

PHY-765 SS18 Gravitational Lensing Week 12

Weak Gravitational Lensing

Kasper B. Schmidt

Leibniz-Institut für Astrophysik Potsdam (AIP)

Last week

- Lens Models:
 - Goal of a model is to minimize disagreement with observations

Parametric

$$\chi_{\text{img}}^2 = \sum_i \left(\frac{\theta_i(\beta) - \theta_i}{\sigma_i} \right)^2$$

Non-Parametric

$$\chi^2 = \frac{|\mathbf{s}_I - P(\text{PSF, lens model}) \mathbf{s}_{\text{source plane}}|^2}{\sigma^2}$$

- The Mass Sheet Degeneracy:
 - ‘classes of models’ provide equally good fit to data
- The Mass Sheet Degeneracy can be broken with:
 - multiple lensed systems at different redshifts behind the lens
 - kinematic information (velocity dispersion measurements) of lens
- Recent studies have assembled multiple teams and compared cluster modeling approaches to both data (SN Refsdal) and model clusters

The aim of today

- Describe weak gravitational lensing
- The ideal spherical case of shearing images
- Relation between ellipticity and the gravitational potential (κ & γ)
- A few examples of effects from imaging to account for
- The Bullet Cluster - a spectacular DM proof from strong and weak lensing

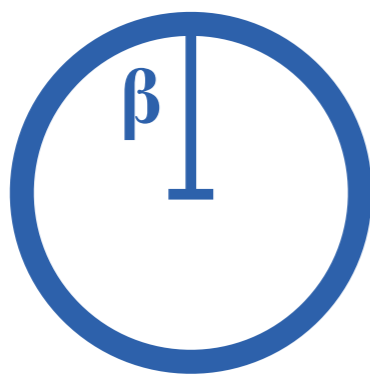
Image Distortions - κ and γ

- Image distortions can be described by the Jacobian matrix (week 6)

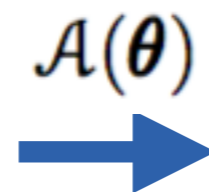
$$\mathcal{A}(\boldsymbol{\theta}) = \frac{\partial \boldsymbol{\beta}}{\partial \boldsymbol{\theta}} = \begin{pmatrix} \frac{\partial \beta_i}{\partial \theta_i} & \frac{\partial \beta_i}{\partial \theta_j} \\ \frac{\partial \beta_j}{\partial \theta_i} & \frac{\partial \beta_j}{\partial \theta_j} \end{pmatrix} = \begin{pmatrix} 1 - \frac{\partial \alpha_i}{\partial \theta_i} & -\frac{\partial \alpha_i}{\partial \theta_j} \\ -\frac{\partial \alpha_j}{\partial \theta_i} & 1 - \frac{\partial \alpha_j}{\partial \theta_j} \end{pmatrix} = (\delta_{ij} - \Psi_{ij})$$

$$\Psi_{ij} \equiv \begin{pmatrix} \kappa + \gamma_1 & \gamma_2 \\ \gamma_2 & \kappa - \gamma_1 \end{pmatrix} \quad \mu = \frac{1}{(1 - \kappa)^2 - \gamma^2} \quad \gamma^2 \equiv \gamma_1^2 + \gamma_2^2$$

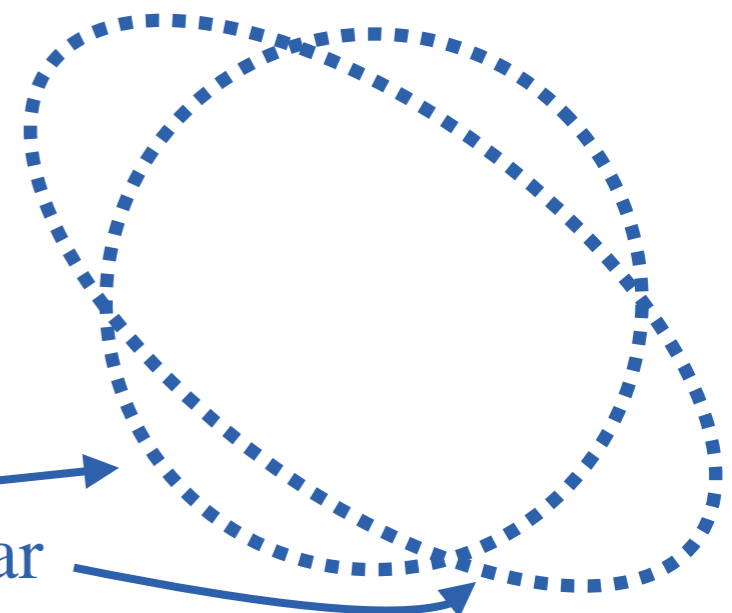
Source Plane



Lens (Image) Plane



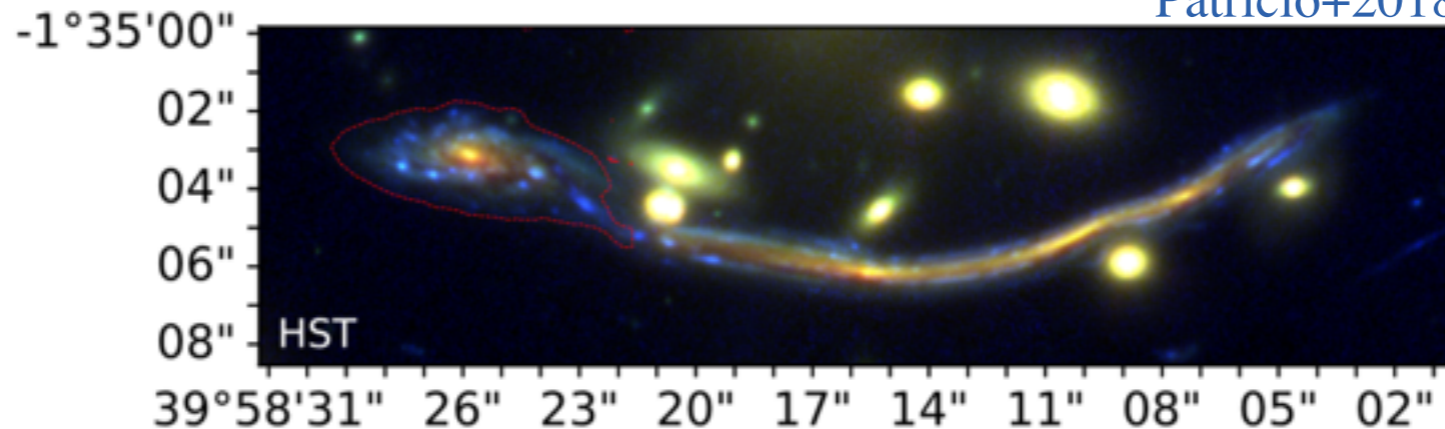
Convergence only
Convergence + Shear



Weak Lensing

- Already in the early days of lensing, arcs were observed, e.g. in A370

Patricio+2018



- Clear that this effect, i.e. shearing of images must happen on all scales
- Weak lensing is the regime where the distortions become small

$$\mathcal{A}(\boldsymbol{\theta}) = (\delta_{ij} - \Psi_{ij}) \quad \mathcal{A}(\boldsymbol{\theta}) \rightarrow \delta_{ij}$$

- To measure the effect, statistical approaches need to be applied
 - Collect as many galaxies as possible → go faint (dig into the noise)
 - Digging into the noise requires good handle of telescope and obs. effects
- Photographic plates not linear, so not possible before the CCD (early 90s)

Idealized Weak Lensing

- Consider an un-lensed spherical object with radius β_0
- If $\kappa = 0$, $\gamma_2 = 0$ and $\gamma_1 > 0$ (but small) then $\mathcal{A}(\boldsymbol{\theta}) = \begin{pmatrix} 1 - \gamma_1 & 0 \\ 0 & 1 + \gamma_1 \end{pmatrix}$
- So the projection from source (β) to image (θ) plane is

$$\frac{d\beta_1}{d\theta_1} = 1 - \gamma_1 \qquad \frac{d\beta_2}{d\theta_2} = 1 + \gamma_1$$
- Solving and adding in quadrature gives an ellipse on the form $1 = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

$$1 = \frac{(1 - \gamma_1)^2}{\beta_0^2} \theta_1^2 + \frac{(1 + \gamma_1)^2}{\beta_0^2} \theta_2^2$$
- If γ_2 is the only non-zero component the distortion is given by

$$1 = \frac{(\theta_1 - \gamma_2 \theta_2)^2}{\beta_0^2} + \frac{(\theta_2 - \gamma_2 \theta_1)^2}{\beta_0^2} \quad \text{(Exercise 3)}$$

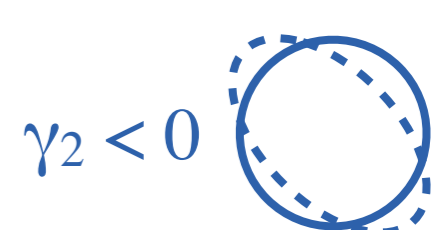
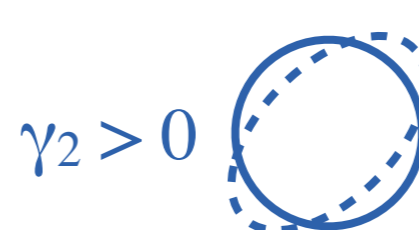
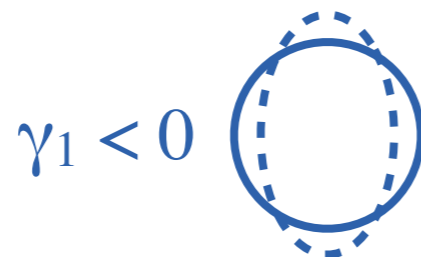
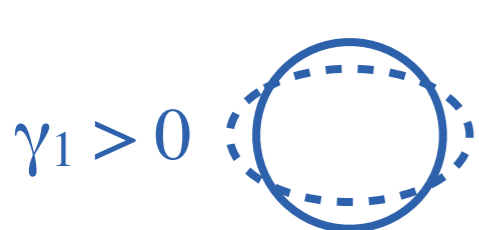


