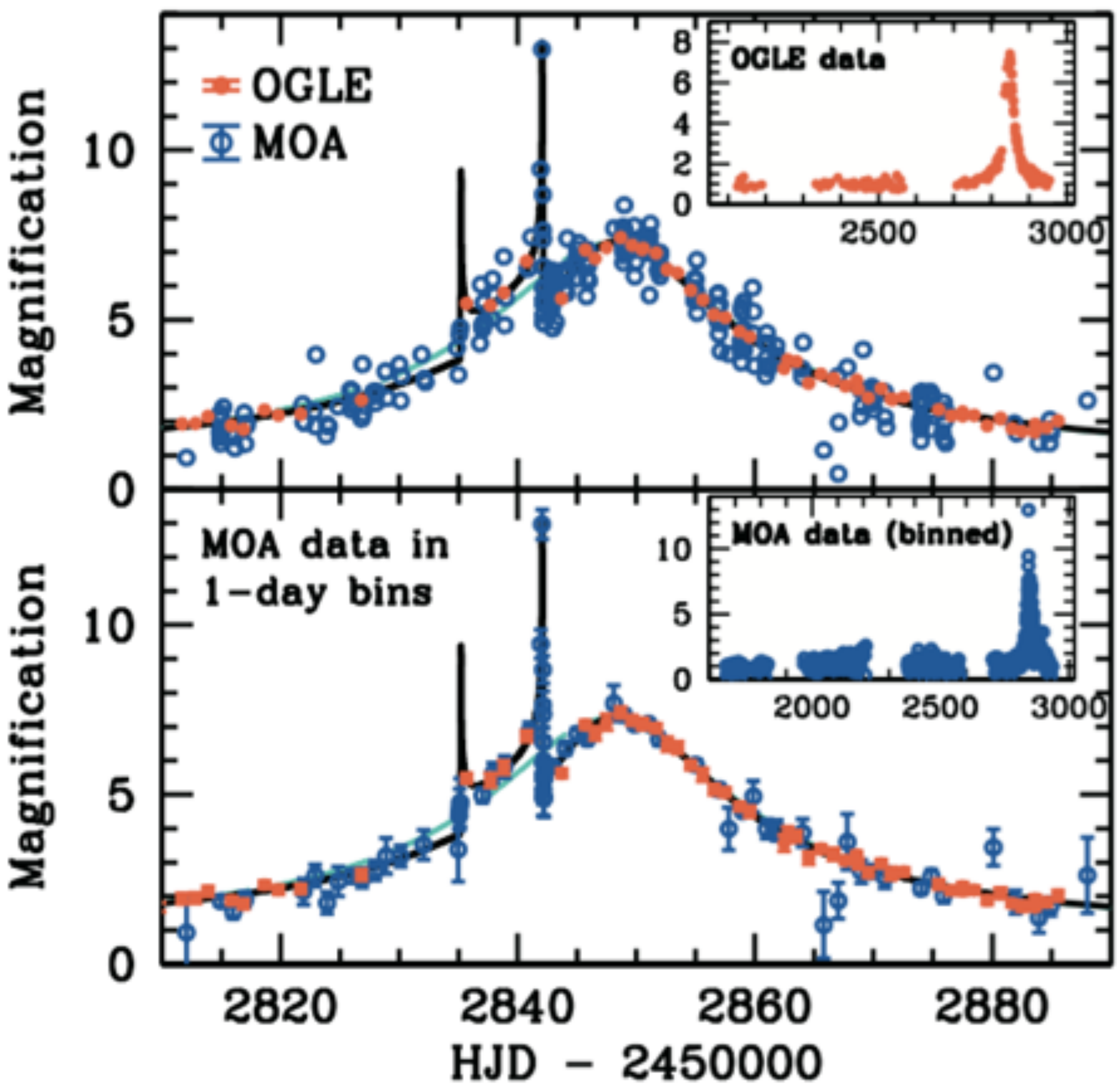


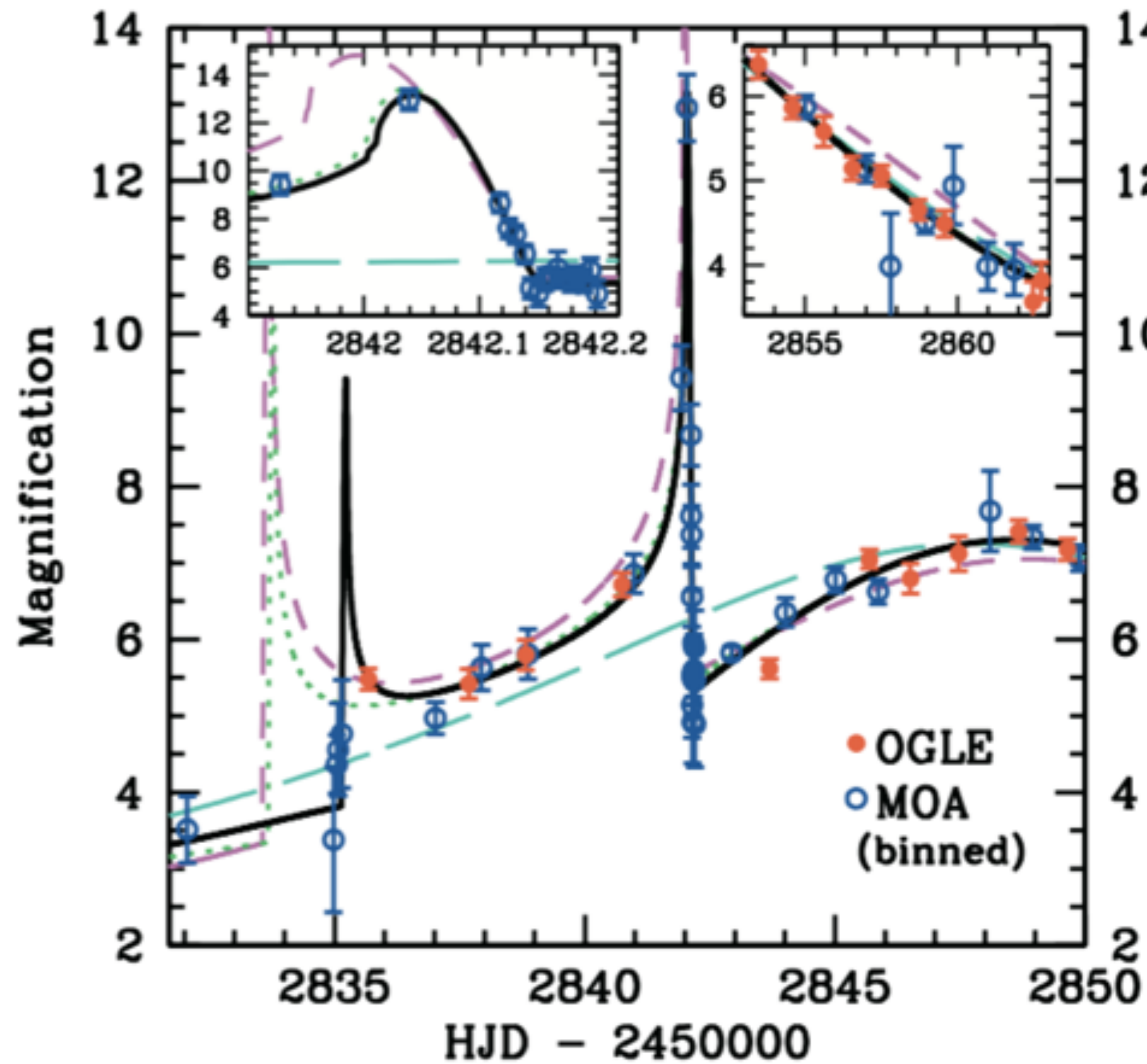


The First Clear Case - Bond et al. (2004)

