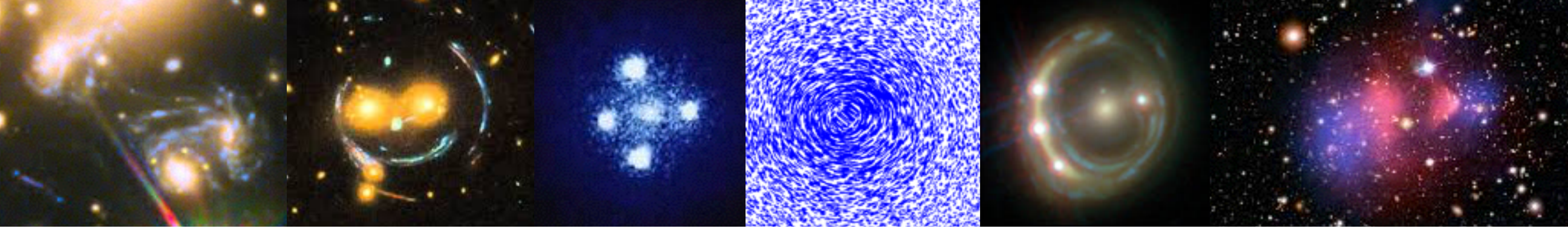

PHY-765 SS18 Gravitational Lensing Week 13

Cosmic Shear & the CMB

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Last week

- Talked about the weak lensing shearing of objects
- Used Jacobian Matrix and assumptions about sphericity to see that
  - (simple) shearing corresponds to ellipticity

$$1 = \frac{(1-\gamma_1)^2}{\beta_0^2}\theta_1^2 + \frac{(1+\gamma_1)^2}{\beta_0^2}\theta_2^2$$

- Described the surface brightness moments ellipticity

$$q_{ij} \equiv \int d^2\theta \, \mathcal{S}^{\mathrm{obs}}(\boldsymbol{\theta})\theta_i\theta_j \qquad \epsilon_1 \equiv \frac{q_{11}-q_{22}}{q_{11}+q_{22}} \qquad \epsilon_2 \equiv \frac{2q_{12}}{q_{11}+q_{22}}$$

- Using the Jacobian Matrix this can be expressed in terms of κ and γ

$$\epsilon_i = \frac{2\gamma_i}{1-\kappa}\left[1 - \frac{\gamma^2}{(1-\kappa)^2}\right]^{-1}$$

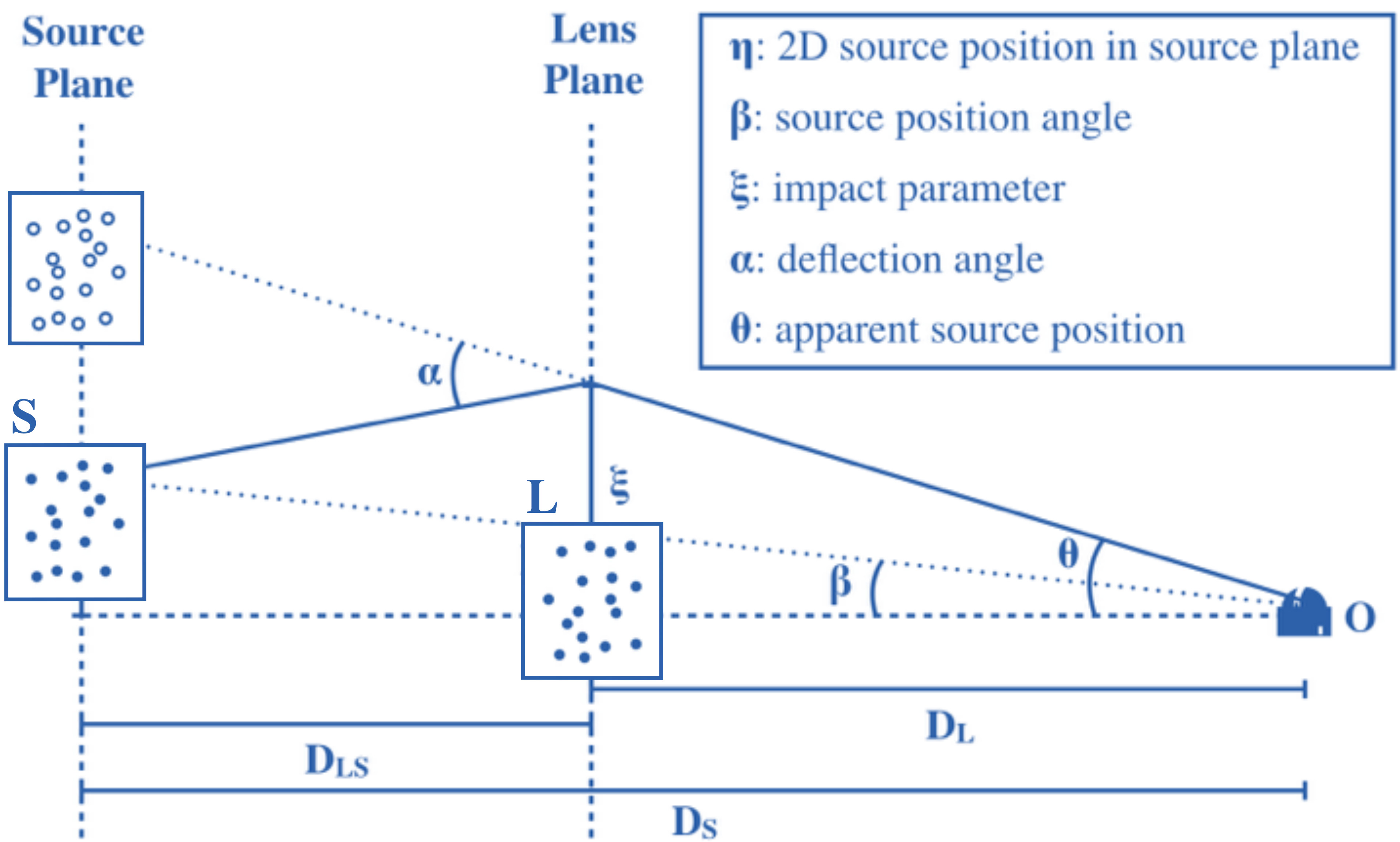
- Considered challenges with determining weak lensing
  - Intrinsic ε, weighing of images, accounting for PSF, etc.
- The Bullet Cluster as a proof of the existence of dark matter

The aim of today

- Deflection of ‘diffuse mass’ by ‘diffuse mass’
- The concept of cosmic shear
- Fourier space description of cosmic shear lensing effects
- The power spectrum as a tool
- Lensing of the Cosmic Microwave Background