Image Moments and Ellipticity

- A convenient method to obtain shape from images is via “moments”
 - standard image processing tool to recognize/describe shapes in images
- The moment of an image is the (intensity) weighted sum of image pixels

$$M_{pq} = \sum_i \sum_j i^p j^q I(i, j)$$

- The 0th order moment is:

$$M_{00} = \sum_i \sum_j i^0 j^0 = N_{\text{pixels}}$$

- The first order moments are:

$$M_{10} = \sum_i \sum_j i^1 j^0 \Rightarrow \bar{x} = M_{10}/M_{00}$$

$$M_{01} = \sum_i \sum_j i^0 j^1 \Rightarrow \bar{y} = M_{01}/M_{00}$$

- 2nd order moments M_{20} , M_{02} and M_{11} provides covariance matrix
 - Shape information (read: ellipticity)

Image Moments and Ellipticity

- This image moment corresponds to integrating over the surface brightness

$$q_{ij} \equiv \int d^2\theta \mathcal{S}^{\text{obs}}(\boldsymbol{\theta}) \theta_i \theta_j$$

- Then ellipticity can be defined in terms of surface brightness moments as

$$\epsilon_1 \equiv \frac{q_{11} - q_{22}}{q_{11} + q_{22}} \quad \epsilon_2 \equiv \frac{2q_{12}}{q_{11} + q_{22}}$$

- For example, elongation along the x-axis: $q_{11} > q_{22}$ so $\epsilon_1 > 0$
 - We are assuming that the objects are spherical: $\mathcal{S}(\boldsymbol{\beta}) = \mathcal{S}(|\boldsymbol{\beta}|)$
 - Preferable to integrate in the source plane (no distortions)
 - Transforming coordinates is done with the Jacobian elements $\mathcal{A}_{ij} = \frac{\partial \beta_i}{\partial \theta_j}$
- $$d^2\theta \rightarrow \left| \frac{\partial \theta_j}{\partial \beta_i} \right| d^2\beta \quad \beta_i = \mathcal{A}_{ij} \theta_j$$
- have used that the Jacobian is roughly constant over the galaxy size ($\sim \text{arcsec}$)

Image Moments and Ellipticity

- Hence, the moments can be calculated in the source plane with

$$q_{ij} = \mathcal{N}(\mathcal{A}^{-1})_{ii'}(\mathcal{A}^{-1})_{jj'} \int d^2\beta \mathcal{S}(\beta) \beta_{i'} \beta_{j'}$$

- This is independent of direction for spherically symmetric sources
- Inserting into the definition for the ellipticities gives (integral drops out)

$$\epsilon_1 \equiv \frac{(\mathcal{A}^{-1})_{1i}(\mathcal{A}^{-1})_{1i} - (\mathcal{A}^{-1})_{2i}(\mathcal{A}^{-1})_{2i}}{(\mathcal{A}^{-1})_{1i}(\mathcal{A}^{-1})_{1i} + (\mathcal{A}^{-1})_{2i}(\mathcal{A}^{-1})_{2i}}$$

$$\epsilon_2 \equiv \frac{2(\mathcal{A}^{-1})_{1i}(\mathcal{A}^{-1})_{2i}}{(\mathcal{A}^{-1})_{1i}(\mathcal{A}^{-1})_{1i} + (\mathcal{A}^{-1})_{2i}(\mathcal{A}^{-1})_{2i}}$$

- Using the inverse of the Jacobian matrix we get (Exercise 4)

$$\epsilon_1 \equiv 2\gamma_1 \frac{1 - \kappa}{(1 - \kappa)^2 + \gamma_1^2 + \gamma_2^2}$$

$$\epsilon_2 \equiv 2\gamma_2 \frac{1 - \kappa}{(1 - \kappa)^2 + \gamma_1^2 + \gamma_2^2}$$

$$\epsilon_i = \frac{2\gamma_i}{1 - \kappa} \left[1 - \frac{\gamma^2}{(1 - \kappa)^2} \right]^{-1}$$

The reduced shear
(1- κ) independent of shape

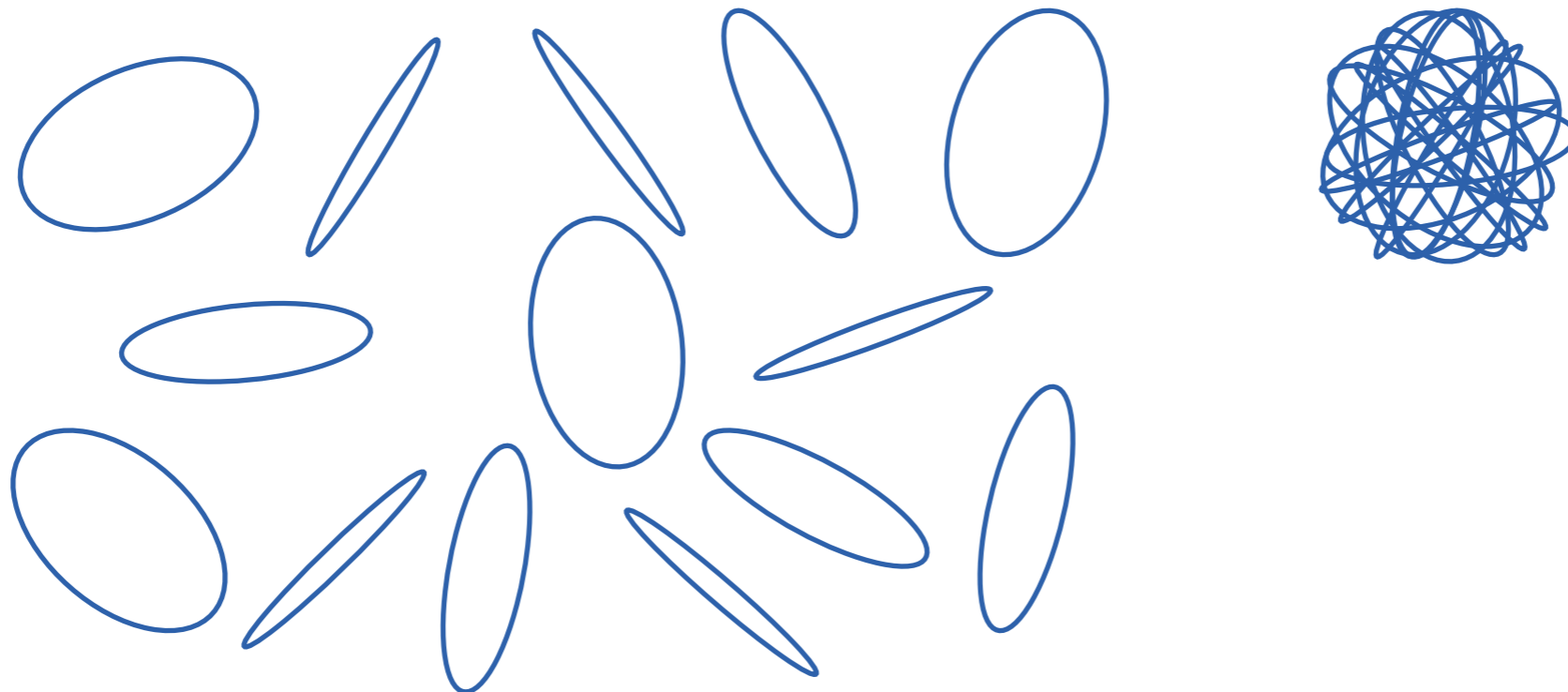
- So the ellipticity relates to γ and κ , and hence the gravitational potential

Accounting for Intrinsic Ellipticity

- Determining the ellipticity is straight forward in the ideal case
- Which of these is an elliptical and which one is spherical but sheared?



- Intrinsic ε averages to 0 as they are random

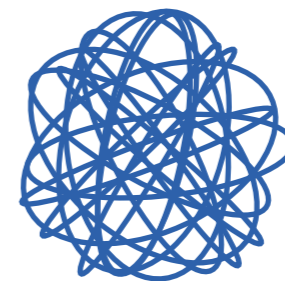
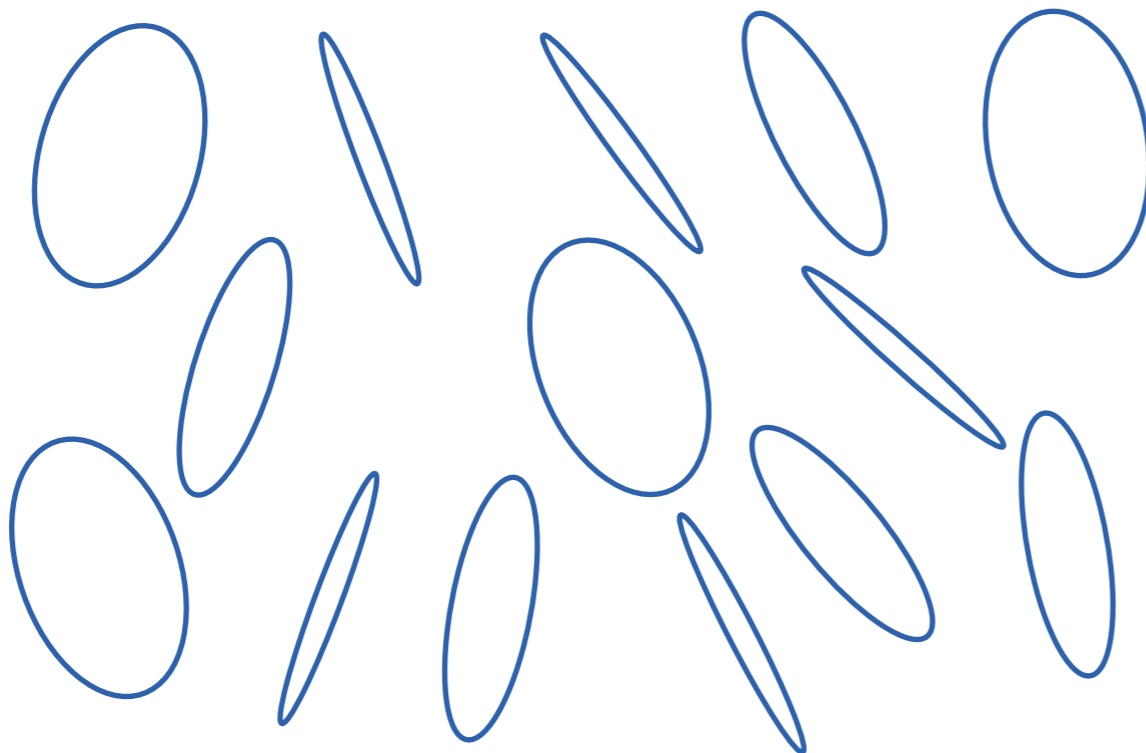


Accounting for Intrinsic Ellipticity

- Determining the ellipticity is straight forward in the ideal case
- Which of these is an elliptical and which one is spherical but sheared?