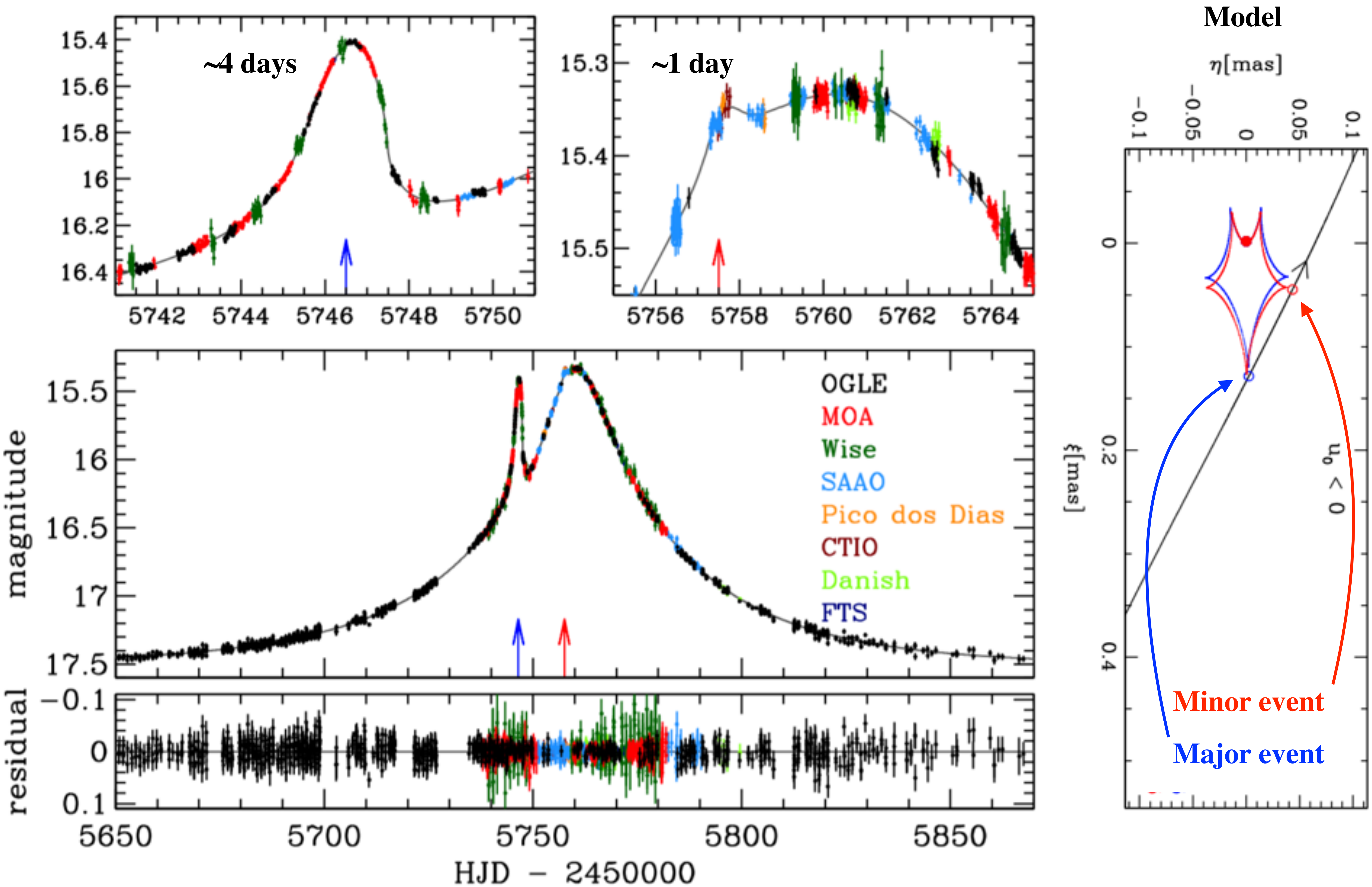






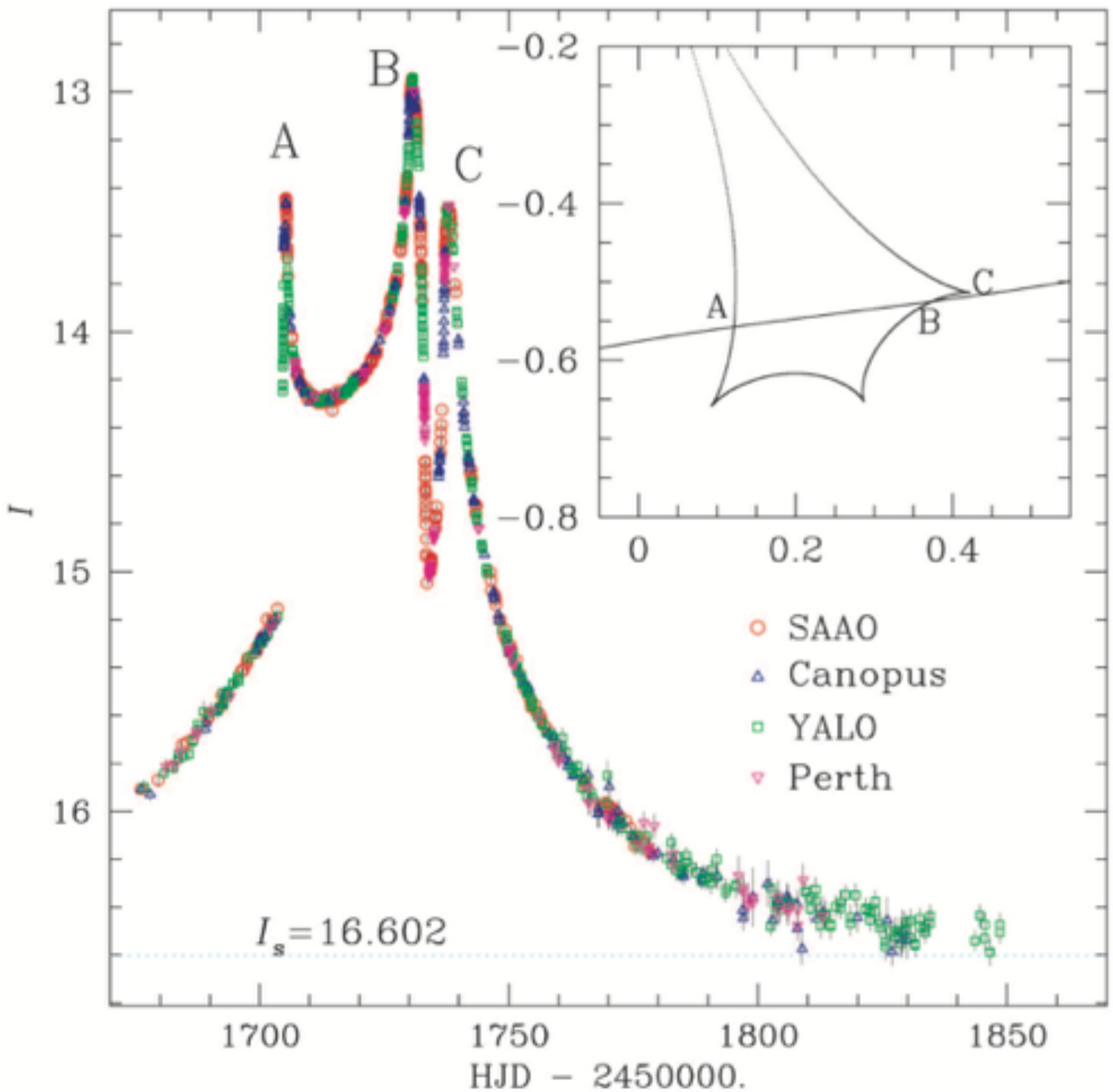


Today's data quality - Showron et al. 2015

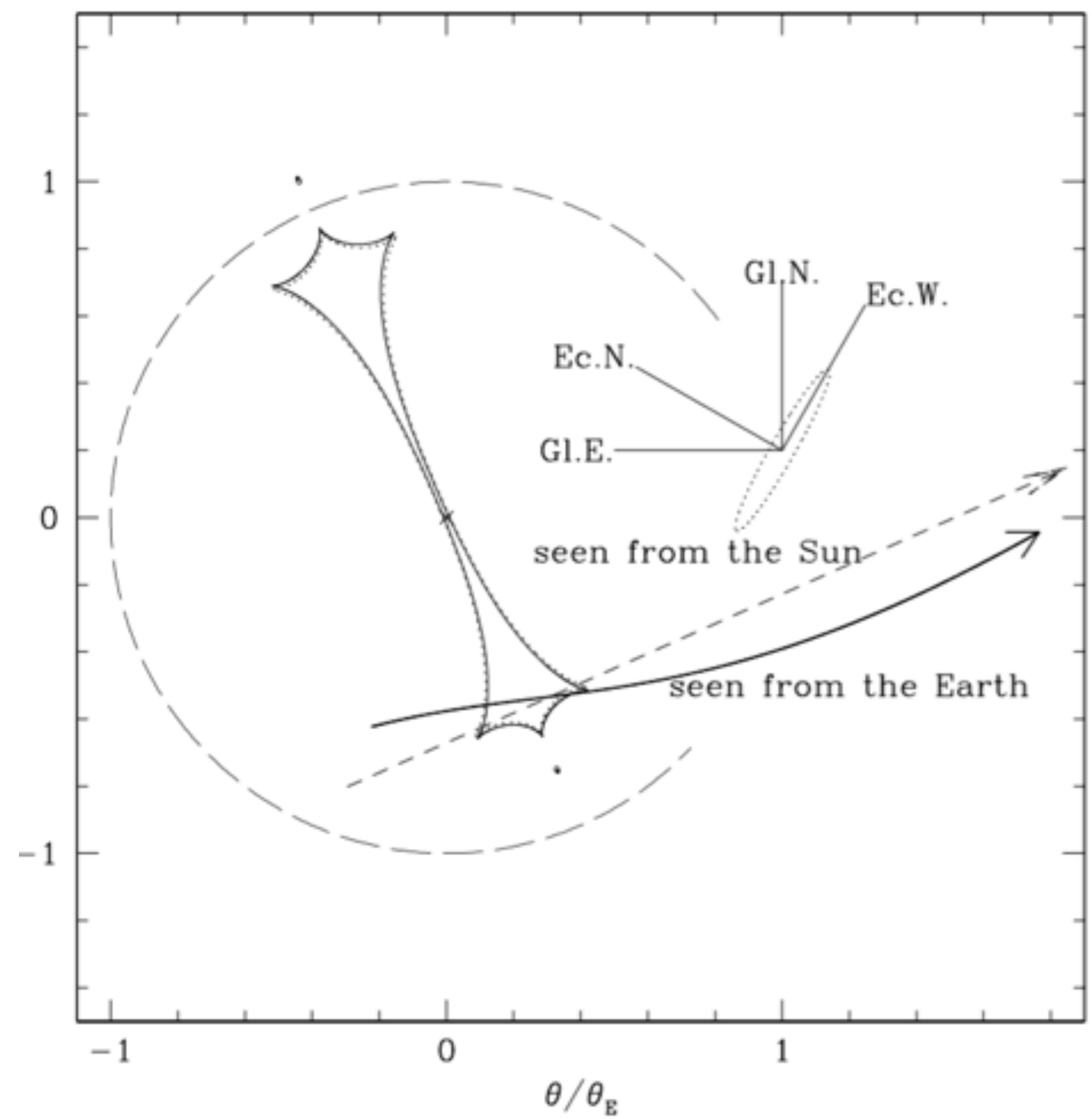


Modeling Sensitive to the Parallax of the Earth

An+00



An+00