Light Deflection (Lensing) Regime

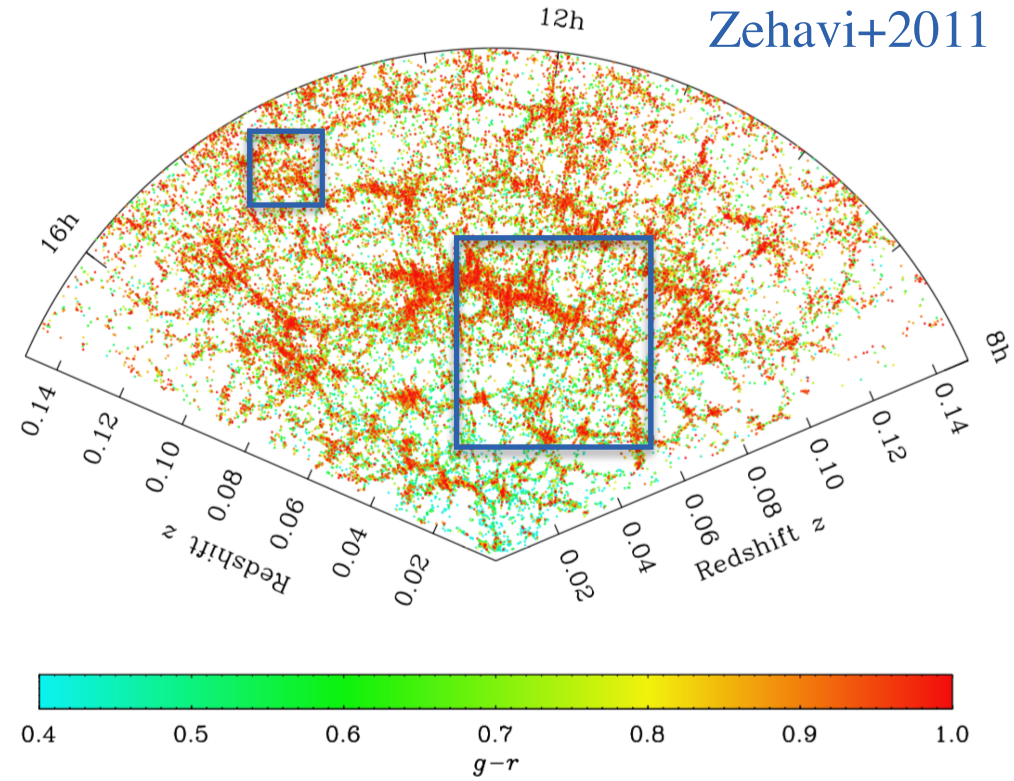
- Strong lensing: Concentrated source deflected by concentrated lens
- Weak lensing: Concentrated source deflected by diffuse lens
- ‘Cosmic weak lensing’: Diffuse source deflected by diffuse lens



Cosmic Shear

- The large scale structure of the Universe
- Web of mass affecting background sources
- Lensing (shearing) from cosmic structure
- Assessment done in a statistical manner
- LSS predicted by cosmological models

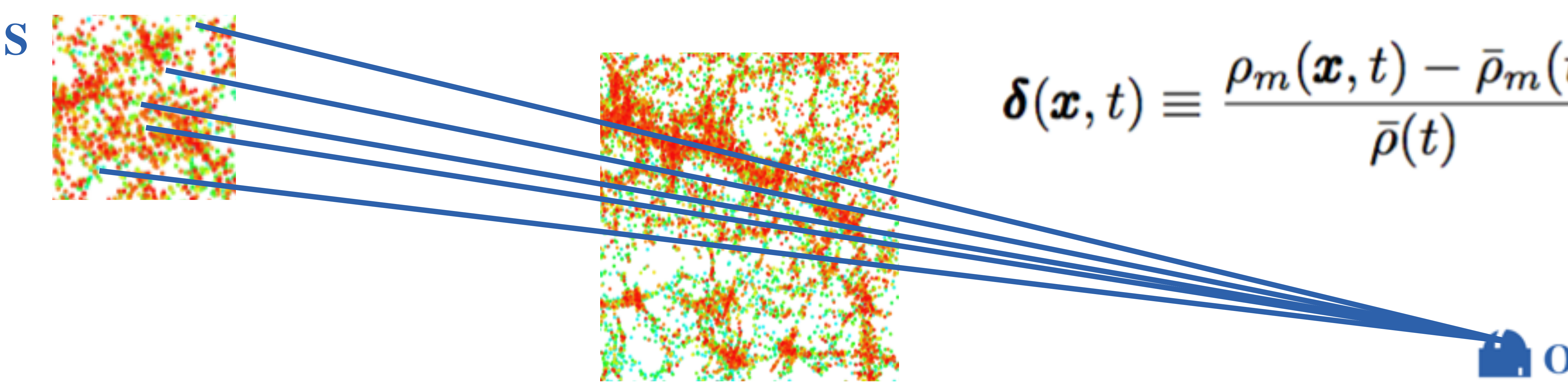
Zehavi+2011



$$p(t) = \sum w \times \rho_w(t) \quad ; \quad w = m, r, \Lambda$$

$$\rho(t) = \rho_m(t) + \rho_r(t) + \rho_\Lambda(t)$$

- Cosmic shear probes cosmology via the density contrast defined as



$$\boldsymbol{\delta}(\boldsymbol{x}, t) \equiv \frac{\rho_m(\boldsymbol{x}, t) - \bar{\rho}_m(t)}{\bar{\rho}(t)}$$

Shear and the Gravitational Potential

- The Jacobina relates κ and γ to the gravitational potential (week 6)

$$\mathcal{A}(\boldsymbol{\theta}) = \begin{pmatrix} 1 - \frac{\partial \alpha_i}{\partial \theta_i} & -\frac{\partial \alpha_i}{\partial \theta_j} \\ -\frac{\partial \alpha_j}{\partial \theta_i} & 1 - \frac{\partial \alpha_j}{\partial \theta_j} \end{pmatrix} = \begin{pmatrix} 1 - \frac{\partial^2 \psi}{\partial \theta_i^2} & -\frac{\partial^2 \psi}{\partial \theta_i \partial \theta_j} \\ -\frac{\partial^2 \psi}{\partial \theta_j \partial \theta_i} & 1 - \frac{\partial^2 \psi}{\partial \theta_j^2} \end{pmatrix}$$

$$\kappa \equiv \frac{1}{2}\left(\frac{\partial^2 \psi}{\partial \theta_i^2} + \frac{\partial^2 \psi}{\partial \theta_j^2}\right) \qquad \gamma_1 \equiv \frac{1}{2}\left(\frac{\partial^2 \psi}{\partial \theta_i^2} - \frac{\partial^2 \psi}{\partial \theta_j^2}\right)$$

$$\gamma_2 \equiv \frac{\partial^2 \psi}{\partial \theta_i \partial \theta_j}$$

- The projected gravitational potential along the line of sight is defined as

$$\psi(\boldsymbol{\theta}) \equiv \frac{2}{D_\mathrm{S}} \int_0^{D_\mathrm{S}} dD_\mathrm{L}\, \Phi\left(x^i = D_\mathrm{L}\theta^i, D_\mathrm{L}; t = t_0 - \frac{D_\mathrm{L}}{c}\right) \frac{D_\mathrm{S} - D_\mathrm{L}}{D_\mathrm{L}}$$

The Project Gravitational Potential

Gives deflection angle as

$$\alpha^i(\boldsymbol{\theta}) = \frac{\partial\psi(\boldsymbol{\theta})/c^2}{\partial\theta^i}$$

Integrate along line of sight to source

Grav. potential evaluated at time t when photons pass the lens plane D_L

$$\psi(\boldsymbol{\theta}) \equiv \frac{2}{D_\mathrm{S}} \int_0^{D_\mathrm{S}} dD_\mathrm{L}\, \Phi\left(x^i = D_\mathrm{L}\theta^i, D_\mathrm{L}; t = t_0 - \frac{D_\mathrm{L}}{c}\right) \frac{D_\mathrm{S} - D_\mathrm{L}}{D_\mathrm{L}}$$

Mass inhomogeneities close to source ignored

Newtonian grav. potential

x^i is the (2D) spatial component of four vector $x^\alpha = (t, x, y, z)$ (week 2)

Grav. potential sampled at transverse distance $D_\mathrm{L}\theta^i$

Born Approximation

- Gravitational potential directly relates to the density contrast via the Poisson equation:

$$\nabla^2\Phi(\boldsymbol{x}, t) = \frac{4\pi G}{c^2}\bar{\rho}_m(t)\; a^2(t)\; \delta(\boldsymbol{x}, t) \qquad \frac{1}{a(t)} = \frac{a(t_\mathrm{obs})}{a(t_\mathrm{emit})} = (1+z)$$

Matter Distribution Described in Fourier Space

- As γ relates to the gravitational potential it relates to the density contrast
- We are interested in the statistics of these density fluctuations
- Convenient to perform such investigations in Fourier space
- Any field can be expressed in terms of its Fourier transform:

$$f(x) = \int \frac{dk}{2\pi}\, \tilde{f}(k) e^{ikx}$$

- In 2D taking κ from Fourier (***l***) space to real (***θ***) space would then be

$$\kappa(\boldsymbol{\theta}) = \int \frac{d^2 l}{(2\pi)^2}\, \tilde{\kappa}(\boldsymbol{l}) e^{i\boldsymbol{l}\cdot\boldsymbol{\theta}}$$