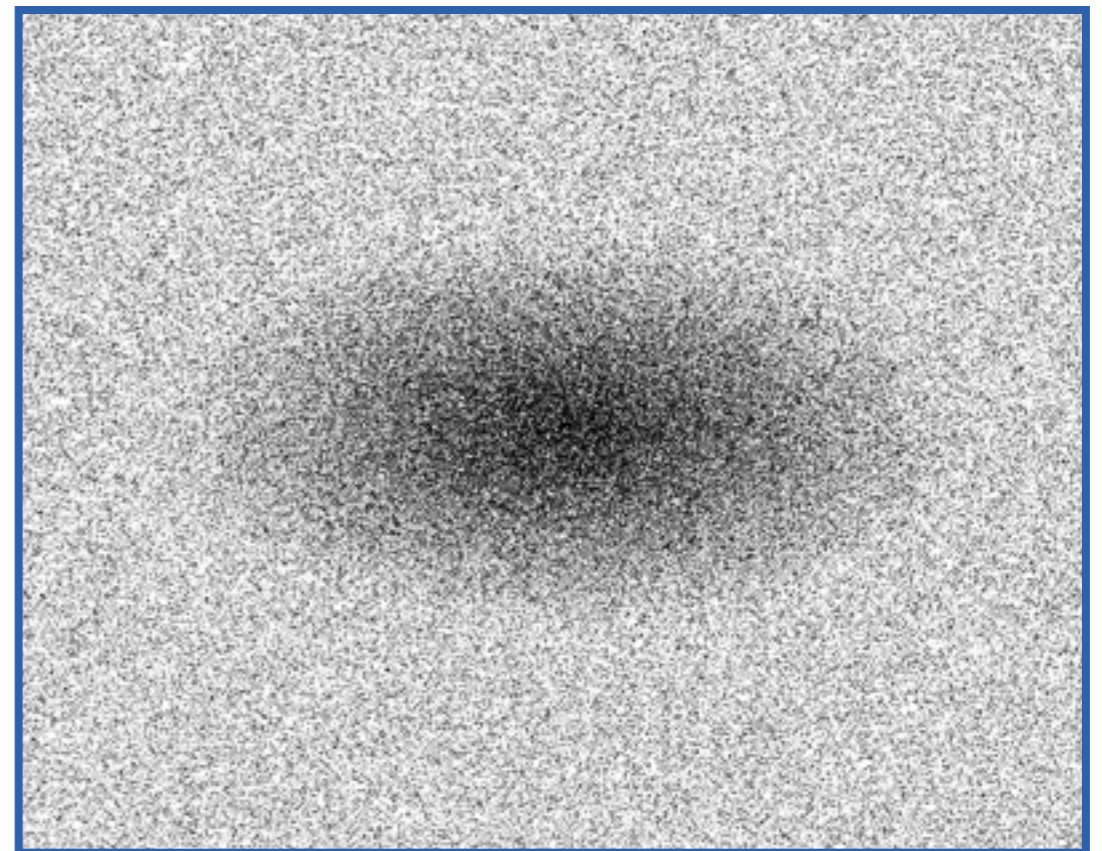
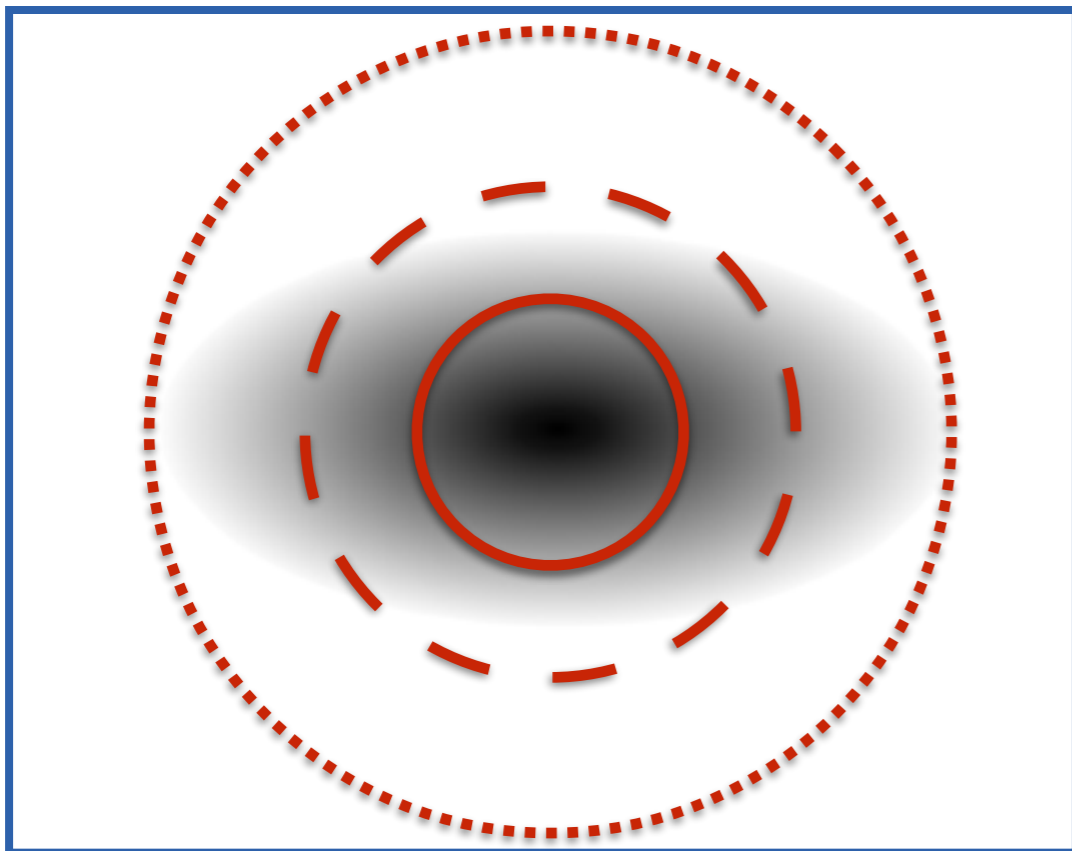


- Intrinsic ϵ averages to 0 as they are random
- Shear is correlated with the mass, i.e., not random.
 - At a given position the same for all galaxies



Weighting of pixels

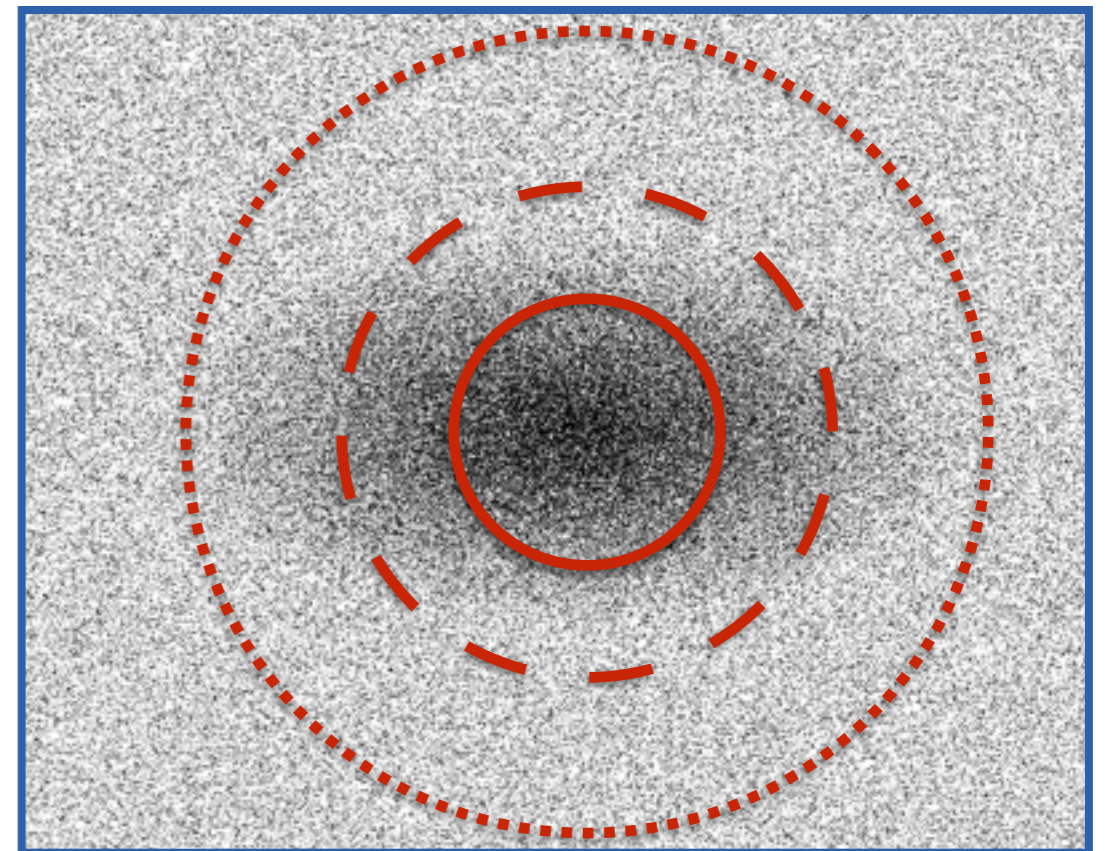
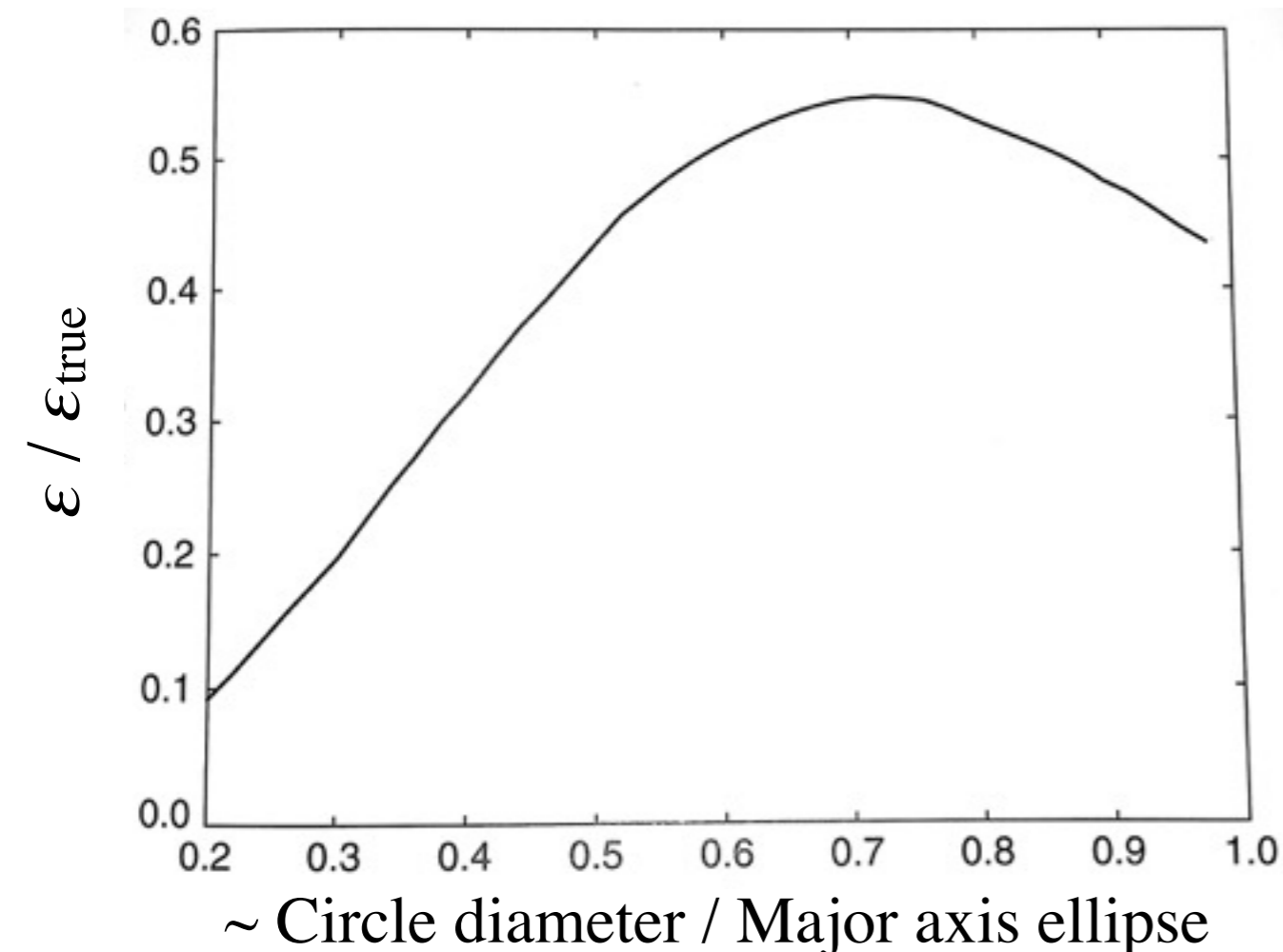
- Noise on observations also affects our ability to determine ellipticity
- Which pixels should be used for calculating moments?
 - Want to get as reliable an estimate of ϵ as possible
 - Want to optimize the amount of signal compared to noise, i.e., S/N