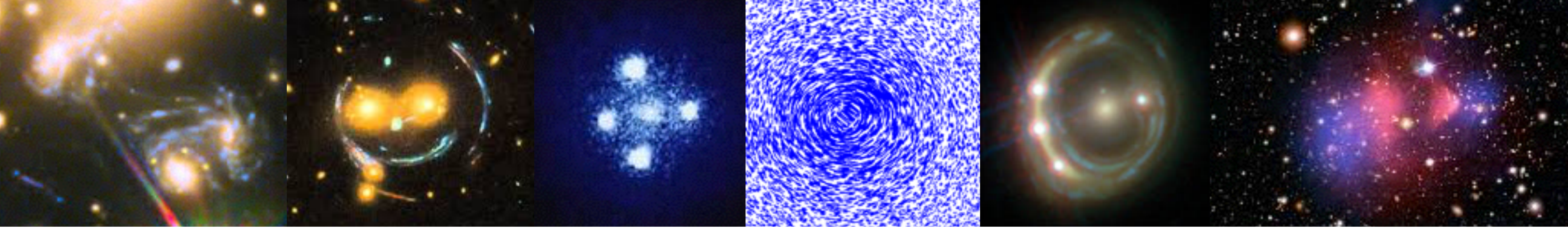


So in summary…

- The deflection from multiple point masses $$\boldsymbol{\alpha}(\boldsymbol{\theta}) = \sum_i \theta_{\mathrm{E},i}^2 \frac{\boldsymbol{\theta} - \boldsymbol{\theta}_i}{|\boldsymbol{\theta} - \boldsymbol{\theta}_i|^2}$$
- With the correspond lens equation $$\boldsymbol{y} = \boldsymbol{x} - \sum_i \frac{m_i}{M} \frac{\boldsymbol{x} - \boldsymbol{x}_i}{|\boldsymbol{x} - \boldsymbol{x}_i|^2}$$
- Using the definitions:

$$\boldsymbol{y} = \frac{\boldsymbol{\beta}}{\theta_{\mathrm{E}}} \qquad \boldsymbol{x} = \frac{\boldsymbol{\theta}}{\theta_{\mathrm{E}}} \qquad \theta_{\mathrm{E}}^2 \equiv \sum_i \theta_{\mathrm{E}_i}^2 = \frac{MGD_{\mathrm{LS}}}{D_{\mathrm{L}} D_{\mathrm{S}} c^2} \qquad M \equiv \sum_i m_i$$

- For a lens consisting of a star and a planet with a mass ratio $q = \frac{m_{\mathrm{p}}}{M_\star}$
  - the lens equation becomes $$\boldsymbol{y} \simeq \boldsymbol{x} - \frac{\boldsymbol{x}}{|\boldsymbol{x}|^2} - q \frac{\boldsymbol{x} - \boldsymbol{x}_{\mathrm{p}}}{|\boldsymbol{x} - \boldsymbol{x}_{\mathrm{p}}|^2}$$
- Microlensing is complimentary to other methods to find planets:
  - Can probe larger distances
  - Not biased towards high mass planets or large-orbit planets
  - Works best for planets close to the ‘habitable zone’
- The microlensing surveying teams are well-coordinated and produce well-sampled data presenting planet microlensing events of high quality



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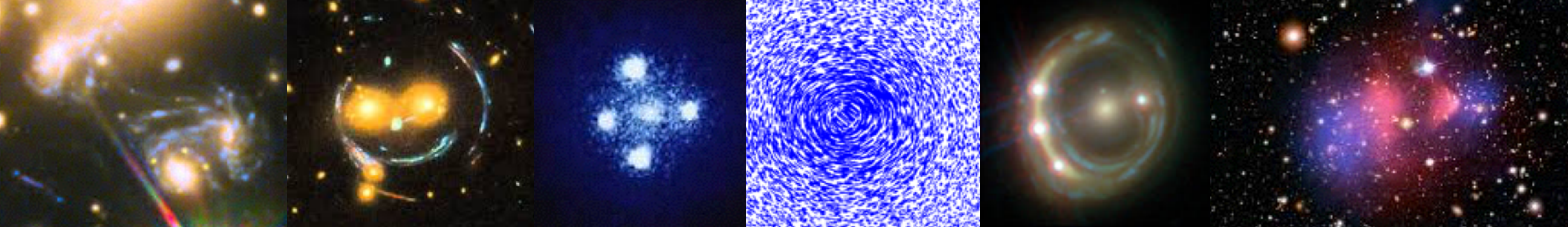
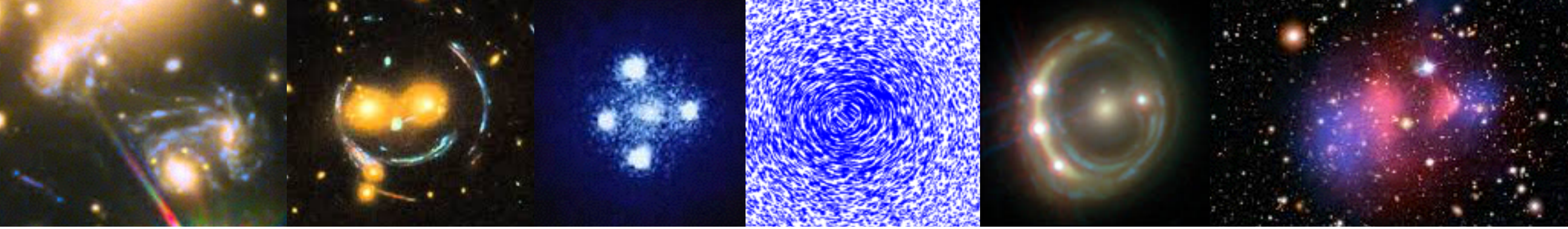
Questions?



Last Week's Worksheet



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This Week's Worksheet