- and you can go back with

$$\tilde{\kappa}(\boldsymbol{l}) = \int d^2\theta\, \kappa(\boldsymbol{\theta}) e^{-i\boldsymbol{l}\cdot\boldsymbol{\theta}}$$

- Using the relations between κ, γ and Φ, you get Fourier space expressions:

$$\tilde{\kappa}(\boldsymbol{l}) = \frac{-l^2}{2c^2}\, \tilde{\psi}(\boldsymbol{l}) \qquad \tilde{\gamma}_1(\boldsymbol{l}) = \frac{-l_x^2 + l_y^2}{2c^2}\, \tilde{\psi}(\boldsymbol{l}) \qquad \tilde{\gamma}_2(\boldsymbol{l}) = \frac{-l_x l_y}{c^2}\, \tilde{\psi}(\boldsymbol{l})$$

- In Fourier space, real-space derivatives appear as powers of conjugate, ***l***

The Two-Point Correlation Function

- Combining these three equations gives expression of Fourier convergence

$$\tilde{\boldsymbol{\kappa}}(\boldsymbol{l}) = \frac{(l_x^2 - l_y^2)\tilde{\gamma}_1(\boldsymbol{l}) + 2 l_x l_y \tilde{\gamma}_2(\boldsymbol{l})}{l^2}$$ (Exercise 3)

  - *l*=0 corresponds to a Fourier mode with no variation, i.e., a constant
  - So up to some constant value of κ this expression holds (MSD week 10)

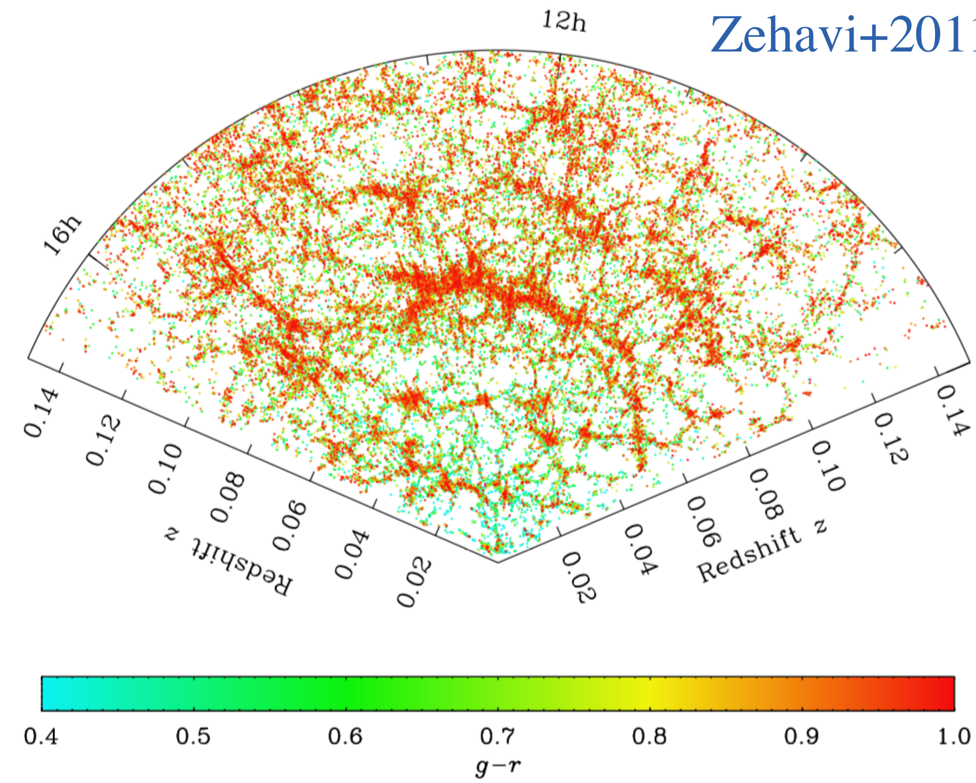
- We want to describe the matter density field statistically:
  - 1st order statistic: $\langle \delta(\boldsymbol{x}) \rangle = 0$
  - 2nd order statistic: $\sigma^2 \equiv \langle \delta(\boldsymbol{x})^2 \rangle$

- Define the two point correlation as

$$\xi(\boldsymbol{x}, \boldsymbol{y}) \equiv \langle \delta(\boldsymbol{x}) \delta(\boldsymbol{y}) \rangle$$

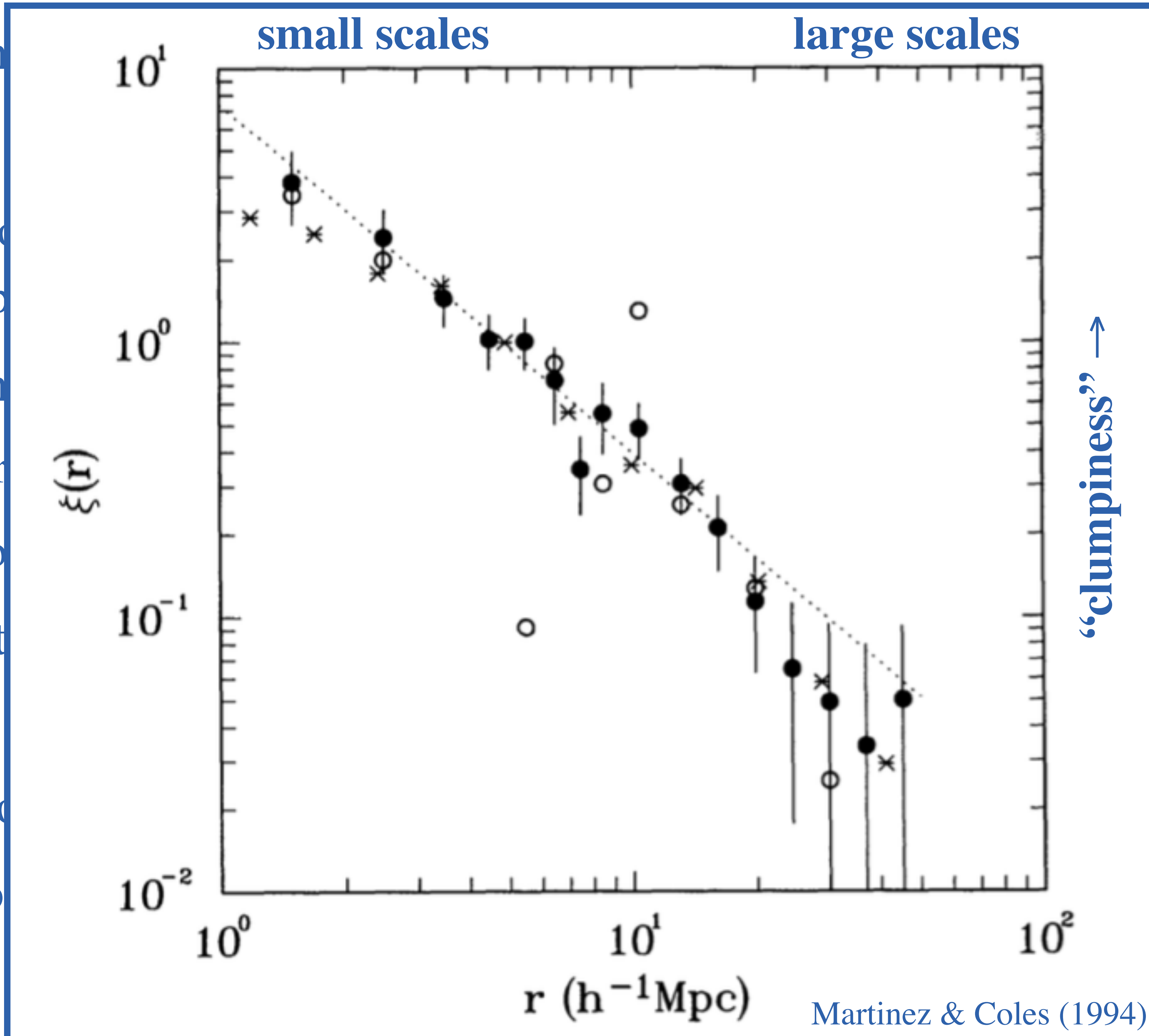
- Then from homogeneity: $\xi(\boldsymbol{x}, \boldsymbol{y}) = \xi(\boldsymbol{x} - \boldsymbol{y})$
- And from isotropy: $\xi(\boldsymbol{x} - \boldsymbol{y}) = \xi(|\boldsymbol{x} - \boldsymbol{y}|)$