Adapted from Dodelson (2017)

Weighting of pixels

- Noise on observations also affects our ability to determine ellipticity
- Which pixels should be used for calculating moments?
 - Want to get as reliable an estimate of ε as possible
 - Want to optimize the amount of signal compared to noise, i.e., S/N



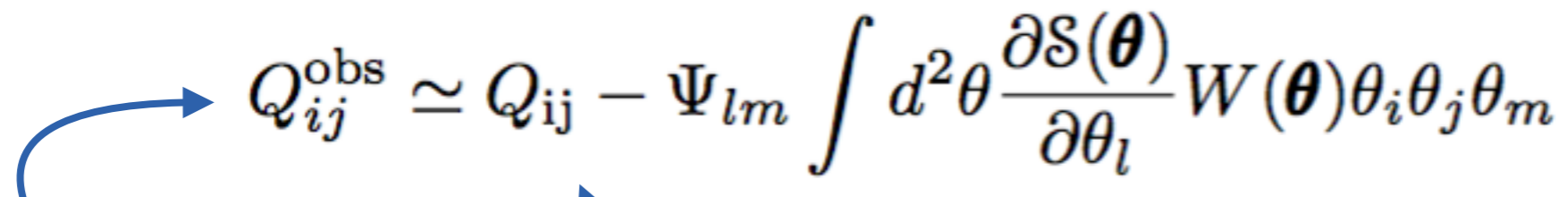
Adapted from Dodelson (2017)

Ellipticities accounting for W and $\epsilon_{\text{intrinsic}}$

- Introducing a weighting of the pixels (not just by surface brightness)

$$Q_{ij} \equiv \int d^2\theta \mathcal{S}^{\text{obs}}(\boldsymbol{\theta}) W(\boldsymbol{\theta}) \theta_i \theta_j$$

- So for the circular binary source, $W(\boldsymbol{\theta})$ is a circular step function
- Defining ϵ_1 and ϵ_2 in terms of Q_{11} and Q_{22} one can express these including the weight function in a way similar to the simplified spherical case
- The observed moments can be expressed as


$$Q_{ij}^{\text{obs}} \simeq Q_{ij} - \Psi_{lm} \int d^2\theta \frac{\partial \mathcal{S}(\boldsymbol{\theta})}{\partial \theta_l} W(\boldsymbol{\theta}) \theta_i \theta_j \theta_m$$

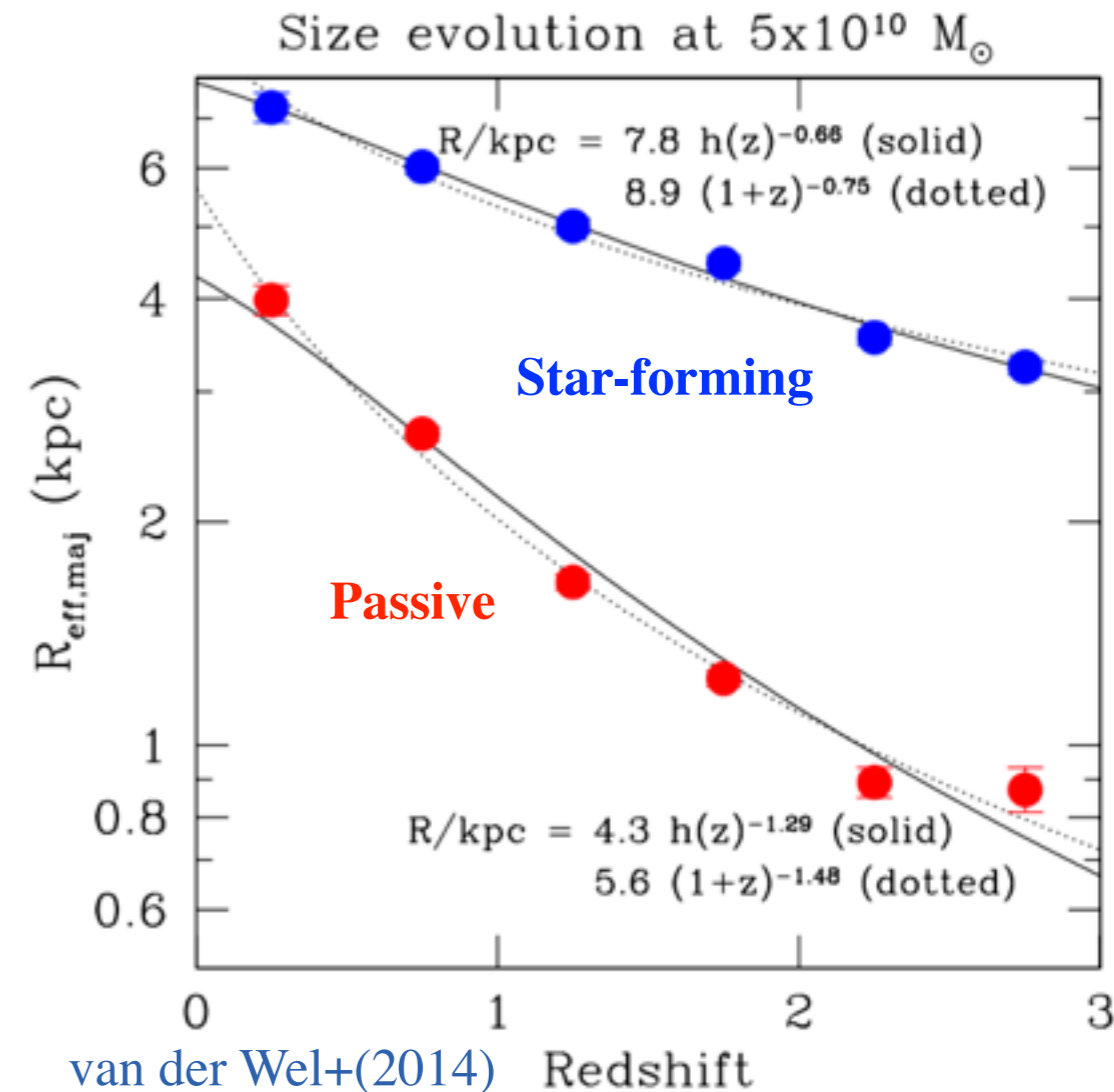
Gives the
observed ellipticity

expresses the
intrinsic ellipticity

Accounts for the
distortion from shearing

Assembling large samples of galaxies

- Decrease the noise of the weak lensing signal by large galaxy samples
 - Wide-field and deep observations
- Distant faint galaxies are small (Exercise 5)
 - ~a few arcsec comparable to seeing/PSF



Assembling large samples of galaxies

- Decrease the noise of the weak lensing signal by large galaxy samples
 - Wide-field and deep observations
- Distant faint galaxies are small (Exercise 5)
 - ~a few arcsec comparable to seeing/PSF