Zehavi+2011

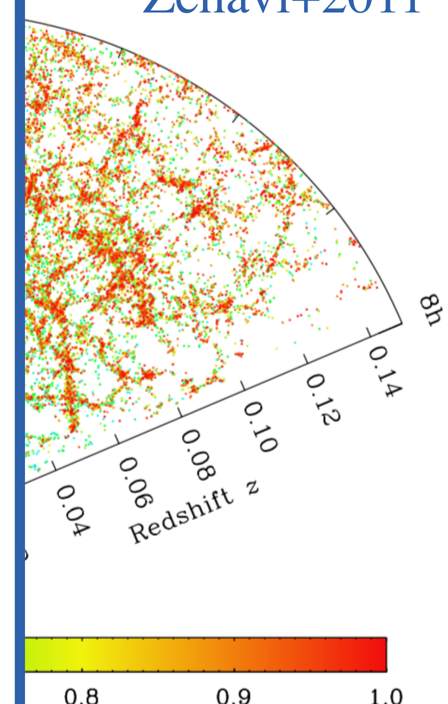


The Two-Point Correlation Function

- Combin … vergence
  … (Exercise 3)
  - l=0 c … t
  - So up … k 10)
- We wan …
  - 1st or …
  - 2nd o …
- Define t …
- Then fr …
- And fro …



Martinez & Coles (1994)



Zehavi+2011

The Power Spectrum

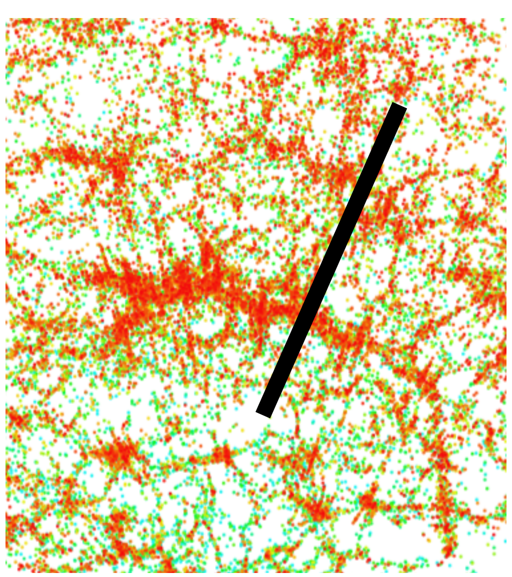
- The Fourier transform of the correlation function, is the Power Spectrum

$$
\begin{aligned}
\langle \tilde{\delta}(\boldsymbol{k})\tilde{\delta}(\boldsymbol{k'})\rangle &= \int d^3x e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \int d^3y e^{-i\boldsymbol{k'}\cdot\boldsymbol{y}} \xi(\boldsymbol{x}-\boldsymbol{y}) \\
\ldots &= (2\pi)^3 \delta^3_{\mathrm{Dirac}}(k-k')P(k) \quad \text{for} \quad \boldsymbol{x}_- = \boldsymbol{x}-\boldsymbol{y} \\
\text{with} & \\
P(|\boldsymbol{k}|) &\equiv \int d^3x_- e^{i\boldsymbol{k}\cdot\boldsymbol{x}_-} \xi(\boldsymbol{x}_-)
\end{aligned}
$$

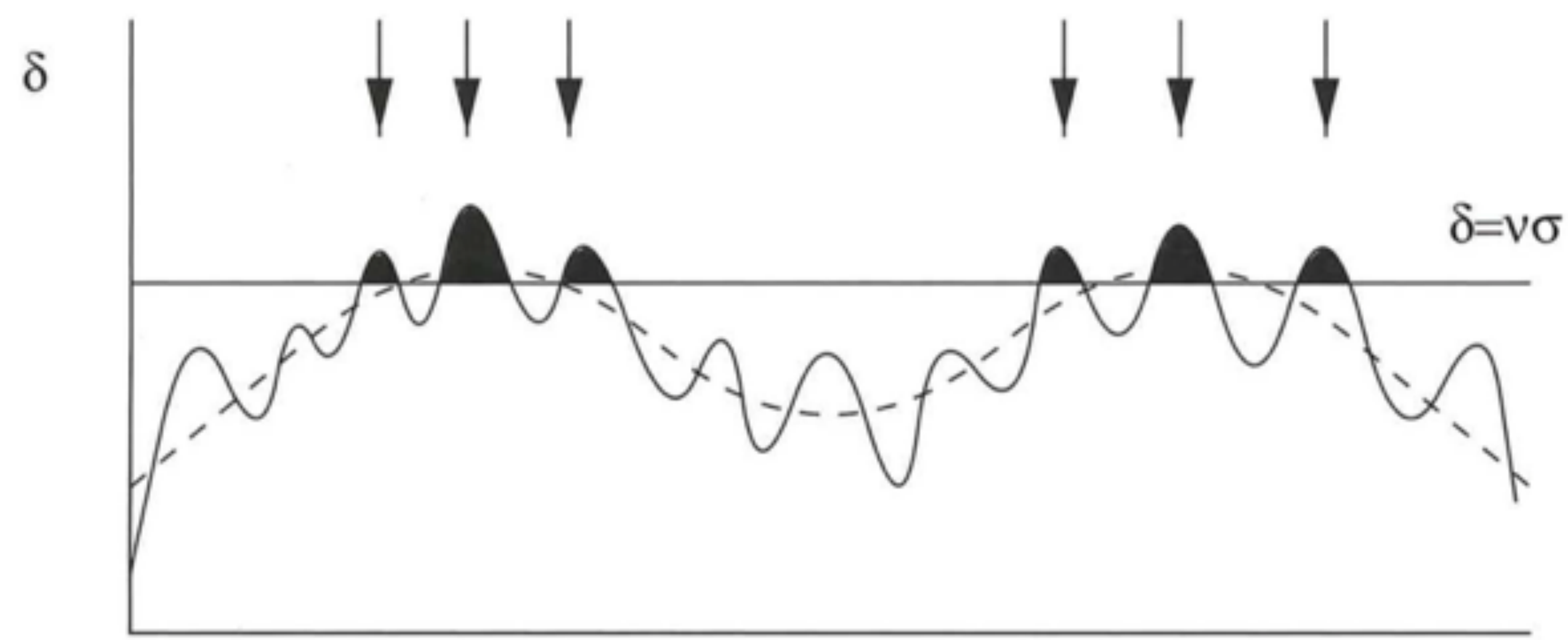
(Exercise 4)