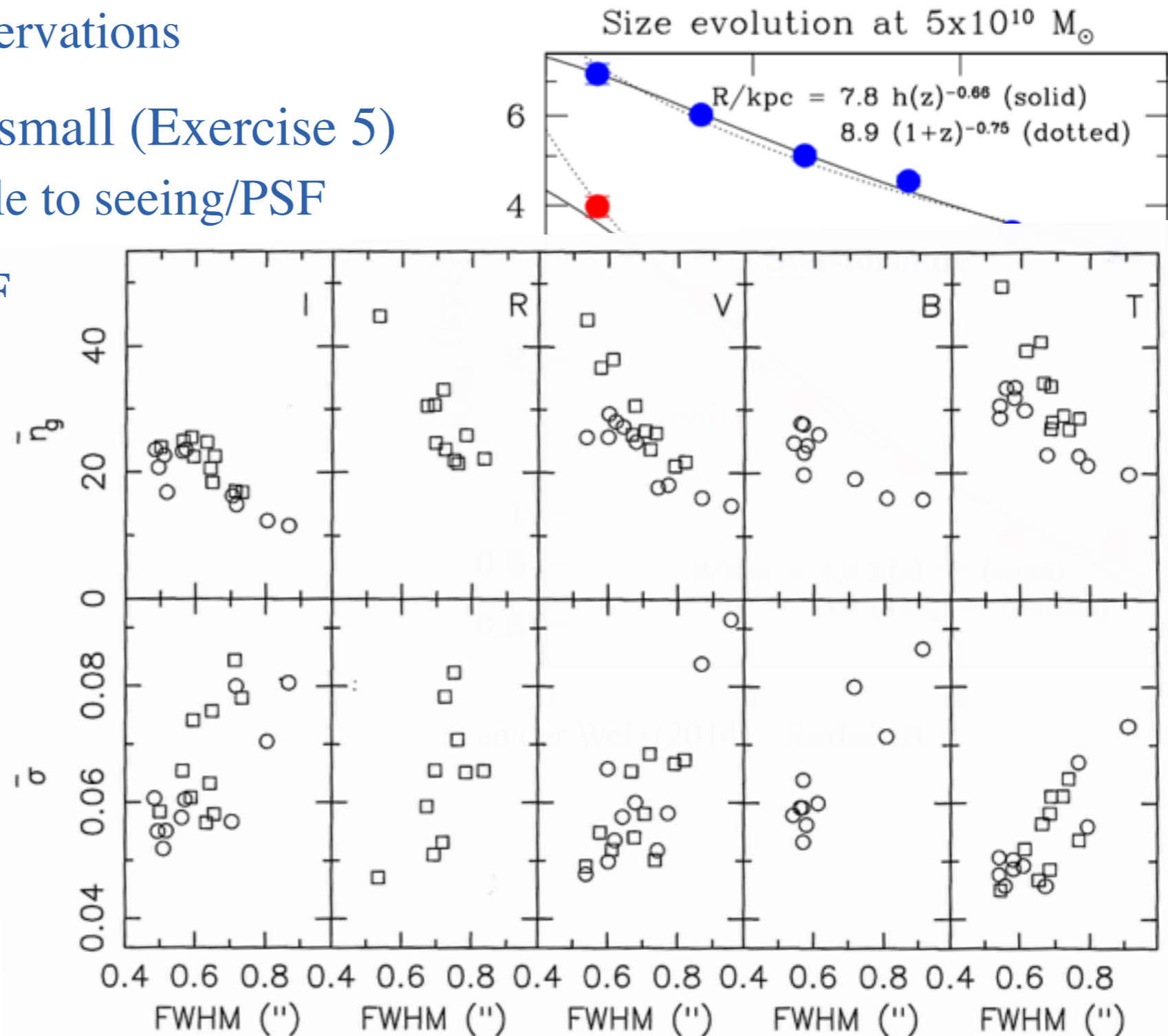
- Results dependent on PSF

Number of galaxies with reliable shear measurement

The noise in signal from intrinsic ellipticity

□ $t_{\text{exp}} \sim 45 \text{ min}$

○ $t_{\text{exp}} \sim 2 \text{ hours}$



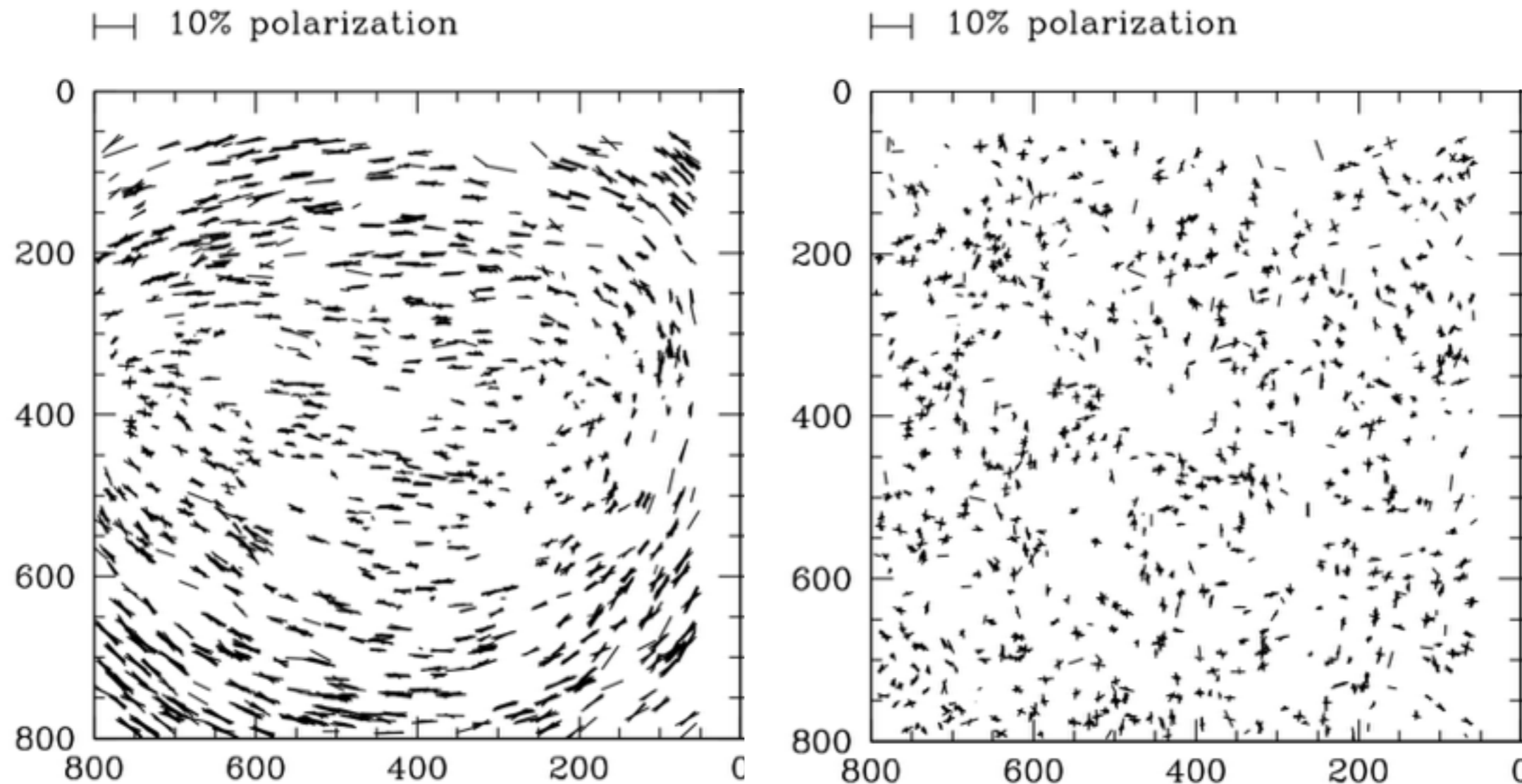
Schneider+(2006)

Correcting for the PSF

- PSF can be treated as another weight functions varying over the FoV

$$Q_{ij} \equiv \int d^2\theta \mathcal{S}^{\text{obs}}(\boldsymbol{\theta}) P(\boldsymbol{\theta}) \theta_i \theta_j \quad \Rightarrow \quad Q'_{ij} = Q_{ij} + P_{lm} Y_{lmij}$$

- Iterative modeling similar to for strong lensing minimizing χ^2

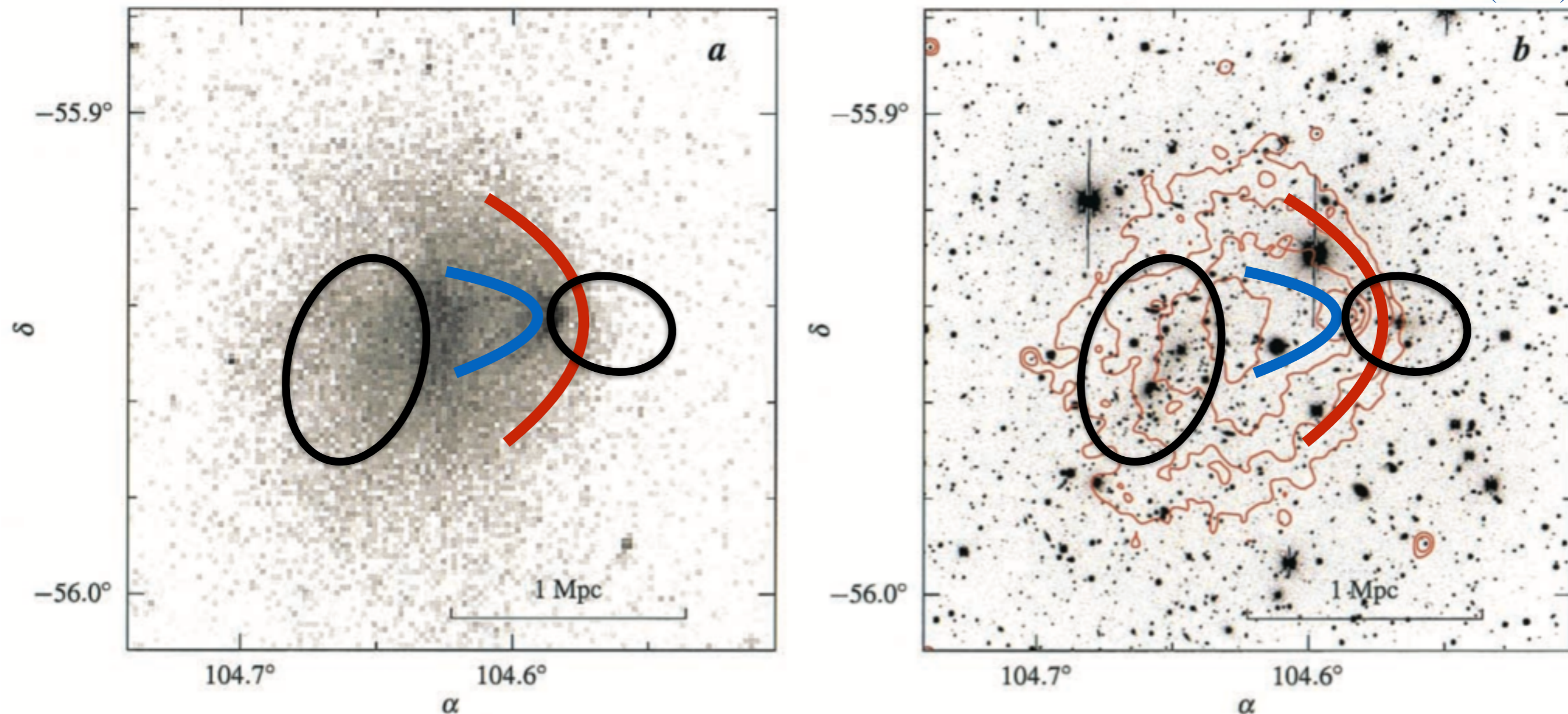


Hoekstra+(1998)