- Describes the scales (1/*k*) at which there is "power" in the density field

2D



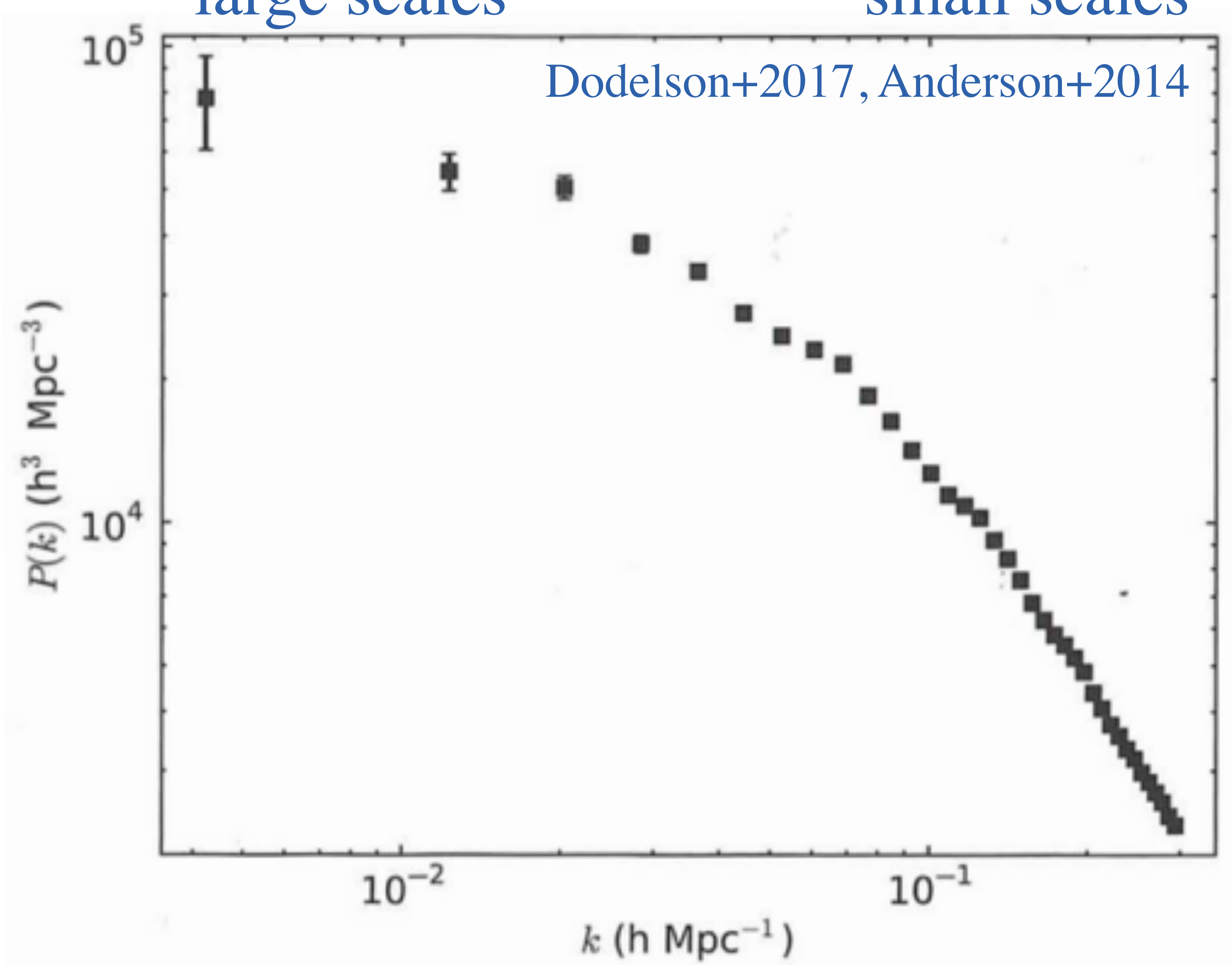
1D

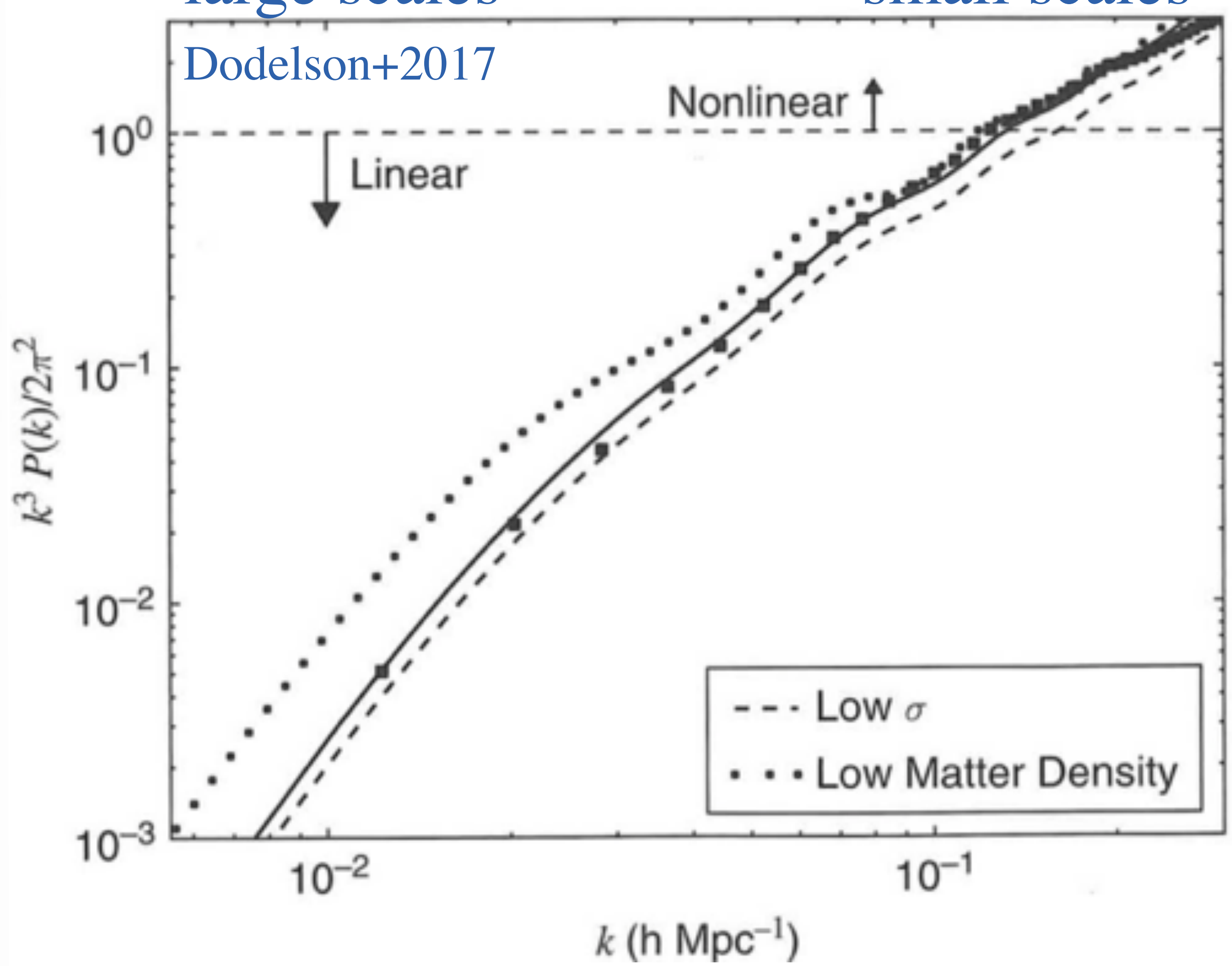


Schneider+2006

The Power Spectrum

- Baryon Oscillation Spectroscopic Survey (BOSS) power spectrum
  - P(k) hides the expected correlation (power) at small scales
- Power spectrum needs to be unit-less to reveal correlation
  - Scaled by dimensionality; in this case k^3





Cosmic Shear Decomposition

- Considering the shear components in Fourier space

$$\tilde{\gamma}_1(\boldsymbol{l}) = \frac{-l_x^2 + l_y^2}{2c^2}\,\tilde{\psi}(\boldsymbol{l}) \qquad \tilde{\gamma}_2(\boldsymbol{l}) = \frac{-l_x l_y}{c^2}\,\tilde{\psi}(\boldsymbol{l})$$

- The defining and angle ϕ that $\boldsymbol{l}$ makes with the (arbitrary) x-axis we have

$$\tilde{\gamma}_1(\boldsymbol{l}) = -\frac{l^2\tilde{\psi}(\boldsymbol{l})}{2c^2}\cos(2\phi) \quad \text{as} \quad \cos(2\phi) = \cos^2(\phi) - \sin^2(\phi)$$