The Bullet Cluster: 1E 0657 558

- X-rays obsessed with *Chandra* probe the hot gas in the cluster
 - See a clear bow shock of hot gas in front of ‘bullet’ from the cluster collision
- Displacement of $\sim 90\%$ of the mass \rightarrow galaxy clumps contain less mass

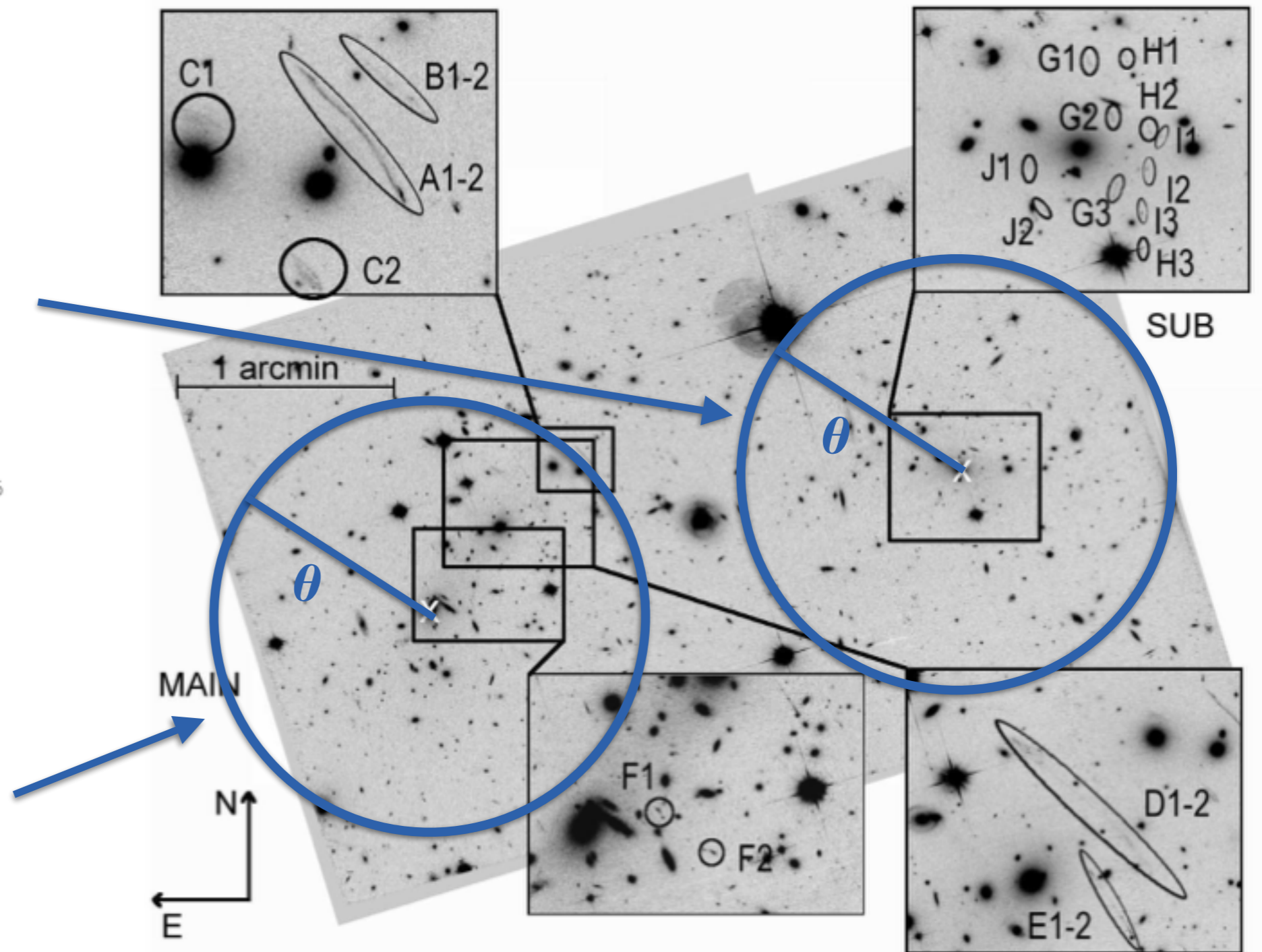
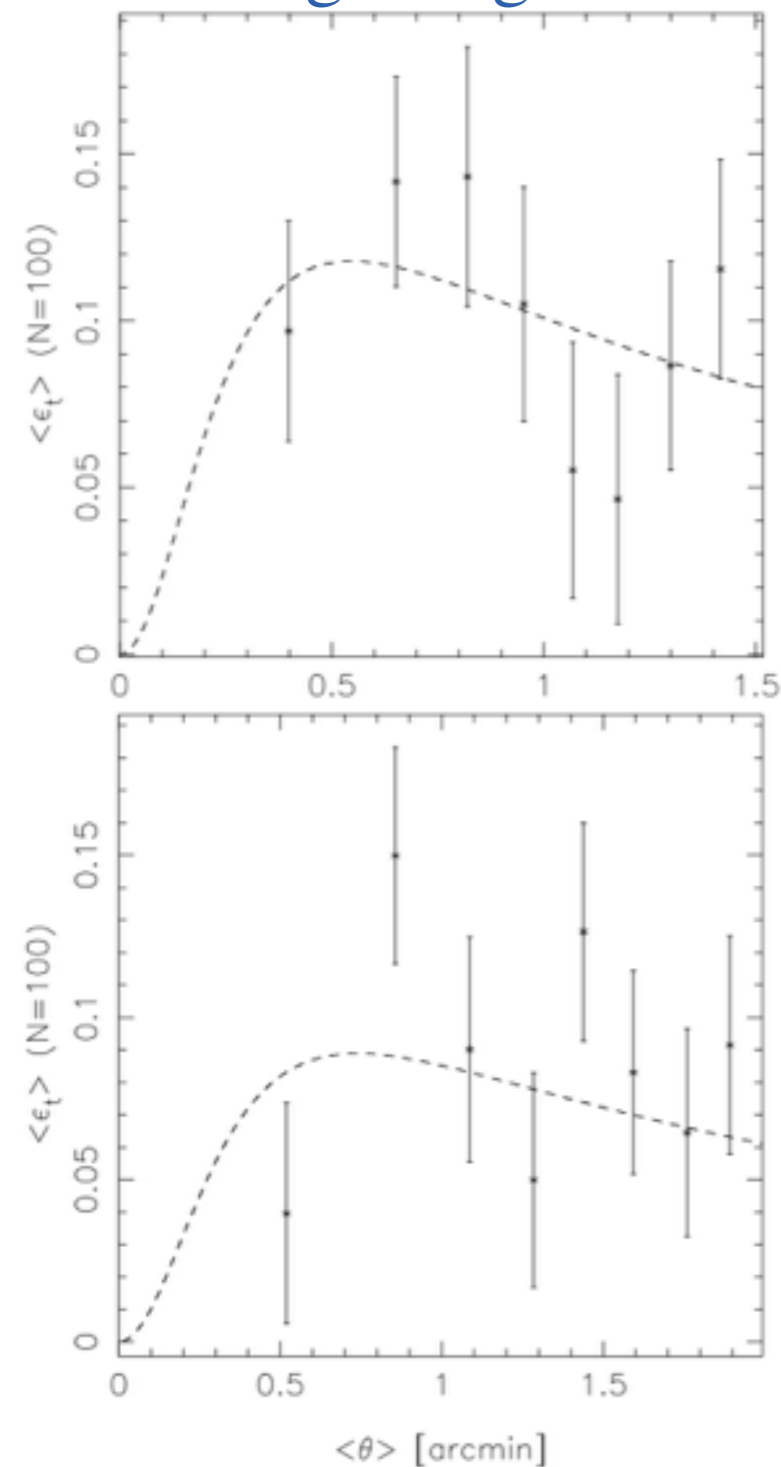
Markevitch+(2002)