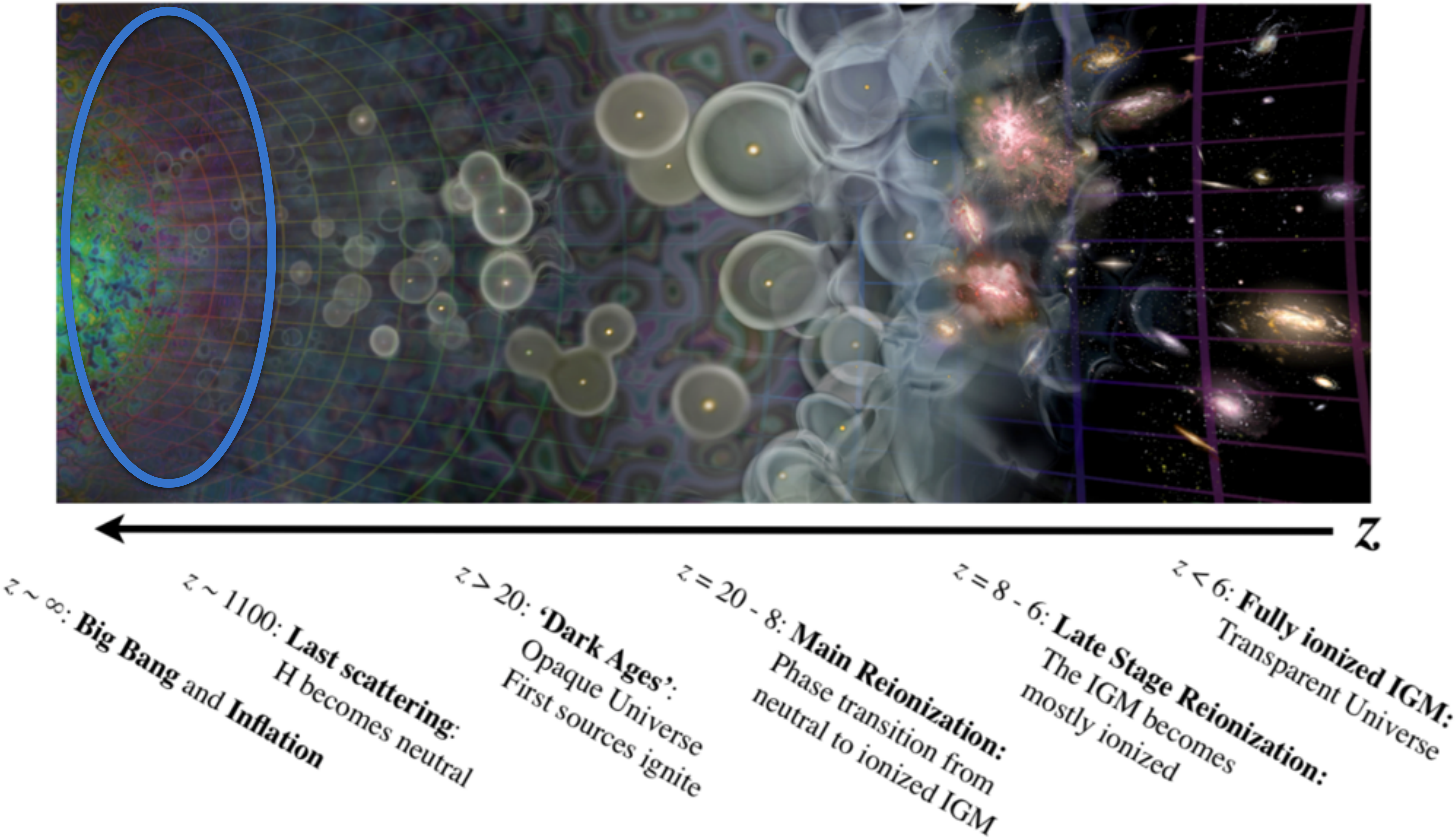
$$\tilde{\gamma}_2(\boldsymbol{l}) = -\frac{l^2\tilde{\psi}(\boldsymbol{l})}{2c^2}\sin(2\phi) \quad \text{as} \quad \sin(2\phi) = 2\cos(\phi) + \sin(\phi)$$

- Considering linear combinations of the shear components we arrive at

$$\begin{aligned} \tilde{E} &\equiv -\tilde{\gamma}_1(\boldsymbol{l})\cos(2\phi) - \tilde{\gamma}_2(\boldsymbol{l})\sin(2\phi) &= -\frac{l^2\tilde{\psi}(\boldsymbol{l})}{2c^2} \\ \tilde{B} &\equiv \tilde{\gamma}_1(\boldsymbol{l})\sin(2\phi) - \tilde{\gamma}_2(\boldsymbol{l})\cos(2\phi) &= 0 \end{aligned}$$

- So any survey has to prove:
  - *B*-mode consistent with 0
  - *E*-mode > 0

The Cosmic Microwave Background



Schmidt 2016, Loeb 2006

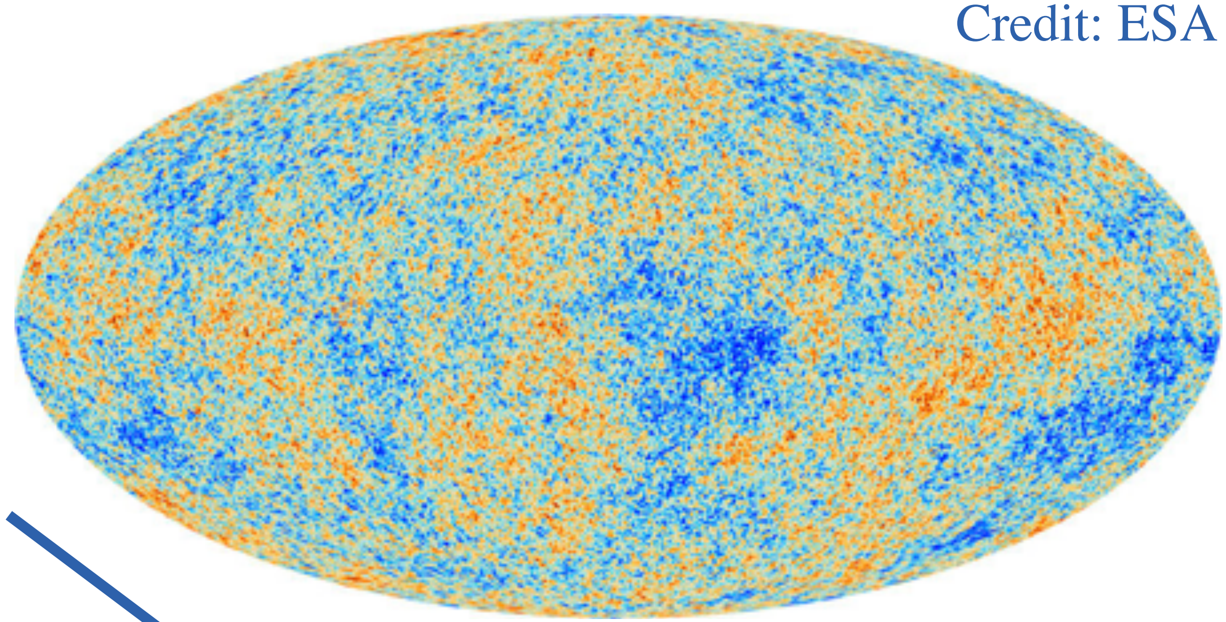
The Cosmic Microwave Background

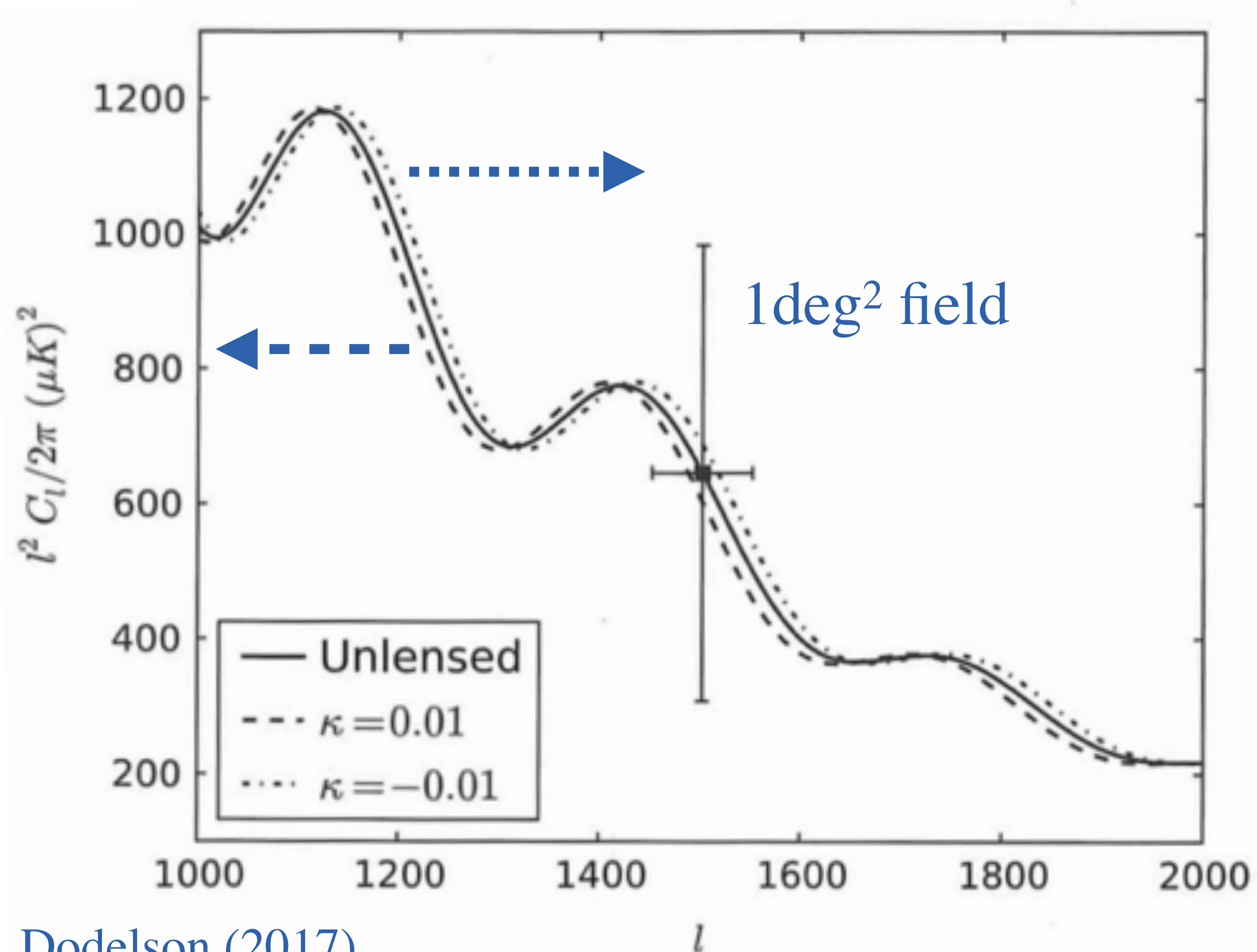
- Black body with T = 2.725K
- Peak (maximum power) at 160Ghz in the microwave today
  - But has been redshifted from universe expansion
- At recombination T~3000K ($z \sim 1100$)
  - CMB is snapshot of Universe when photons started traveling freely
- Surface of last scattering - the edge of the observable Universe
- Temperature contrast, i.e., the CMB temperature fluctuations are $\delta_T \sim 10^{-4}$

The Cosmic Microwave Background

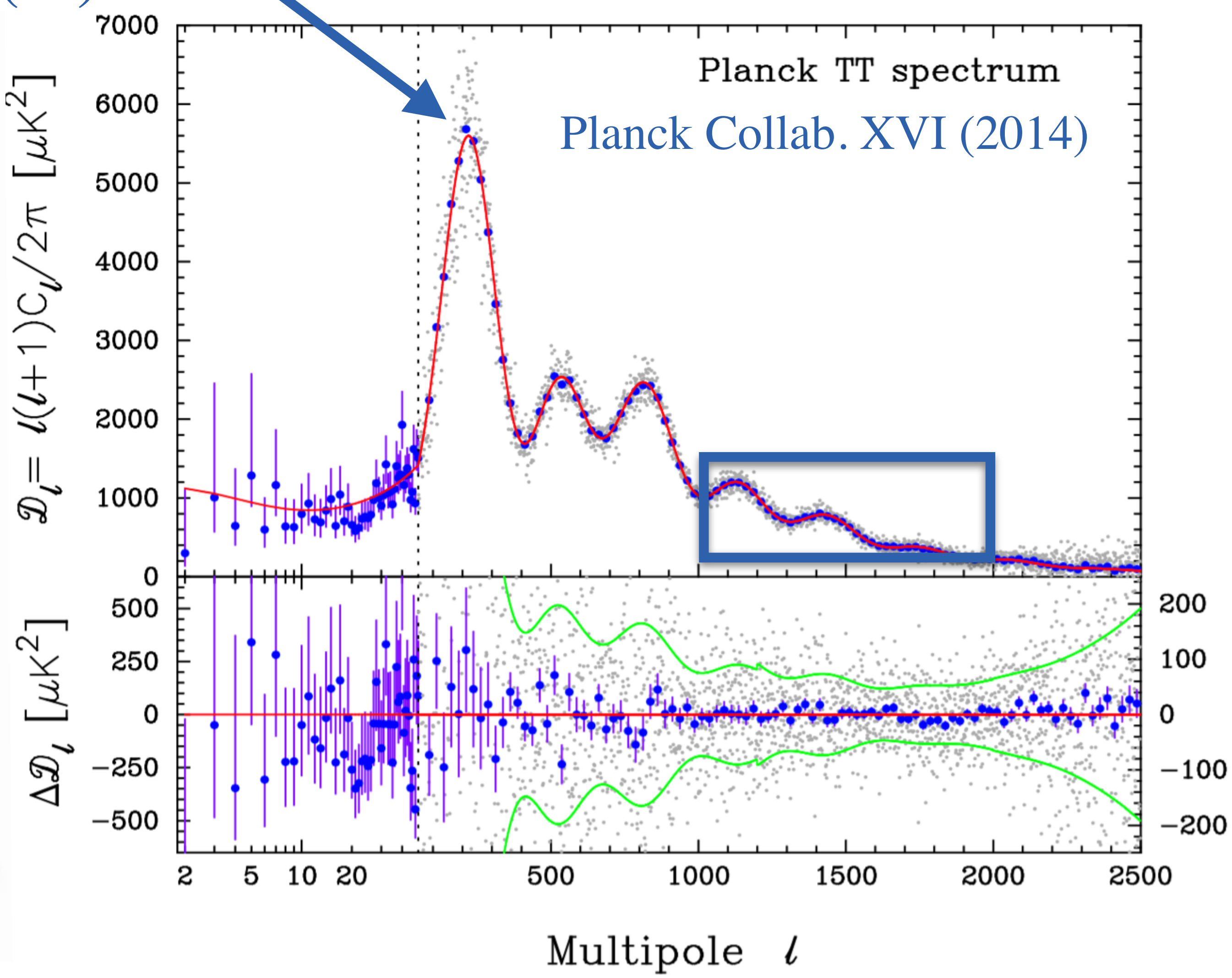
- Map shows temperature fluctuations
- Power spectrum describes their oscillations around average density
- Hot and cold spots are of the order 1deg
- Shape sensitive to lensing potential (κ)