The Bullet Cluster: 1E 0657 558

- Bradac+2006 modeled mass distribution combining strong and weak lensing

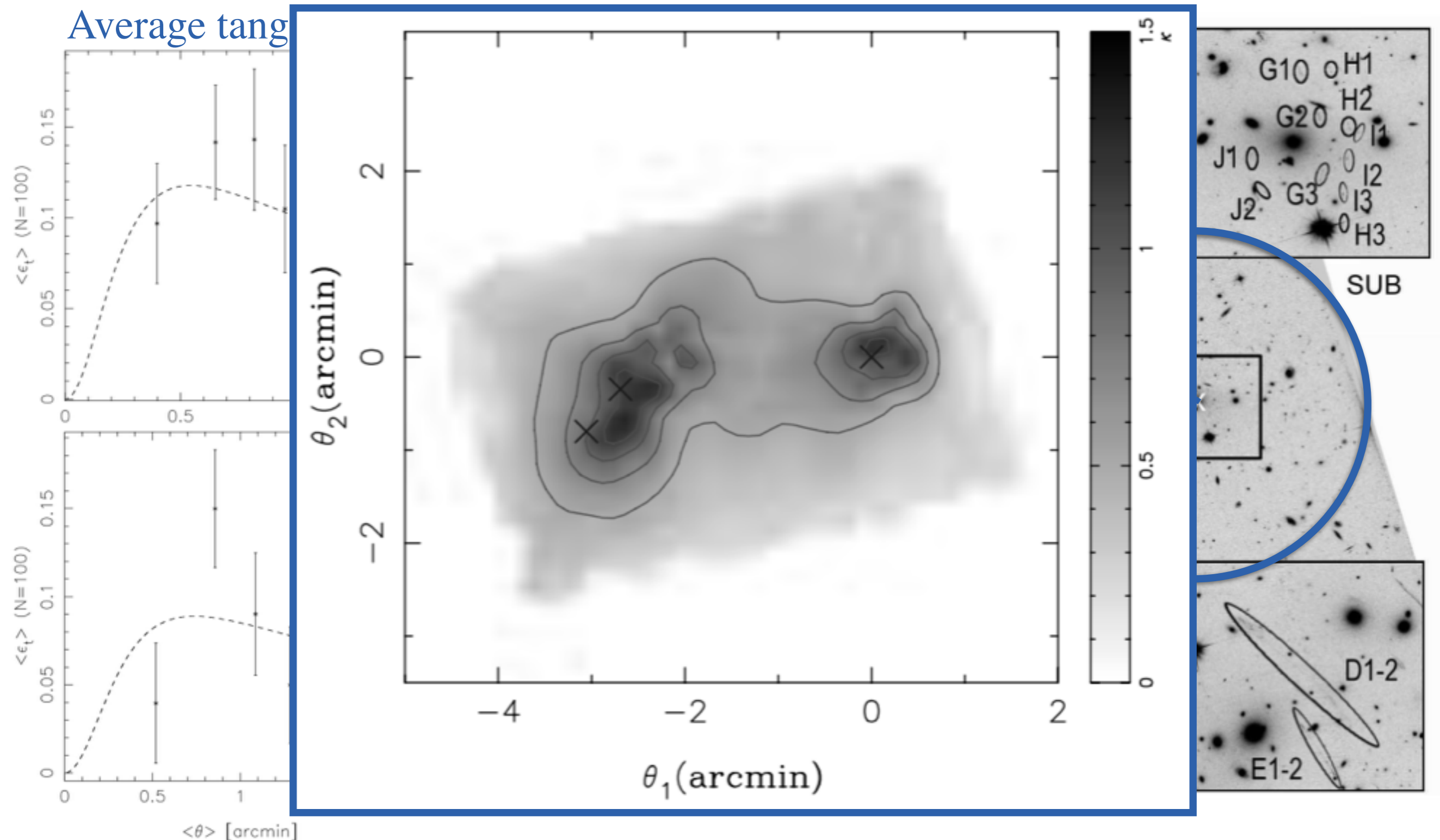
Average tangential ϵ



The Bullet Cluster: 1E 0657 558

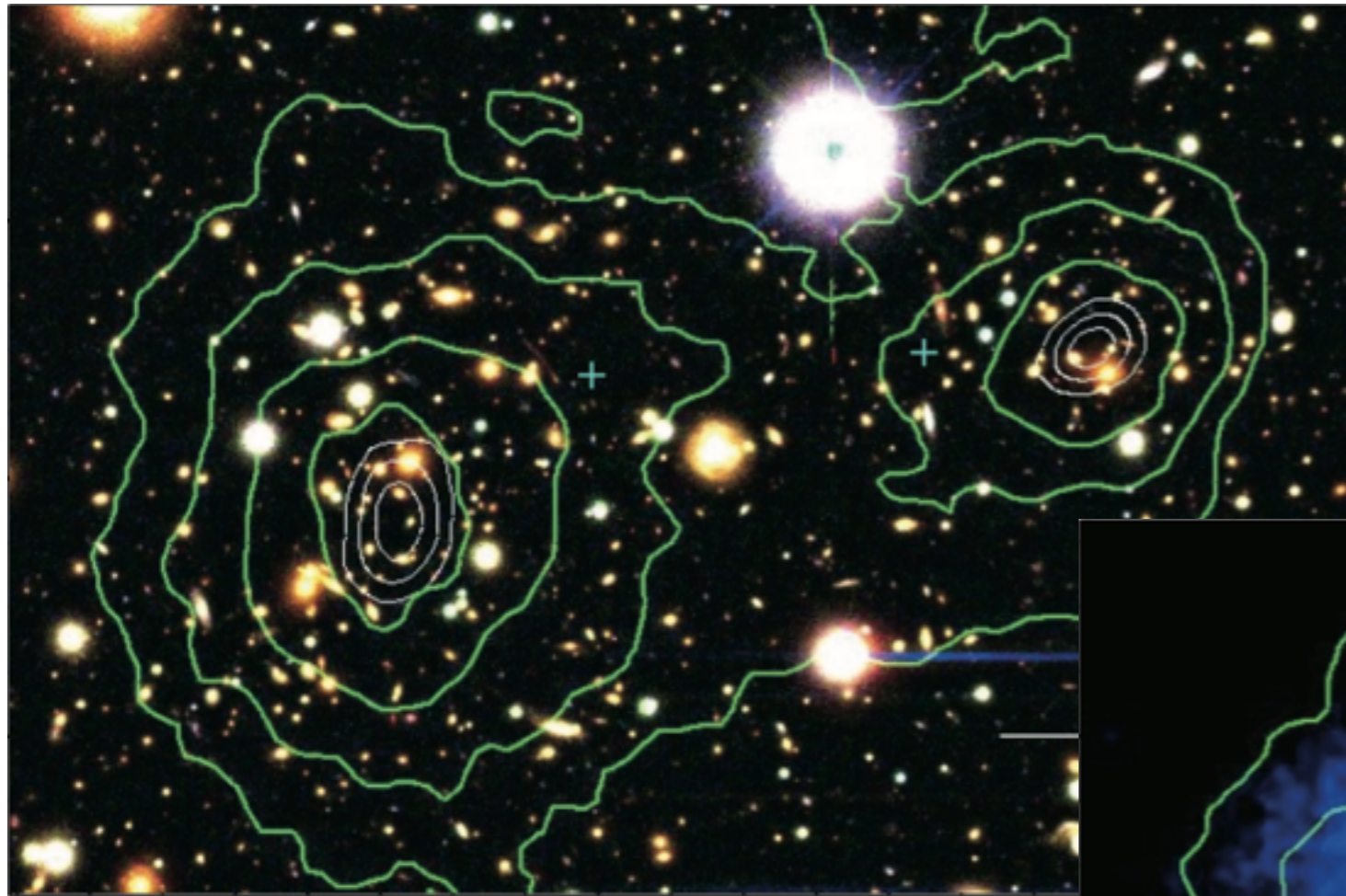
- Bradac+2006 modeled mass distribution combining strong and weak lensing