Credit: ESA



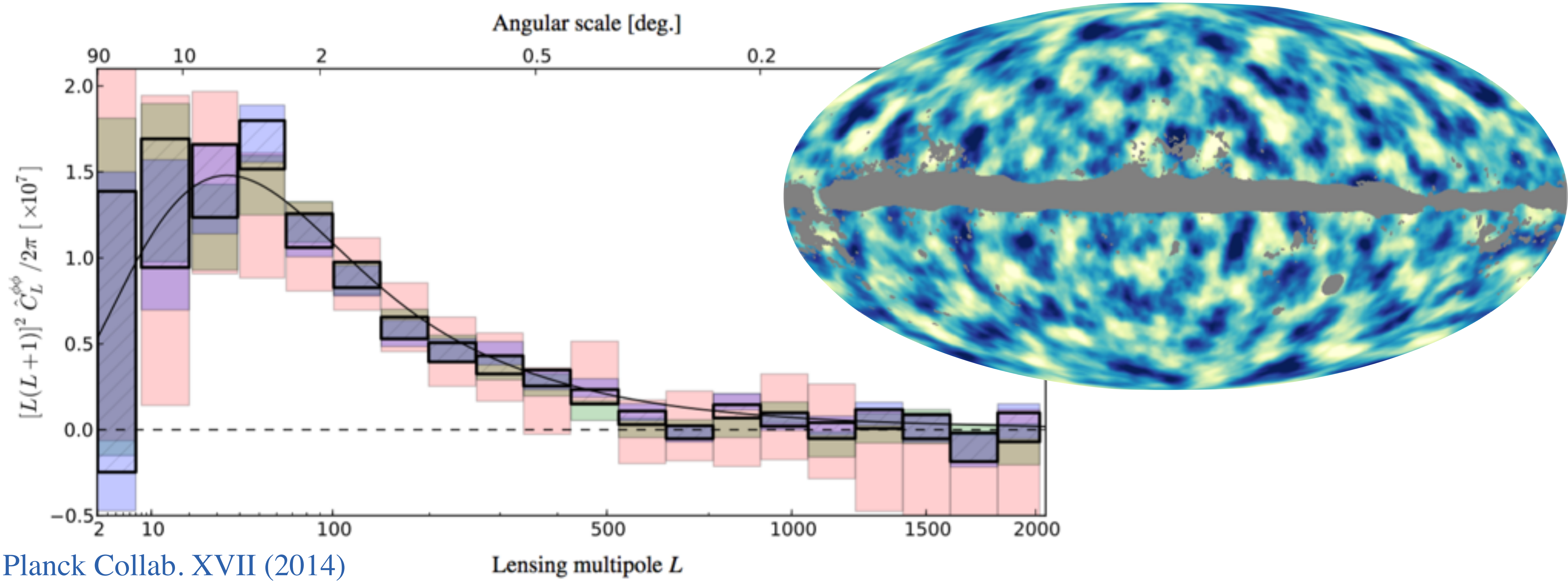


Dodelson (2017)



Lensing Potential from CMB

- Relate T-T CMB map to the gravitational potential, Φ
- As we have seen Φ is related to δ, κ, γ_1, and γ_2
  - Hence a lensing power spectrum can be estimated
- Measured lensing deflection angles: Due to density fluctuations of ~1'
- Structures that perform these deflections are of the order degrees on sky



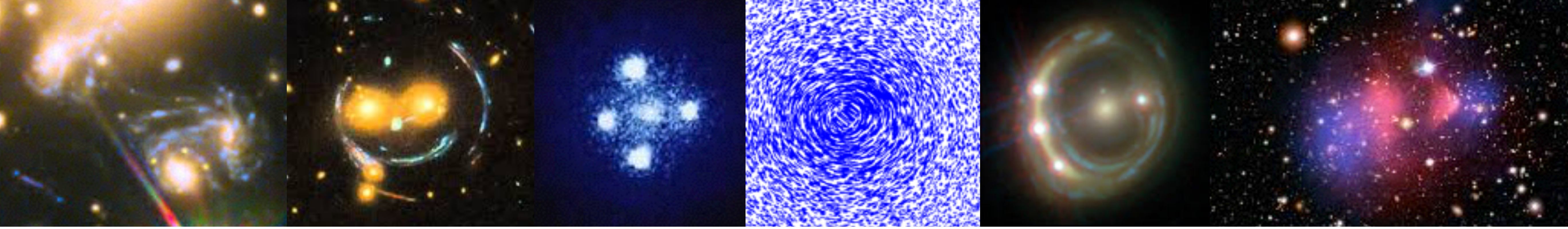
Planck Collab. XVII (2014)

So in summary…

- The cosmic energy density maps are lensed by matter along line of sight
  - Lensing of diffuse source by diffuse lens
- The density contrast, δ is related to κ and γ via the gravitational potential

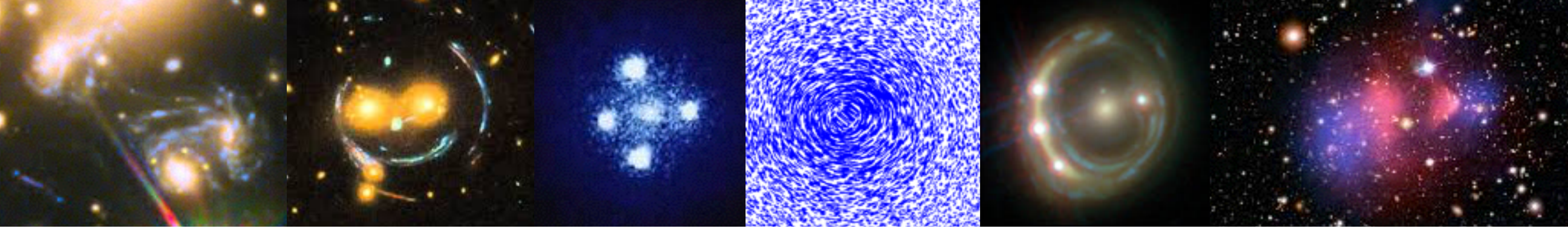
$$\boldsymbol{\delta}(\boldsymbol{x},t) \quad \longleftrightarrow \quad \Phi(\boldsymbol{x},t) \quad \longleftrightarrow \quad \psi(\boldsymbol{\theta}) \quad \longleftrightarrow \quad \kappa \quad \gamma_1 \quad \gamma_2$$

- The variance (2-point correlation function) of δ provides statistic on pattern
- In Fourier space, this results in the Power Spectrum
  - Describing (size) scales containing most power, i.e., correlation
- E and B mode decomposition of γ are useful sanity checks of results
- Cosmic Microwave Background T maps provide information about
  - Cosmology via TT Power Spectrum
  - Lensing potential (matter distribution) via matter Power Spectrum

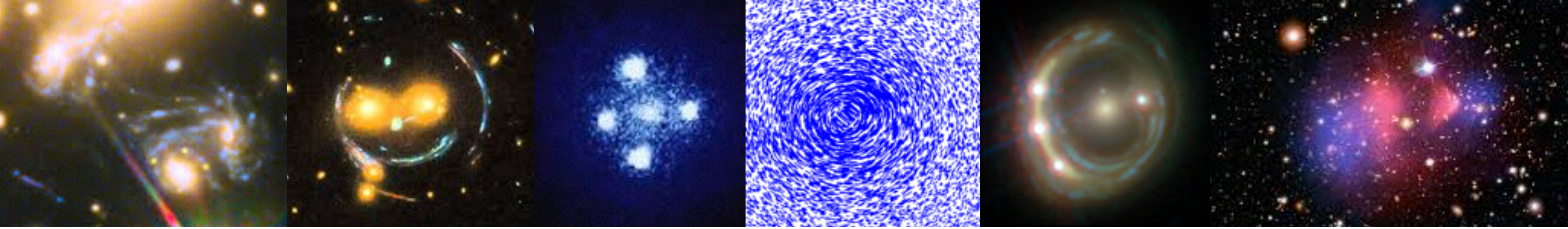


PHY-765 SS18 Gravitational Lensing Week 13

Questions?



Last Week’s Worksheet



This Week’s Worksheet