Average tang



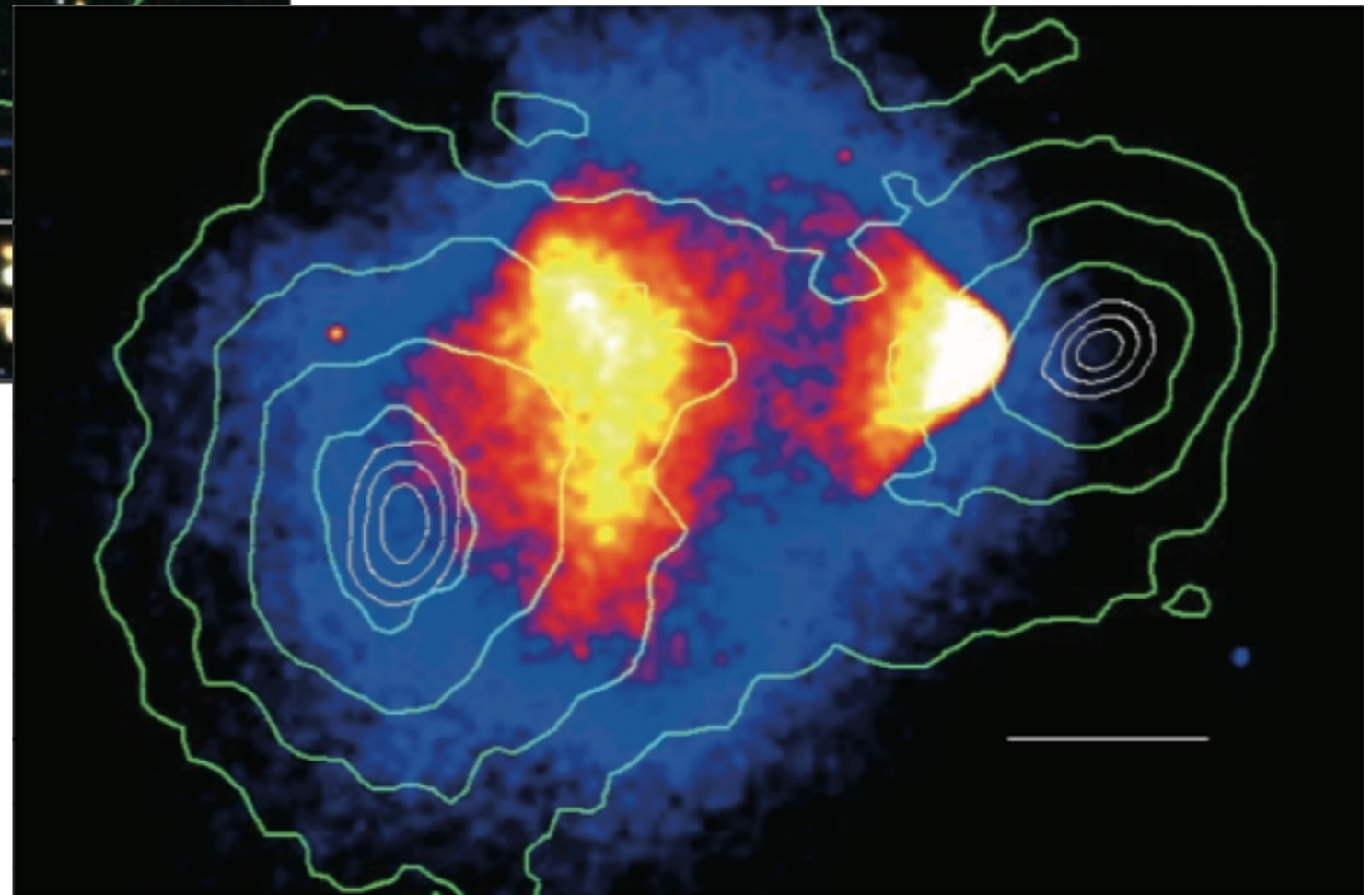
The Bullet Cluster: 1E 0657 558

- Combining the X-ray and lensing info provides a strong proof for DM

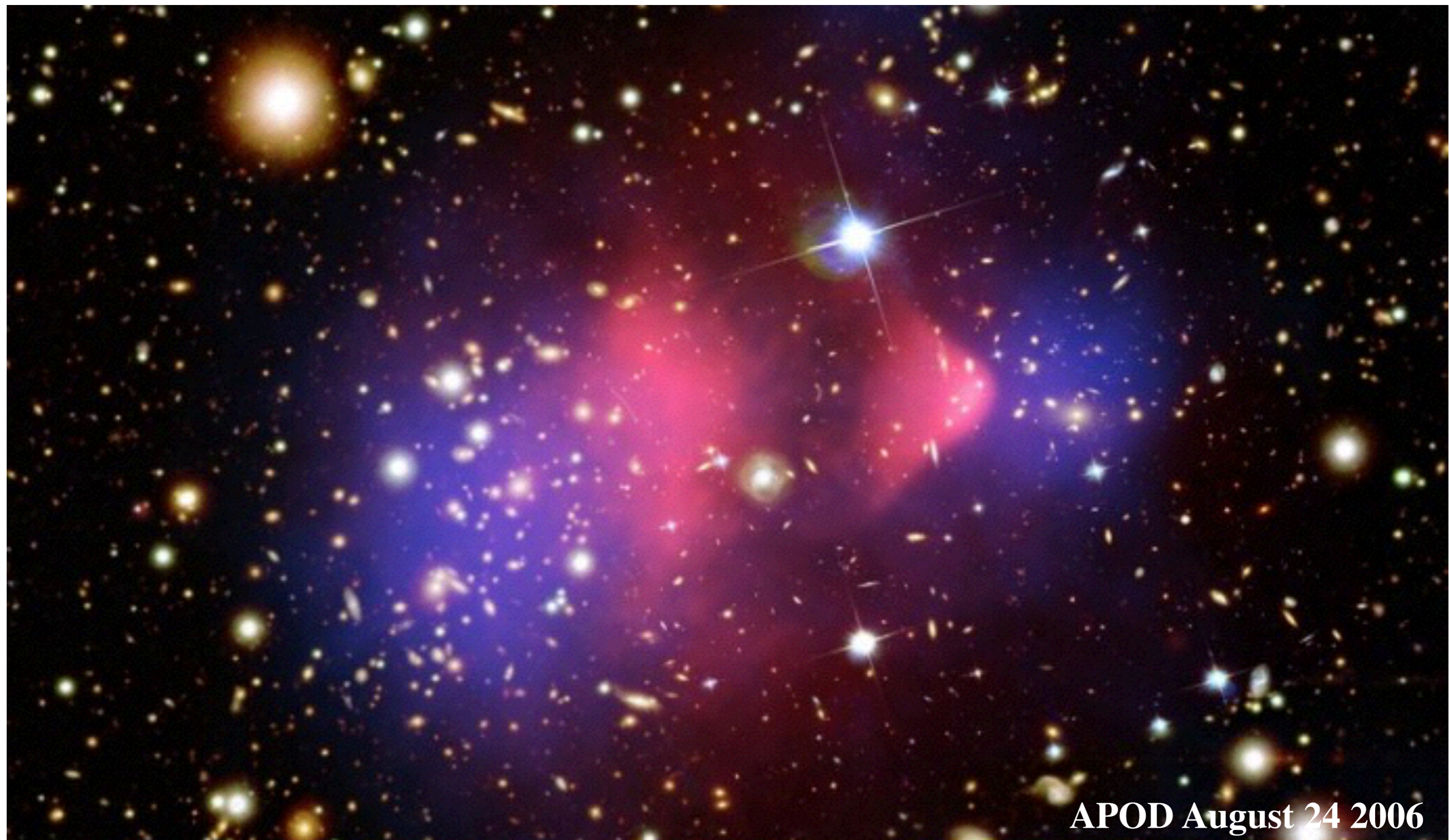


Clowe+(2006)

Clowe+(2006)



The Bullet Cluster: 1E 0657 558



So in summary...

- The 2nd order surface brightness moments are defined as

$$q_{ij} \equiv \int d^2\theta \mathcal{S}^{\text{obs}}(\boldsymbol{\theta}) \theta_i \theta_j$$

- For the ideal spherical galaxy case, they can be used to express the ellipticity of an object as

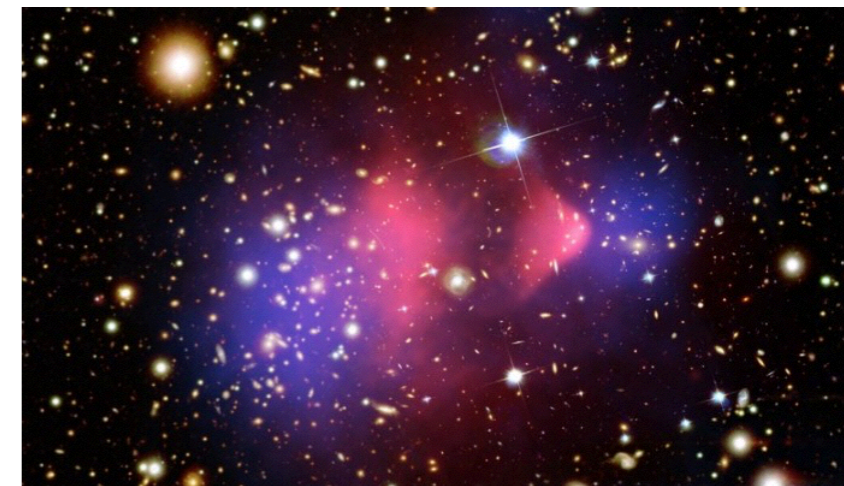
$$\epsilon_i = \frac{2\gamma_i}{1 - \kappa} \left[1 - \frac{\gamma^2}{(1 - \kappa)^2} \right]^{-1}$$

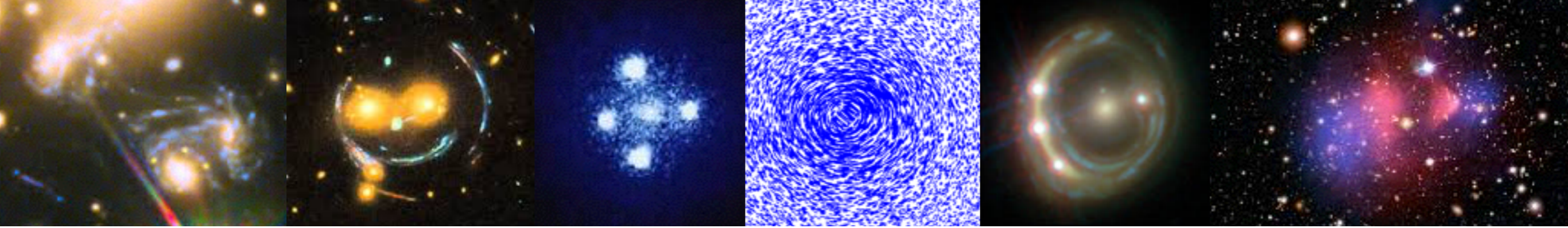
- This depends directly on the gravitational potential through κ and γ
- In the general case also noise, PSF, intrinsic ellipticity (among other things) needs to be accounted for.

$$Q_{ij}^{\text{obs}} \simeq Q_{ij} - \Psi_{lm} \int d^2\theta \frac{\partial \mathcal{S}(\boldsymbol{\theta})}{\partial \theta_l} W(\boldsymbol{\theta}) \theta_i \theta_j \theta_m$$

$$Q'_{ij} = Q_{ij} + P_{lm} Y_{lmij}$$

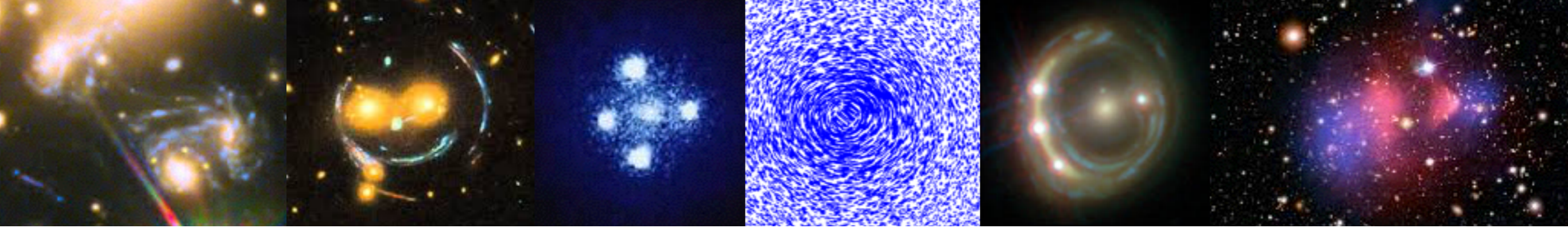
- The bullet cluster provides direct empirical proof of the existence of DM





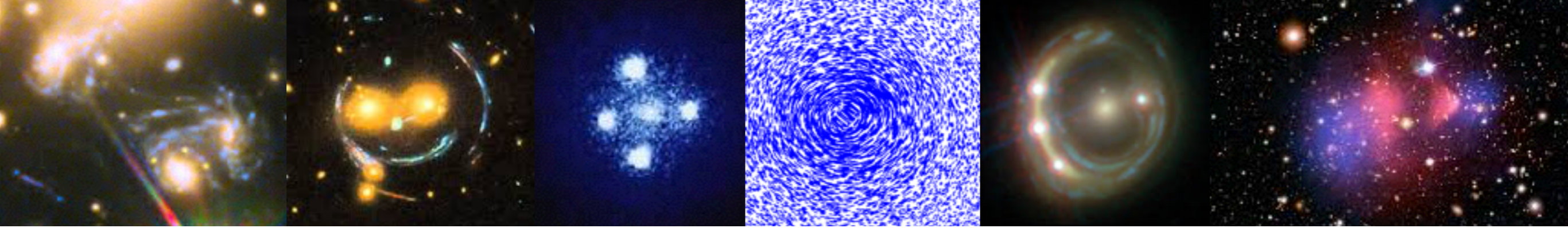
PHY-765 SS18 Gravitational Lensing Week 12

Questions?



PHY-765 SS18 Gravitational Lensing Week 12

Last Week's Worksheet



PHY-765 SS18 Gravitational Lensing Week 12

This Week's Worksheet

Giving “a good talk”

- Assume spending 2 minutes per slide (at least) when estimating total time
- Tell a story, i.e., answer the “whys”, “hows” and “whats” of the work
- Limit the use of colors on slides → No “rainbow” slides
 - Avoid poor color combinations like “blue on black” and “green on red”
- Don’t write text paragraphs unless you are going to read them out loud
 - Otherwise the audience will be reading instead of listening to you
- Use builds to avoid displaying the whole slide at once
 - Otherwise the audience will be reading instead of listening to you
- Use cartoons/sketches to explain concepts instead of just waving you hands
- Don’t have slides where you say “just ignore this and that”.
 - If things should be ignored, just remove them from the slides.
- **Rehearse the talk before giving it!**

Disclaimer: “A good talk” according to K. B. Schmidt... others might (surely will) disagree